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Ultraviolet modified photons and anisotropies in the cosmic microwave background radiation

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Abstract

We discuss a minimal canonical modification of electrodynamics in order to account for ultraviolet Lorentz violating effects. This modification creates a birefringence that rotates the polarization planes from different directions. Such effects might be detectable in the anisotropic polarization of the Cosmic Microwave Background radiation.

The cornerstone of modern cosmology is the cosmological principle, which is based on the notion that spacetime is locally Lorenz invariant. The analysis, however, of problems such as the matter-antimatter asymmetry, the origin of dark matter/energy, or even the nature of the primordial magnetic field, calls for a critical reconsideration of the principles underlying the cosmological standard model [1].

In this direction, several authors have put to the test the validity of Lorentz symmetry [2] in the propagation of light from faraway galaxies [3–5]. The theoretical framework of this analysis has mostly been a Maxwell–Chern–Simons model [3,6], which introduces a parameter with dimensions of energy and would represent a correction to electrodynamics at very large scales and very low energies.

A distinct possibility would be to investigate Lorentz violating effects due to highly energetic processes in light propagation. An excellent candidate to test such a phenomenon would be the study of anisotropies in the Cosmic Microwave Background (CMB) radiation. This is because, even though the mean temperature of CMB is only 2.275 K, it is just a relic of events which happened at the first epochs of our Universe—at the decoupling era or much earlier—where the typical energies were high enough to spur new ultraviolet effects.[1] Furthermore, even a tiny asymmetric effect on the shifting of polarization planes would be amplified due to the very large distances that those photons have traveled around to reach us.

In this Letter, we will explore the reasonable possibility that in the early epochs of our Universe, when it was mainly dominated by radiation, the electromagnetic processes were not necessarily described by standard relativistic theory. In other words, Maxwell theory should be modified in the ultraviolet regime. From this point of view our work would be closer to the one of Myers and Pospelov [9].

One approach to modifying Lorentz symmetry at high energies invokes deformations of the Lorentz group involving an invariant length (of the order of the Planck length) [10]. Such deformations are, in fact, non-linear realizations of the Lorentz group and they can be mapped to standard Lorentz transformations by a non-linear map of the momenta. The single-particle dispersion relations, then, are mapped to the standard ones under this map. These modified symmetries, however, do not possess a non-trivial co-product; that is, there is no way to

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[1] For a recent discussion about this topic in the context of COBE and WMAP see [7,8].

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compose two representations into a new representation of the group, other than the standard one as obtained by the usual addition of momenta via the non-linear momentum map. As such, there are no interacting field theories that realize the modified transformations in a non-trivial way and therefore no physically interesting effects.

We shall consider, instead, modifying the electromagnetic theory by including small Lorentz violating terms in the Lagrangian. The main issue, of course, is what these modifications might be and what are the criteria for selecting and including relevant and reasonable contributions. An effective field theory deriving from an underlying fundamental theory (string theory or other), would involve in principle many possible ultraviolet terms in the effective action, leaving the question for identifying the relevant ones wide open.

In this Letter we take the approach of including a minimal modification to the *canonical structure* of the electromagnetic field theory, amounting to adding a tiny violation of the microcausality principle. This procedure, proposed in [11,12], includes small modifications to the canonical commutators in the Maxwell theory

$$\left[A_i(x), A_j(y)\right] = \epsilon_{ijk}\theta_k \delta^{(3)}(x-y), \tag{1}$$

$$\left[\pi_i(x), \pi_j(y)\right] = \epsilon_{ijk}\gamma_k \delta^{(3)}(x-y), \tag{2}$$

$$\left[A_i(x), \pi_j(y)\right] = \delta_{ij} \delta^{(3)}(x-y), \tag{3}$$

where $i, j, \ldots$ are spatial indices, θ_i and γ_i are two given Lorentz violating vectors which play the role of ultraviolet and infrared energy scales respectively.[2]

If we are interested in ultraviolet effects, we can neglect the infrared scale ($\gamma \sim 0$), and retain the corresponding ultraviolet parameter.

The action that reproduces this modified electrodynamics and is consistent with the commutators (1)–(3) is given by the general form

$$S = \int d^4x\, \mathcal{L} = \int d^4x\, \frac{1}{2}\Omega_{ab}\psi^a\dot{\psi}^b - V(\psi), \tag{4}$$

where ψ^a and $\dot{\psi}^a$ are the coordinates and velocities with $a = 1, 2, \ldots, 2n$, and Ω_{ab} is a constant antisymmetric and regular matrix. Here, V is a potential which modifies the free theory. From this Lagrangian the Poisson structure obtains as

$$\left\{\psi^a, \psi^b\right\} = \left(\Omega^{-1}\right)^{ab}. \tag{5}$$

We take, therefore, a set of fields $\{A_i(x), F_j(x)\}$ with $i, j = 1, 2, 3$, whose Poisson brackets are,

$$\begin{aligned} &\left\{A_i(x), A_j(y)\right\} = \epsilon_{ijk}\theta_k \delta^{(3)}(x-y), \\ &\left\{A_i(x), F_j(y)\right\} = \delta_{ij}\delta^{(3)}(x-y), \\ &\left\{F_i(x), F_j(y)\right\} = 0. \end{aligned} \tag{6}$$

[2] In fact, the $|\vec{\theta}|$ and $|\vec{\gamma}|$ respectively.

In the basis $\psi^a(x) = \{A_1, A_2, A_3, F_1, F_2, F_3\}$, then, Ω_{ab} becomes

$$\Omega_{ab} = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & \theta_3 & -\theta_2 \\ 0 & 1 & 0 & -\theta_3 & 0 & \theta_1 \\ 0 & 0 & 1 & \theta_2 & -\theta_1 & 0 \end{pmatrix} \delta^3(x-y). \tag{7}$$

In terms of these variables the Lagrangian can be written as

$$L = \frac{1}{2}\int d^3x\, d^3y\, \Omega_{ab}(x-y)\psi^a\dot{\psi}^b + V(\psi), \tag{8}$$

$$= \int d^3x \left(F_i\dot{A}_i + \frac{1}{2}\epsilon_{ijk}\theta_k F_i\dot{F}_j\right) - V(A, F). \tag{9}$$

Let us define $F_i = F_{0i} = -F_{i0} = F^{i0} = -F^{0i}$, $F_{ij} = \partial_i A_j - \partial_j A_i = F^{ij}$, and a new auxiliary variable $A^0 = -A_0$. Then we can choose as potential V the following,

$$V(A, F) = \int d^3x \left(-\frac{1}{2}F_{0i}F^{0i} - F^{0i}\partial_i A_0 + \frac{1}{4}F_{ij}F^{ij}\right). \tag{10}$$

This is the minimal potential that regains the standard electrodynamics when the θ parameters vanish. Or equivalently,

$$L = \int d^3x \left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}F^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{2}\epsilon_{ijk}\theta^k F^{0i}\partial_0 F^{0j}\right). \tag{11}$$

The equations of motion as obtained by Hamilton's principle are

$$F_{0i} = \partial_0 A_i - \partial_i A_0 - \epsilon_{ijk}\theta_j\partial_0 F_{0k}, \tag{12}$$

$$F_{ij} = \partial_i A_j - \partial_j A_i, \tag{13}$$

$$\partial_\nu F^{\mu\nu} = J^\mu, \tag{14}$$

where J^μ represents a matter current coupled to A_μ.

By construction, this theory is gauge invariant if $\partial_\mu J^\mu = 0$, in the sense that a transformation of $A_\mu \to A_\mu + \partial_\mu\Delta$ for any arbitrary function Δ, keeps the action and equations of motion invariant. Then, the expression of F's in terms of A's is given by,

$$F_{0i} = \left(\frac{1}{I_3 + \Theta\partial_t}\right)_{ij}(\partial_0 A_j - \partial_j A_0),$$

$$F_{ij} = \partial_i A_j - \partial_j A_i,$$

where I_3 is the 3×3 identity matrix and $\Theta_{ij} = \epsilon_{ijk}\theta_k$. Here it is explicit that in terms of the A_i alone the theory is non-local because of the non-local operator $(I + \Theta\partial_t)^{-1}$ in the equation of F_{0i} in terms of A_i.

Let us define, as usual, the magnetic field,

$$B_i = \frac{1}{2}\epsilon_{ijk}F^{ij},$$

and the electric field,

$$E_i = F^{0i}.$$

This is the electric field that couples to matter, according to the equations of motion. Note, thought, that this is not the usual electric field as defined in terms of the gauge potential fields. We can define another electric field, which we will call “old electric field”, as

$$\tilde{E}_i = -\partial_t A^i - \partial_i A^0 = (\delta_{ij} + \epsilon_{ijk}\theta_k\partial_t)E_j.$$

Then, the equations of motion without matter are,

$$\partial_i E_i = 0, \tag{15}$$

$$\dot{E}_i = (\vec{\nabla} \times \vec{B})_i, \tag{16}$$

$$\dot{B}_i = -\big(\vec{\nabla} \times \vec{\tilde{E}}\big)_i, \tag{17}$$

where the last equation is just the Bianchi identity. This identity can be read in terms of the electric field, i.e.,

$$\dot{B}_i = -(\vec{\nabla} \times \vec{E})_i - \vec{\theta} \cdot \vec{\nabla}\dot{E}_i, \tag{18}$$

where we have made use of the Gauss law. Then, differentiating one of the equations of motion with respect to time, we get,

$$\partial^2 E_i = -\theta_m \epsilon_{ijk}\partial_m\partial_j \dot{E}_k, \tag{19}$$

where $\partial^2 = \partial_t^2 - \vec{\nabla}^2$. Expressing E_i in terms of its Fourier transform,

$$E_i(x) = \int d^3k\, \varepsilon_i(k) e^{i(\omega t - \vec{k}\cdot\vec{x})}$$

with the Gauss law implying that $\vec{k} \cdot \vec{\varepsilon}(k) = 0$, we obtain

$$\big[(\omega^2 - \vec{k}^2)\delta_{ij} - i\omega(\vec{k}\cdot\vec{\theta})k_k\epsilon_{ijk}\big]\varepsilon_j(k) = 0.$$

Diagonalizing this expression, and taking into account the Gauss law, we obtain a dispersion relation with two different modes,

$$\omega^2 - k^2(1 \pm \omega\theta\cos\alpha_{k\theta}) = 0, \tag{20}$$

where $\alpha_{k\theta}$ is the angle between $\vec{k}$ and $\vec{\theta}$, and θ is the length of $\vec{\theta}$. Equivalently,

$$\omega_\pm = k\left[\sqrt{1 + \frac{1}{4}(\vec{k}\cdot\vec{\theta})^2} \pm \frac{1}{2}(\vec{k}\cdot\vec{\theta})\right].$$

This theory, then, presents a birefringence, or Faraday-like rotation effect with polarization planes shifted by an amount proportional to $\Delta k \approx \omega^2\theta\cos\alpha_{k\theta}$. This fact is similar to the one in the model studied in [3–6]. However, in that model a tiny Lorentz symmetry violating parameter affected equally the whole spectrum, while here the effect is increasingly important for higher frequencies. This is also unlike Lorenz violation induced by space non-commutativity, which induces no Faraday-like rotation [13].

Though the analysis of experimental data is beyond the scope of our Letter, we think that it would be very interesting to look for the effects of the above dipole anisotropy in the CMB polarization—above and beyond the dipole anisotropy—due to the relative motion of our galaxy with respect to the CMB rest frame [15]. Other anisotropy effects on the polarization of the CMB have been recently pointed out by Kosowsky et al. [14]. A randomized cosmological magnetic field could produce multipolar anisotropies in the CMB polarization. So, in order to detect a Lorentz violation in the CMB polarization we should take into account anisotropy effects of different origin, such as this one.

The possibility of a tiny dipolar anisotropy at large scale in the propagation of light through the Universe was pointed out by Ralston and Nodland [4] who argued this fact by analyzing data from polarized light coming from far galaxies. Carroll and Field [5] reanalyzed the Ralston and Nodland method using another procedure and suggested that, even though observational data are not complete, the possibility of such anisotropy cannot be totally ruled out.

As we have pointed out in the introduction, in the analyses of the above works the authors used a theoretical model based on a Chern–Simons like coupling as a test for a possible Lorentz symmetry violation. In this work, we have considered another possible scenario in which the dipolar anisotropy arises from short distance effects. Which effect, if any, is backed by observational data is yet to be discovered.

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