Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Physics Letters B 633 (2006) 362–367
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Casimir effect in a six-dimensional vortex scenario

Román Linares *, Hugo A. Morales-Técotl [1], Omar Pedraza

Departamento de Física, Universidad Autónoma Metropolitana Iztapalapa, San Rafael Atlixco 186, CP 09340, México DF, Mexico

Received 21 May 2005; received in revised form 8 November 2005; accepted 21 November 2005

Available online 29 November 2005

Editor: G.F. Giudice

Abstract

Recently Randjbar-Daemi and Shaposhnikov put forward a 4-dimensional effective QED coming from a Nielsen–Olesen vortex solution of the Abelian Higgs model with fermions coupled to gravity in $D = 6$. However, exploring possible physical consequences of such an effective QED was left open. In this Letter we study the corresponding effective Casimir effect. We find that the extra dimensions yield fifth and third inverse powers in the separation between plates for the modified Casimir force which are in conflict with known experiments, thus reducing the phenomenological viability of the model.

1. Introduction

The idea that our observable 4-dimensional universe may be a brane extended in some higher-dimensional space–time has been attracting interest for many years [1–3]. Roughly speaking, there exist two different approaches to implement this idea. One approach is to start with theories that incorporate gravity in a reliable manner such as string theory/M-theory [4,5]. Almost all the known examples of these kind of theories are naturally and consistently formulated in higher dimensions. For instance, it is possible to include chiral fermions by considering intersecting D-branes [6–8]. The second approach follows more phenomenological lines and is often based on simplified field-theoretical models which have recently led to new insights on whether they may help to solve long-standing problems of particle theory such as the hierarchy problem, the cosmological constant problem, etc. [9–16] (see, for instance, the comprehensive reviews [17,18]).

An important problem in the field theory approach is to find natural mechanisms for localization of the different fields to 4-dimensional space–time. There exist many models that can achieve the localization of scalar and fermionic fields, however, the localization of gauge fields is not an easy challenge to tackle [19,20]. Recently, starting from a Higgs model with fermions coupled to gravity in $D = 6$, Randjbar-Daemi and Shaposhnikov [21] constructed an effective quantum electrodynamics in 4-dimensional space–time, with fermionic and gauge functions spread on the transverse direction in a small region in the vicinity of the core of a Nielsen–Olesen vortex. This construction is possible because the vortex solution [22–24], admits gravity localization [23] and contains the massless $U(1)$ gauge field, which is a mixture of a graviton fluctuation and the original $U(1)$ gauge field fluctuation forming the Nielsen–Olesen vortex.

Since the 4-dimensional effective QED owns many nontrivial properties, despite all the theoretical interest it is natural to ask ourselves how far we can go with this model and compute its consequences in low/high energy physics. In doing this there exists the additional possibility of saying something about the potential detectability of extra dimensions by measuring effects which for this particular model have not been discussed to the best of our knowledge. The aim of this Letter is to analyze the Casimir effect between parallel plates in the context of the effective QED of [21].

The standard Casimir effect between parallel, uncharged, perfectly conducting plates is understood on the basis of the

* Corresponding author.
E-mail addresses: lirr@xanum.uam.mx (R. Linares), hugo@xanum.uam.mx (H.A. Morales-Técotl), omp@xanum.uam.mx (O. Pedraza).
[1] Associate member of the Abdus Salam ICTP, Trieste, Italy.

0370-2693
doi:10.1016/j.physletb.2005.11.057

ordinary 4-dimensional QED. For flat plates separated by a distance l, the force per unit area A is given by $F(l)/A = -\frac{\pi^2}{240}\frac{\hbar c}{l^4}$. This relationship is derived considering the electromagnetic mode structure between the two parallel plates, as compared to the mode structure when the plates are infinitely far apart, and by assigning a zero-point energy of $\hbar\omega/2$ to each electromagnetic mode (photon) [25]. The change in the total energy density between the plates, as compared to the free space, as a function of the separation l, leads to the force of attraction. The only fundamental constants that enter into the expression of the force are $\hbar$ and c. The electron charge e is absent, implying that the electromagnetic field is not coupled to matter. The role of c is to convert the electromagnetic mode wavelength, as determined by l, to a frequency, while $\hbar$ converts the frequency to an energy. The Casimir effect has also been obtained for other fields and other geometries of the bounding surfaces which may be described by real material media, with electromagnetic properties [26].

The Casimir effect, on the other hand, has received great deal of attention within theories and models with extra dimensions. For example, it has been discussed in the context of string theory [27–30]. In the Randall–Sundrum model, the Casimir effect has been considered to stabilize the radion [31–35] as well as within the inflationary brane world universe models [36]. More recently the effect was analyzed in the presence of compactified universal extra dimensions [37]. In all these cases the boundaries in the extra dimensions are associated to the topology of space.

In general the Casimir effect may be defined as the stress on the bounding surface when a quantum field is confined to a finite volume of space. In any case, the boundaries restrict the modes of the quantum field giving rise to a force which can be either attractive or repulsive, depending on the model, the field and the space–time dimension.

In this Letter we start with the 4-dimensional effective QED of Randjbar-Daemi and Shaposhnikov (Section 2) in order to determine the dispersion relations of the electromagnetic modes (Section 3). This is done near the core of the vortex scenario that is meant to represent our world. Next we proceed with the standard approach of analysis of the Casimir effect. Namely we add up the electromagnetic mode contributions to the energy between two parallel conducting plates (Section 4). Finally we discuss our results in Section 5.

2. $D = 4$ effective QED

The starting point in our analysis is the 4-dimensional effective QED of Randjbar-Daemi and Shaposhnikov [21]. This theory emerges from considering a Nielsen–Olesen vortex-type solution of the Abelian Higgs model with fermions coupled to gravity in $D = 6$. The various field configurations of the solution are [23]

$$\begin{aligned} ds^2 &= e^{A(r)}\eta_{\mu\nu}\,dx^\mu\,dx^\nu + dr^2 + e^{B(r)}a^2\,d\theta^2, \\ \Phi &= f(r)e^{in\theta}, \qquad aeA_\theta = \big(P(r)-n\big)\,d\theta, \end{aligned} \tag{2.1}$$

where $\eta_{\mu\nu}$ is the $D = 4$ flat metric, e is the 6-dimensional gauge coupling and a is the radius of S^1 covered by the θ coordinate. To avoid confusion below notice the difference between e and e. The boundary conditions that $f(r)$ and $P(r)$ must satisfy are

$$\begin{aligned} f(0) &= 0, \qquad f(\infty) = f_0, \\ P(0) &= n, \qquad P(\infty) = 0. \end{aligned} \tag{2.2}$$

On the other hand, there are solutions with different boundary conditions for the metrical functions $A(r)$ and $B(r)$ [19]. Among all of them, the one that localize fields of spin 0, 1/2 and 1, near the core of the vortex satisfy the boundary conditions

$$\begin{aligned} &A(0) = 1, \qquad B(r\to 0) = 2\ln\frac{r}{a}, \\ &A(r\to\infty) = B(r\to\infty) = -2cr, \quad c > 0, \end{aligned} \tag{2.3}$$

where the parameters a and c are combinations of the 6-dimensional gravitational constant κ, the cosmological constant and of the parameters of the Abelian Higgs model [19]. In this case as $r \to 0$ the flat space geometry is recovered whereas for $r \to \infty$ the metric becomes AdS.

The effective QED action in this background results from a specific mixture of the fluctuation of the 6-dimensional vector potential and the $\theta\mu$ component of the metric [24]. Its explicit form is

$$\begin{aligned} S(W) = -\frac{\pi}{ae^2}\int_0^\infty dr\,F(r) \\ \times \int d^4x\,\big[(\partial_\mu W_\nu)^2 + e^{-A}\partial_r W_\mu \partial_r W^\mu\big], \end{aligned} \tag{2.4}$$

where

$$F(r) = e^{\frac{B(r)}{2}}\left(P^2(r) + \frac{a^2e^2}{\kappa^2}e^{B(r)}\right). \tag{2.5}$$

Notice that the quotient $\ell \equiv \kappa/e$ has dimension of length. The equations of motion for the gauge fields are

$$F\partial_\mu\partial^\mu W^\nu + \partial_r\big(Fe^{-A}\partial_r W^\nu\big) = 0. \tag{2.6}$$

3. Dispersion relations

As discussed in [24], a 4-dimensional effective low energy theory can arise if two conditions are satisfied:

(i) The spectrum contains normalizable zero (or small mass) modes of graviton, gauge, scalar and fermion fields, with wave functions of the type $e^{ip_\mu x^\mu}\psi(y^m)$; y^m here represent the extra dimensional coordinates.

(ii) The effects of higher modes should be experimentally unobservable at low enough energies, i.e., there should be a mass gap between the zero modes and excited states. Another possibility is that extra, unwanted modes may be light but interact very weakly with the zero modes.

In this section we shall analyze the implications of the first condition for the zero modes of the gauge fields, i.e., we shall consider a wave function of the form $W^\mu(x,r) = R^\mu(r)\mathrm{e}^{ip_\nu x^\nu}$, where p_μ is the 4-dimensional wave vector. With this form of the wave function and proposing the change of variable $R = H(F\mathrm{e}^{-A})^{-1/2}$, the equation of motion for each component of W, Eq. (2.6), is rewritten as a Schrödinger-like equation

$$\left(-\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + V(r)\right)H(r) = -\frac{p^2\mathrm{e}^A}{2}H(r), \tag{3.1}$$

where the potential is given by

$$V(r) = \frac{1}{4}\frac{\mathrm{d}^2}{\mathrm{d}r^2}\ln\left(F\mathrm{e}^{-A}\right) + \frac{1}{8}\left(\frac{\mathrm{d}}{\mathrm{d}r}\ln\left(F\mathrm{e}^{-A}\right)\right)^2. \tag{3.2}$$

Because the localization of the gauge fields is given near the core of the vortex, we are interested in solutions where $r \to 0$. In this limit the potential is

$$V(r \to 0) \approx -\frac{1}{8r^2} + \frac{1}{n^2\ell^2} \tag{3.3}$$

and Eq. (3.1) becomes

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{4r^2} + k_r^2\right)H(r) = 0, \quad \text{where } k_r^2 = -\left(p^2\mathrm{e} + \frac{2}{n^2\ell^2}\right). \tag{3.4}$$

Hence, in order to find the explicit functional form of the gauge fields near the core of the vortex it is necessary to solve the 1-dimensional radial Schrödinger equation with an attractive potential proportional to $1/r^2$. The solutions to Eq. (3.4) have been already discussed [38] and it has been shown that their properties depend strongly on the value λ—the coefficient of the term $1/r^2$. When $\lambda < 1/4$ and the boundary condition is integrability—not finiteness—all negative energies are allowed [39]. For $\lambda > 1/4$, the requirement that the state functions for bound states be a mutually orthogonal set imposes a quantization of energy which does not uniquely fix the levels but the levels relative to each other [38].

It is remarkable that the value of λ in (3.4) takes the critical value $1/4$. The solution to the differential equation in this case is given by

$$H = c_1 r h^{(1)}_{-\frac{1}{2}}(irk_r) + c_2 r h^{(2)}_{-\frac{1}{2}}(irk_r), \tag{3.5}$$

where h^1 and $h^{(2)}$ are the spherical Hankel functions. These functions behave in the limit $r \to 0$ as

$$h^{(1)}_{-\frac{1}{2}}(r) \approx \sqrt{\frac{\pi}{2r}}\frac{2i}{\pi}\ln r, \qquad h^{(2)}_{-\frac{1}{2}}(r) \approx -\sqrt{\frac{\pi}{2r}}\frac{2i}{\pi}\ln r. \tag{3.6}$$

Thus if the boundary condition is integrability [39]

$$\int \mathrm{d}r\, R^2 \sim \int r(\ln rk_r)^2 < \infty, \tag{3.7}$$

all negative energies are allowed and therefore the dispersion relation is

$$\frac{\omega^2}{c^2} = \vec{k}^2 + \frac{2}{\mathrm{e}}k_r^2 + \frac{2}{\mathrm{e}}\frac{1}{n^2\ell^2}, \tag{3.8}$$

where $\vec{k}$ is the 3-dimensional wave vector. Notice that the dependence of the extra dimensions in the dispersion relations comes into two different ways. There is a continuum contribution k_r that comes from the radial extra dimension and there is one discrete contribution that goes like n^{-2} coming from the vortex number. This last contribution is of a different type to the one that emerges from a Kaluza–Klein compact extra dimension which goes like n^2 [37].

4. The Casimir effect

Once we have computed the dispersion relations we evaluate the Casimir force between two parallel plates in the $D = 4$ effective QED. Because of the presence of the plates, we impose the standard Dirichlet boundary condition on the wave vector in the direction restricted by the plates: $k_N = \pi N/l$, where l is the distance between the plates. The Casimir energy between the plates is obtained by summing up the zero-point energy per unit area, where the frequency of the vacuum fluctuations is, according to (3.8),

$$\omega_{k_\perp,k_r,N,n} = c\sqrt{k_\perp^2 + \frac{2}{\mathrm{e}}k_r^2 + \frac{\pi^2N^2}{l^2} + \frac{2}{\mathrm{e}}\frac{1}{n^2\ell^2}}, \tag{4.1}$$

with $k_\perp = \sqrt{k_1^2 + k_2^2}$. k_1 and k_2 are the wave vector components in the direction of the unbounded space coordinates along the plates. Each of these modes contributes an energy $\hbar\omega/2$. Therefore the energy between plates reads

$$E_{\text{plates}} = \frac{\hbar L^2 ap}{2}\int\frac{\mathrm{d}^2k_\perp\,\mathrm{d}k_r}{(2\pi)^3}\sum_{n=1,N=1}^{\infty}\omega_{k_\perp,k_r,N,n}, \tag{4.2}$$

where L^2 is the area of the plates and the parameter a, measuring the size of the vortex'es core, appears associated to the integration in k_r. The factor p indicates the possible polarization of the photon. In our case $p = 4$.

There are several ways to extract a finite value from the above divergent sum. We shall use the one that invokes dimensional regularization. To do so we let the transverse dimension be d, which we will subsequently treat as a continuous complex variable following [26]. Let us start with the expression

$$I_1(d) = \frac{1}{2}\int\frac{\mathrm{d}^dk}{(2\pi)^d}\sum_{n=1,N=1}^{\infty}\sqrt{k_d^2 + \frac{\pi^2N^2}{l^2} + \frac{2}{\mathrm{e}}\frac{1}{n^2\ell^2}}, \tag{4.3}$$

which becomes in the limit $d \to 3$ the one in (4.2), namely

$$I_1(d=3) \equiv \frac{1}{2}\int\frac{\mathrm{d}^2k_\perp\,\mathrm{d}k_r}{(2\pi)^3}\sum_{n=1,N=1}^{\infty}\sqrt{k_\perp^2 + k_r^2 + \frac{\pi^2N^2}{l^2} + \frac{2}{\mathrm{e}}\frac{1}{n^2\ell^2}}. \tag{4.4}$$

Using the Euler representation for the gamma function

$$\Gamma(z) = g^z\int_0^\infty \mathrm{e}^{-gt}t^{z-1}\,\mathrm{d}t, \tag{4.5}$$

the integral (4.3) can be rewritten employing the Schwinger proper-time representation for the square root as

$$I_1(d)=\frac{1}{2}\frac{1}{\Gamma(-\frac{1}{2})}\sum_{n=1,N=1}\int\frac{d^dk}{(2\pi)^d}\int_0^\infty\frac{dt}{t}t^{-1/2}\times e^{-t(k_d^2+\frac{\pi^2N^2}{l^2}+\frac{2}{e}\frac{1}{\ell^2n^2})}. \tag{4.6}$$

Performing the Gaussian integral first and using (4.5) again we have

$$I_1(d)=\frac{1}{2}\frac{1}{\Gamma(-\frac{1}{2})}\frac{1}{(4\pi)^{d/2}}\Gamma\left(-\frac{d+1}{2}\right)\times\sum_{n=1}^\infty\sum_{N=1}^\infty\left(\frac{\pi^2N^2}{l^2}+\frac{2}{e}\frac{1}{\ell^2n^2}\right)^{(d+1)/2}. \tag{4.7}$$

The double sum in (4.7) is better handled by factorizing $(\frac{\pi}{l})^{d+1}$ and using the Epstein function [40,41]

$$E_1^{M^2}(z,1)=\sum_{N=1}^\infty\left(N^2+M^2\right)^{-z},$$
$$M^2=\frac{2}{e}\frac{l^2}{\pi^2\ell^2n^2},\qquad z=\frac{d+1}{2}. \tag{4.8}$$

This expression is not well defined for $\Re(z)>1/2$, however, it can be analytically continued into a meromorphic function in the whole complex plane, namely

$$E_1^{M^2}(z,1)=\frac{1}{2M^{2z}}+\pi^{\frac{1}{2}}\frac{1}{2M^{2z-1}\Gamma(z)}\left[\Gamma\left(z-\frac{1}{2}\right)+4\sum_{m=1}^\infty\frac{1}{(\pi Mm)^{\frac{1}{2}-z}}K_{\frac{1}{2}-z}(2\pi Mm)\right], \tag{4.9}$$

where $K_\nu(z)$ is the modified Bessel function of second type. Thus using the Epstein function in (4.7) leads to

$$I_1(d)=\frac{1}{2}\frac{1}{\Gamma(-\frac{1}{2})}\frac{1}{(4\pi)^{\frac{d}{2}}}\left[\frac{1}{2}\Gamma\left(-\frac{d+1}{2}\right)\left(\frac{2}{e}\frac{1}{\ell^2}\right)^{(d+1)/2}\times\zeta(d+1)+\frac{l}{2\sqrt{\pi}}\Gamma\left(-\frac{d+2}{2}\right)\left(\frac{2}{e}\frac{1}{\ell^2}\right)^{(d+2)/2}\zeta(d+2)+\frac{2}{\sqrt{\pi}}\left(\sqrt{\frac{2}{e}}\frac{1}{\ell}\right)^{(d+2)/2}\frac{1}{l^{\frac{d}{2}}}\times\sum_{n,m=1}^\infty\left(\frac{1}{mn}\right)^{(d+2)/2}K_{\frac{d+2}{2}}\left(\sqrt{\frac{2}{e}}\frac{2l}{\ell}\frac{m}{n}\right)\right]. \tag{4.10}$$

Notice the first term in square brackets in (4.10) is independent of l and hence it can be interpreted as a constant energy shift upon substitution in (4.2) [41]. Obviously it will neither yield any contribution to the Casimir force. Hence from now on it will be discarded. The energy between plates takes the explicit form

$$E_{\text{plates}}=\hbar cL^2ap\sqrt{\frac{e}{2}}I_1'(d\to 3), \tag{4.11}$$

with the prime meaning we have dropped the first term in brackets in I_1, Eq. (4.10). Next we compute the vacuum energy without the plates. Appealing again to (4.3) we have that such vacuum energy

$$E_0=\hbar cL^2al\sqrt{\frac{e}{2}}p\frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\sum_{n=1}^\infty\sqrt{k_\perp^2+k_r^2+k_z^2+\frac{2}{e}\frac{1}{n^2\ell^2}} \tag{4.12}$$

becomes

$$E_0=\left[\hbar cL^2al\sqrt{\frac{e}{2}}p\frac{1}{2}\frac{1}{\Gamma(-\frac{1}{2})(4\pi)^{\frac{d}{2}}}\left(\frac{2}{e\ell^2}\right)^{(d+1)/2}\times\Gamma\left(-\frac{d+1}{2}\right)\zeta(d+1)\right]_{d\to 4}. \tag{4.13}$$

Finally, the exact Casimir energy per unit area in the vortex scenario reads

$$\mathcal{E}_{\text{vortex}}=\frac{E_{\text{plates}}-E_0}{L^2}=\mathcal{E}_{\text{Casimir}}f(l,a,\ell),$$
$$\mathcal{E}_{\text{Casimir}}=-\frac{\hbar c\pi^2}{720l^3},$$
$$f(l,a,\ell)=\frac{45p(\sqrt{\frac{2}{e}})^3}{\pi^{\frac{7}{2}}\Gamma(-\frac{1}{2})}\left(\frac{a}{\ell}\right)\left(\frac{l}{\ell}\right)^{3/2}\times\sum_{n,m=1}^\infty\left(\frac{1}{mn}\right)^{5/2}K_{\frac{5}{2}}\left(\sqrt{\frac{2}{e}}\frac{2l}{\ell}\frac{m}{n}\right). \tag{4.14}$$

Here we recognize $\mathcal{E}_{\text{Casimir}}$ as the standard four-dimensional Casimir energy between parallel plates. Moreover, the correction within the vortex scenario is encoded in the function $f(l,a,\ell)$ in the form of a factor rather than an additive term. We should also stress that to arrive to Eq. (4.14) there occurs a cancellation between the second term in (4.10) with the vacuum contribution with no plates, Eq. (4.13). To compare more neatly $\mathcal{E}_{\text{vortex}}$ with the standard result it is easier to get an approximate form of it for $l/\ell\ll 1$ in the argument of the modified Bessel function $K_{5/2}$ [39]. This produces, to leading order in l/ℓ,

$$\mathcal{E}_{\text{vortex}}\approx-\alpha\frac{a}{l^4}+\beta\frac{a}{\ell^2l^2},$$
$$\alpha=\frac{3\hbar c\zeta(5)}{2(4\pi)^{7/2}}\Gamma\left(\frac{5}{2}\right)\sqrt{\frac{2}{e}},$$
$$\beta=\frac{\hbar c\zeta(2)}{32\pi^{7/2}}\zeta\left(\frac{3}{2}\right)\Gamma\left(\frac{5}{2}\right)\left(\frac{2}{e}\right)^{3/2}. \tag{4.15}$$

As for the Casimir force we obtain then

$$F_{\text{vortex}}=-\frac{\partial\mathcal{E}_{\text{vortex}}}{\partial l}\approx-4\alpha\frac{a}{l^5}+2\beta\frac{a}{\ell^2l^3}. \tag{4.16}$$

Experimentally the Casimir force is difficult to measure because parallelism cannot be obtained easily so it is preferable to replace one of the plates by a metal sphere of radius R where

$R \gg l$. For such geometry the Casimir force is modified to

$$F_{\text{sphere}} = 2\pi R L^2 \mathcal{E}_{\text{Casimir}}. \tag{4.17}$$

The force between a metallic sphere of diameter 196 μm and a flat plate is measured using an atomic force microscope for separations l ranging from 0.1 to 0.9 μm [42]. In this case the experimental uncertainty for the Casimir force is 1.6 pN. Due to the factor correction in Eq. (4.14) giving rise to an inverse power in the separation between plates of l^{-5} in the effective Casimir force, Eq. (4.16), it is not possible to reconcile it with the experimental results even within the error bar [42–44] (see [45] for a review of the current experimental situation).

5. Discussion

In this Letter we have obtained the Casimir effect corresponding to the effective QED of Randjbar-Daemi and Shaposhnikov [21]. The latter emerges from a 6-dimensional Abelian Higgs model coupled to gravity in a Nielsen–Olesen vortex background with fermions. The effective 4-dimensional gauge field is a mixture of the original 6-dimensional metric and the vector potential.

We determined the contribution of the extra dimensions to the dispersion relations of the electromagnetic modes near the core of the vortex, our world, and we found two types of contributions, Eq. (3.8): a continuum one, associated with the radial extra dimension and a discrete one corresponding to a vortex number. This behaves as n^{-2} just as in the Casimir force for compact noncommutative extra dimensions [46]. As a result we get an effective Casimir energy, Eq. (4.14), which differs with respect to the standard one by a multiplicative factor rather than an additive term. This correction depends on both parameters of the vortex scenario, namely the size of the core a and the coupling constants length ℓ. In the approximation $l/\ell \ll 1$ the effective force, Eq. (4.16), contains both an attractive and a repulsive contributions with inverse powers of the separation between plates l^{-5} and l^{-3}, respectively. Demanding agreement of this force with the experiment to set bounds for the parameters of the effective QED does not work due to the fact that the correction is multiplicative yielding a different power in l with respect to the standard case. This limits the phenomenological implications of the effective QED here considered.

The appearance of the extra dimensional correction as a multiplicative factor that depends on the separation between plates seems to be a generic feature of noncompact extra dimensions. Further studies in this direction for different models is in progress and will be reported elsewhere.

Acknowledgements

We are grateful to Kei-ichi Maeda for useful discussions and Jorge Alfaro and Kim Milton for enlightening observations. This work was partially supported by Mexico's National Council of Science and Technology (CONACyT), under grants CONACyT-40745-F, CONACyT-NSF-E-120-0837 and SEP-2004-C01-47597. The work of O.P. was supported by CONACyT Scholarship number 162767. He would also like to thank the Young Collaborator Programme of Abdus Salam ICTP Trieste, Italy, for supporting a visit where part of this work was developed.

References

[1] K. Akama, An early proposal of 'brane world', in: Lecture Notes in Physics, vol. 176, 1982, p. 267, hep-th/0001113.
[2] V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B 125 (1983) 136.
[3] M. Visser, Phys. Lett. B 159 (1985) 22, hep-th/9910093.
[4] P. Horava, E. Witten, Nucl. Phys. B 475 (1996) 94, hep-th/9603142.
[5] P. Horava, E. Witten, Nucl. Phys. B 460 (1996) 506, hep-th/9510209.
[6] D. Cremades, L.E. Ibanez, F. Marchesano, Nucl. Phys. B 643 (2002) 93, hep-th/0205074.
[7] C. Kokorelis, Nucl. Phys. B 677 (2004) 115, hep-th/0207234.
[8] R. Blumenhagen, M. Cvetic, P. Langacker, G. Shiu, hep-th/0502005.
[9] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B 429 (1998) 263, hep-ph/9803315.
[10] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B 436 (1998) 257, hep-ph/9804398.
[11] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Rev. D 59 (1999) 086004, hep-ph/9807344.
[12] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370, hep-ph/9905221.
[13] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690, hep-th/9906064.
[14] I. Brevik, K.A. Milton, S. Nojiri, S.D. Odintsov, Nucl. Phys. B 599 (2001) 305, hep-th/0010205.
[15] J.D. Barrow, K.-I. Maeda, Nucl. Phys. B 341 (1990) 294.
[16] E. Elizalde, S. Nojiri, S.D. Odintsov, Phys. Rev. D 70 (2004) 043539, hep-th/0405034.
[17] A. Perez-Lorenzana, hep-ph/0503177.
[18] F. Feruglio, Eur. Phys. J. C 33 (2004) s114, hep-ph/0401033.
[19] T. Gherghetta, M.E. Shaposhnikov, Phys. Rev. Lett. 85 (2000) 240, hep-th/0004014.
[20] S.L. Dubovsky, V.A. Rubakov, P.G. Tinyakov, JHEP 0008 (2000) 041, hep-ph/0007179.
[21] S. Randjbar-Daemi, M. Shaposhnikov, JHEP 0304 (2003) 016, hep-th/0303247.
[22] M. Giovannini, Phys. Rev. D 66 (2002) 044016, hep-th/0205139.
[23] M. Giovannini, H. Meyer, M.E. Shaposhnikov, Nucl. Phys. B 619 (2001) 615, hep-th/0104118.
[24] S. Randjbar-Daemi, M. Shaposhnikov, Nucl. Phys. B 645 (2002) 188, hep-th/0206016.
[25] H.B.G. Casimir, Kon. Ned. Akad. Wetensch. Proc. 51 (1948) 793.
[26] K.A. Milton, The Casimir Effect: Physical Manifestations of Zero-Point Energy, World Scientific, River Edge, 2001.
[27] M. Fabinger, P. Horava, Nucl. Phys. B 580 (2000) 243, hep-th/0002073.
[28] H. Gies, K. Langfeld, L. Moyaerts, JHEP 0306 (2003) 018, hep-th/0303264.
[29] I. Brevik, A.A. Bytsenko, hep-th/0002064.
[30] L. Hadasz, G. Lambiase, V.V. Nesterenko, Phys. Rev. D 62 (2000) 025011, hep-th/9911248.
[31] E. Elizalde, S. Nojiri, S.D. Odintsov, S. Ogushi, Phys. Rev. D 67 (2003) 063515, hep-th/0209242.
[32] J. Garriga, A. Pomarol, Phys. Lett. B 560 (2003) 91, hep-th/0212227.
[33] O. Pujolas, Int. J. Theor. Phys. 40 (2001) 2131, hep-th/0103193.
[34] A. Flachi, D.J. Toms, Nucl. Phys. B 610 (2001) 144, hep-th/0103077.
[35] W.D. Goldberger, I.Z. Rothstein, Phys. Lett. B 491 (2000) 339, hep-th/0007065.
[36] S. Nojiri, S.D. Odintsov, S. Zerbini, Class. Quantum Grav. 17 (2000) 4855, hep-th/0006115.
[37] K. Poppenhaeger, S. Hossenfelder, S. Hofmann, M. Bleicher, Phys. Lett. B 582 (2004) 1, hep-th/0309066.
[38] K.M. Case, Phys. Rev. 80 (1950) 797.
[39] P.M. Morse, H. Feshbach, in: Methods of Theoretical Physics, McGraw–Hill, New York, 1953, p. 1665.

[40] A.C. Aguiar Pinto, T.M. Britto, R. Bunchaft, F. Pascoal, F.S.S. da Rosa, Braz. J. Phys. 33 (2003) 860.
[41] J. Ambjorn, S. Wolfram, Ann. Phys. 147 (1983) 1.
[42] U. Mohideen, A. Roy, Phys. Rev. Lett. 81 (1998) 4549, physics/9805038.
[43] R.S. Decca, et al., Ann. Phys. 318 (2005) 37, quant-ph/0503105.
[44] G.L. Klimchitskaya, et al., Int. J. Mod. Phys. A 20 (2005) 2205, quant-ph/0506120.
[45] S.K. Lamoreaux, Rep. Prog. Phys. 68 (2005) 201.
[46] S.-K. Nam, JHEP 0010 (2000) 044, hep-th/0008083.