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# Aerodynamic design optimization of helicopter rotor blades including airfoil shape for hover performance

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**Abstract** This study proposes a process to obtain an optimal helicopter rotor blade shape for aerodynamic performance in hover flight. A new geometry representation algorithm which uses the class function/shape function transformation (CST) is employed to generate airfoil coordinates. With this approach, airfoil shape is considered in terms of design variables. The optimization process is constructed by integrating several programs developed by author. The design variables include twist, taper ratio, point of taper initiation, blade root chord, and coefficients of the airfoil distribution function. Aerodynamic constraints consist of limits on power available in hover and forward flight. The trim condition must be attainable. This paper considers rotor blade configuration for the hover flight condition only, so that the required power in hover is chosen as the objective function of the optimization problem. Sensitivity analysis of each design variable shows that airfoil shape has an important role in rotor performance. The optimum rotor blade reduces the required hover power by 7.4% and increases the figure of merit by 6.5%, which is a good improvement for rotor blade design.

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## 1. Introduction

There are two common approaches to blade aerodynamic performance design. First, most researchers now focus on

blade shape design to optimize the aerodynamic performance of rotor blades by selecting the point of taper initiation, root chord, taper ratio, and maximum twist which minimize hover power without degrading forward flight performance.<sup>1</sup> This approach usually deals with integration of several programs to build an optimization process. Second, some works have tried to solve this problem using computational fluid dynamics (CFD) methods. These CFD methods are reasonable for the hover case but very time-consuming. Moreover, the application of the CFD method to the flow field passing the blade in forward flight is very complex. Therefore, the CFD method is not suitable for the preliminary design phase because of the need for quick estimation. With the aim of allowing quick estimation in the preliminary design phase, this study follows the

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first approach with advanced improvements. In this study, a new geometry representation algorithm which uses the class function/shape function transformation (CST) method is applied to considering airfoil shape. The advantages of this CST method are high accuracy and use of few variables in geometry representation.<sup>2</sup> Therefore, this work deals with the same problem of blade aerodynamic performance design as mentioned above while some additional design variables are included through consideration of airfoil shape.

Satisfactory aerodynamic performance design has been defined by the following requirements which must be met for any flight condition: the required power must be less than the power available and the rotor blade must be trimmed.<sup>1</sup>

The design process is represented in Fig. 1. This process also includes a sizing module. After setting the size of the helicopter, the helicopter rotor blade shape optimization process is performed as the next step of the design process. Following this process, a set of initial values for design variables is chosen from the sizing module. The airfoil baseline, which is airfoil NACA0012, was chosen for the first step of the design process. Then, blade shape variables such as chord distribution, twist distribution, and airfoil point coordinates are generated. The required power for hover is computed by the program Konkuk Helicopter Design Program (KHDP), and the trim condition is checked. Airfoil analysis is performed by the program 2KFoil to generate airfoil aerodynamic characteristics in C81 format. Some others additional codes to generate airfoil coordinates, chord distribution, and twist distribution are implemented in order to build a full framework for the optimization process in ModelCenter software. ModelCenter is a powerful tool for automating and integrating design codes. Once a model has been constructed, trade studies such as parametric studies, optimization studies, and design of experiment (DOE) studies may be performed.<sup>3</sup>

The required power in hover analysis is determined by using the blade element method, which considers the airfoil characteristics.

## 2. Design process

### 2.1. Design considerations

Helicopter hover performance is expressed in terms of power loading or figure of merit (FM). In this study we assume that the rotor thrust and helicopter weight are equal. Therefore, the required hover power should be made as small as possible. The hover power required to drive the main rotor is formed by two components: induced power and profile power (to overcome viscous losses at the rotor). The induced power and the profile power primarily influence the blade aerodynamics performance design.<sup>4</sup>

The conventional approach to blade aerodynamics performance design starts with the selection of the airfoils which could be applied over various regions of the blade radius. The choice of airfoils is controlled by the need to avoid exceeding the section drag divergence Mach number on the advancing side of the rotor disc or exceeding the maximum section lift coefficients on the retreating side of the rotor disc.<sup>1</sup>

The present work considers the effect of blade airfoil shape on required power. Therefore, a baseline airfoil NACA0012 was chosen as a unique airfoil for the blade to simplify the process of optimum design. Moreover, this approach can deal with various helicopters which operate in various velocity ranges. The considerations of selection of the baseline airfoil are skipped. The airfoil shape is represented by CST function coefficients. These coefficients are also the design variables of the examined optimization problem.

Considering the above-mentioned factors, this approach gives the induced and profile power as functions of twist, taper ratio, point of taper initiation, blade root chord, and coefficients of the airfoil distribution function. Satisfactory

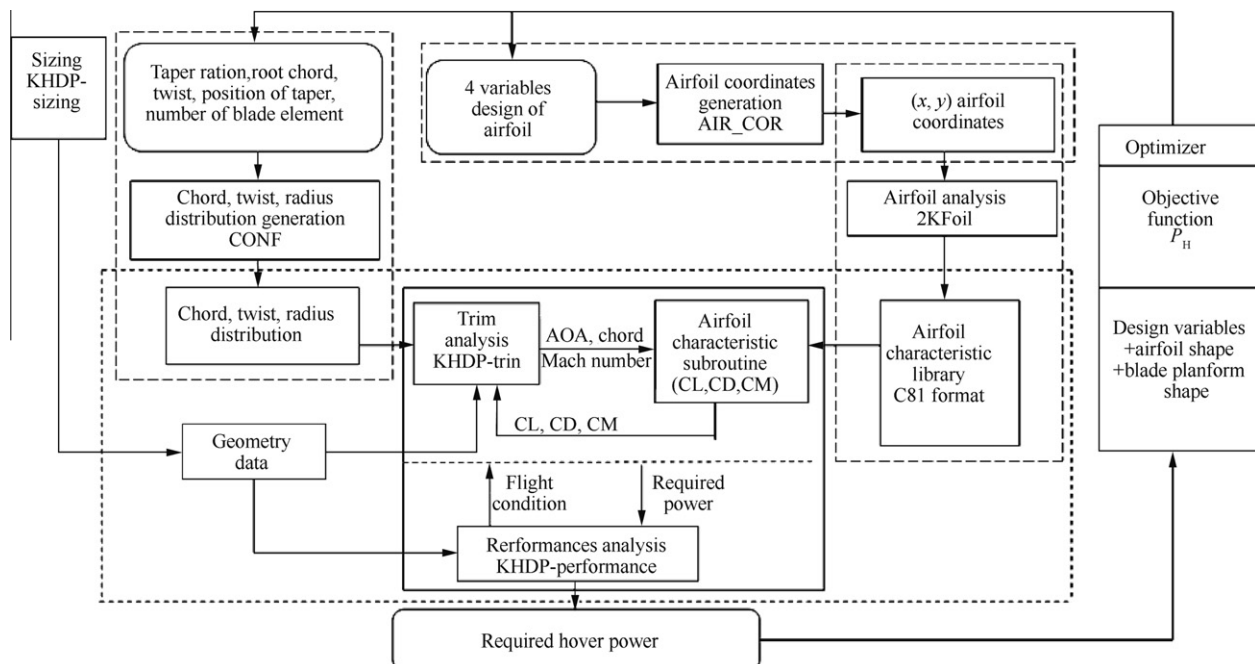


Fig. 1 Design synthesis process.

aerodynamics performance is defined by the following requirements<sup>1</sup>:

- (1) The required power must be less than the power available.
- (2) The helicopter must be able to trim at hover flight condition.

## 2.2. Design synthesis process

The design synthesis process is shown in Fig. 1. Four modules are implemented in this optimization framework: (A) the chord, twist, and radius distribution generation module; (B) the airfoil point coordinates generation module; (C) the airfoil characteristics library with C81 format module; (D) and the sizing, trim, and performance analysis module. The chord, twist, and radius distributions are generated by a code in which the geometry representation can be changed; for example it can be a linear or nonlinear function. In this study, chord distribution is generated based on the root chord, the point of taper initiation, and the taper ratio. Twist distribution is assumed to vary as a linear function along the blade. Radius distribution was divided by the equal annulus area of the rotor disk. These distributions are the input data for the trim code in the trimming process.

### 2.2.1. Geometry representation CST method

The procedure of the CST representation method is shown in Fig. 2. This method is based on analytical expressions to represent and modify the various shapes.<sup>2</sup> The components of this function are “shape function” and “class function”.

Using the CST method, the curve coordinates are distributed by the following equation:

$$y(x/c) = C_{N_2}^{N_1}(x/c)S(x/c) \quad (1)$$

where  $C_{N_2}^{N_1}(x/c) = (x/c)^{N_1}(1-x/c)^{N_2}$  represents class function,  $N_1$  and  $N_2$  are exponents;  $S(x/c) = \sum_{i=0}^N [A_i(x/c)^i]$  is shape function,  $x$  non-dimensional values from 0 to 1, and  $c$  curve length (if the shape is like an airfoil upper curve,  $c$  is the chord length).

For the formulation of the CST method, Bernstein polynomials are used as a shape function.

$$S_i(x) = K_i x^i (1-x)^{n-i} \quad (2)$$

where  $K \equiv \binom{n}{i} = \frac{n!}{i!(n-i)!}$  represents binomial coefficients,  $n$  is the order of Bernstein polynomial, and  $i$  the numbers 0 to  $n$ .

The CST method follows the process shown in Fig. 2. First, the given data points are converted to non-dimensional values. Second, the class function exponents and the degree of the shape function are defined. Then, shape function coefficients are calculated by the fitting process. Finally, by multiplying

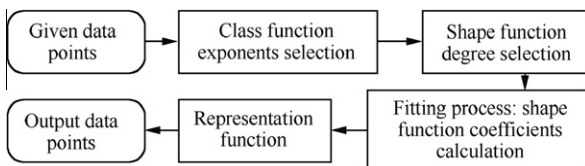


Fig. 2 Representation procedure using CST method.

the shape function and the class function, the distribution function is obtained.

Fig. 3 shows the airfoil geometry represented using the CST method and non-uniform rational basis B-spline (NURBS). In this case, the control variables are the coordinates of control points (five variables for the upper curve and five for the lower curve). The CST method with four control variables fits the existing airfoil better than NURBS, which uses ten control variables.<sup>5</sup>

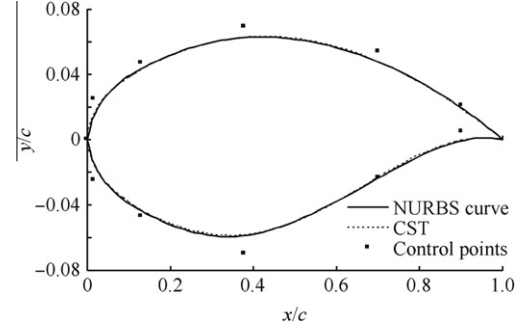


Fig. 3 RAE2822 airfoil representation.

Fig. 4 shows the absolute errors  $\varepsilon$  of airfoil generation using CST and NURBS (five control points for each curve, fourth order blending functions). Generation by NURBS gives bigger errors at the tail part of the airfoil.

The advantage of the CST method in comparison with other methods such as Spline, B-Splines, or NURBS is that it can represent curves and shapes very accurately using few scalar control parameters.

In this study, the airfoil baseline was chosen as NACA0012. With the given data coordinate points in Cartesian coordinate space, a curve fitting was generated using fourth order Bernstein polynomials.

The class function for the airfoils is

$$C(x) = x^{0.5}(1-x) \quad (3)$$

The airfoil distribution functions defined as upper and lower curves are presented sequentially as

$$\begin{cases} y_1(x) = C(x)[A_{10}(1-x)^4 + A_{11}4x(1-x)^3 + A_{12}6x^2(1-x)^2 \\ \quad + A_{13}4x^3(1-x) + A_{14}x^4] \\ y_u(x) = C(x)[A_{u0}(1-x)^4 + A_{u1}4x(1-x)^3 + A_{u2}6x^2(1-x)^2 \\ \quad + A_{u3}4x^3(1-x) + A_{u4}x^4] \end{cases} \quad (4)$$

where  $A_{u0} = 0.1718$ ;  $A_{u1} = 0.15$ ;  $A_{u2} = 0.1624$ ;  $A_{u3} = 0.1211$ ;  $A_{u4} = 0.1671$ ;  $A_{10} = -0.1718$ ;  $A_{11} = -0.15$ ;  $A_{12} = -0.1624$ ;  $A_{13} = -0.1211$ ;  $A_{14} = -0.1671$ .

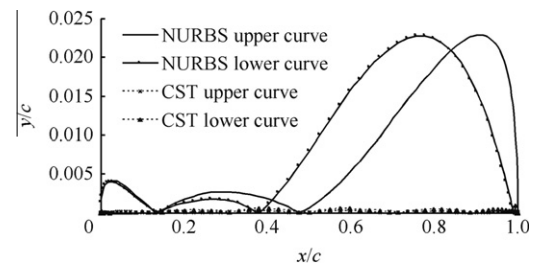


Fig. 4 Absolute errors in airfoil generation.

Changes in the coefficients  $A_0$  and  $A_4$  in the CST method are sufficient for airfoil shape modification.<sup>2</sup> These coefficients are also the design variables of the examined optimization problem.

Four coefficients of the airfoil distribution function are defined as the initial input data of the design process after obtaining the fitting curve of the airfoil baseline NACA0012. Then, airfoil coordinate points are generated by the CST function.

### 2.2.2. 2KFoil program

2KFoil, an airfoil analysis program for subsonic isolated airfoils, was adapted from the well-known XFOIL program to be suitable for the present study. The main algorithm of this code is a combination of high-order panel methods with a fully coupled viscous/inviscid interaction method.<sup>6</sup>

The inviscid formulation of 2KFoil is a linear vorticity stream function panel method. A Karman-Tsien compressibility correction is incorporated, allowing good compressible predictions all the way to sonic conditions.<sup>6</sup>

The viscous formulations come from the boundary layers and wake which are described with a two-equation lagged dissipation integral boundary layer and an envelope en transition criterion.<sup>6</sup>

A sequence of angle of attack (AoA) from  $-20^\circ$  to  $20^\circ$  is calculated for each Mach number  $Ma_\infty$  from 0.05 to 0.70. The starting AoA of each calculation is set to  $0^\circ$ , and the AoA step is set to  $0.5^\circ$ , thereby ensuring that the Newton solution method using the last available solution as a starting guess for a new solution works well.<sup>6</sup> Moreover, an algorithm has been implemented in order to recognize any impossible predictions such as a very high AoA in the stall condition. Detected errors are handled by halting the calculation and proceeding to the next calculation at another  $Ma_\infty$ . Therefore, the algorithm

ensures good predictions and always completes sequence calculations automatically.

The airfoil to be analyzed will be input into 2KFoil as airfoil coordinate points, and then 2KFoil will generate the lift, drag, and moment coefficients  $C_L$ ,  $C_D$ , and  $C_M$  corresponding to a specific angle of attack,  $Ma_\infty$  (from 0.05 to 0.70), and Reynolds number.

### 2.2.3. Konkuk helicopter design program (KHDP)

KHDP is a helicopter sizing, performance analysis, and trim analysis program that was developed at Konkuk University. These codes were developed for use in the conceptual design phase and hence they used empirical formulas to reduce computing times.<sup>7</sup> The sizing process was based on graphical design techniques method called the fuel ratio or  $R_F$  method developed during the 1950s and 1960s and initially utilized with nomographs.<sup>8,9</sup>

To quickly understand and image the helicopter behavior, the performance analysis module was developed. An analytical method was used to provide the designer with a reliable tool of sufficient fidelity to assist in the design process. The module is based on an energy approach and it has been written to yield results quickly and inexpensively.<sup>4,10</sup>

Blade element theory was implemented to calculate the required power in different helicopter operations, namely hover, climb, cruise, descent, and autorotation.<sup>10,11</sup>

Fig. 5 shows an integrated algorithm that was developed to predict the performance behavior of a helicopter by momentum theory and blade element theory (BET). BET needs to call trim module analysis to obtain the required power. Therefore, the required power is a function of the airfoil shape, and the blade platform.

The program KHDP with the performance analysis module provides many options for the objective function. The objec-

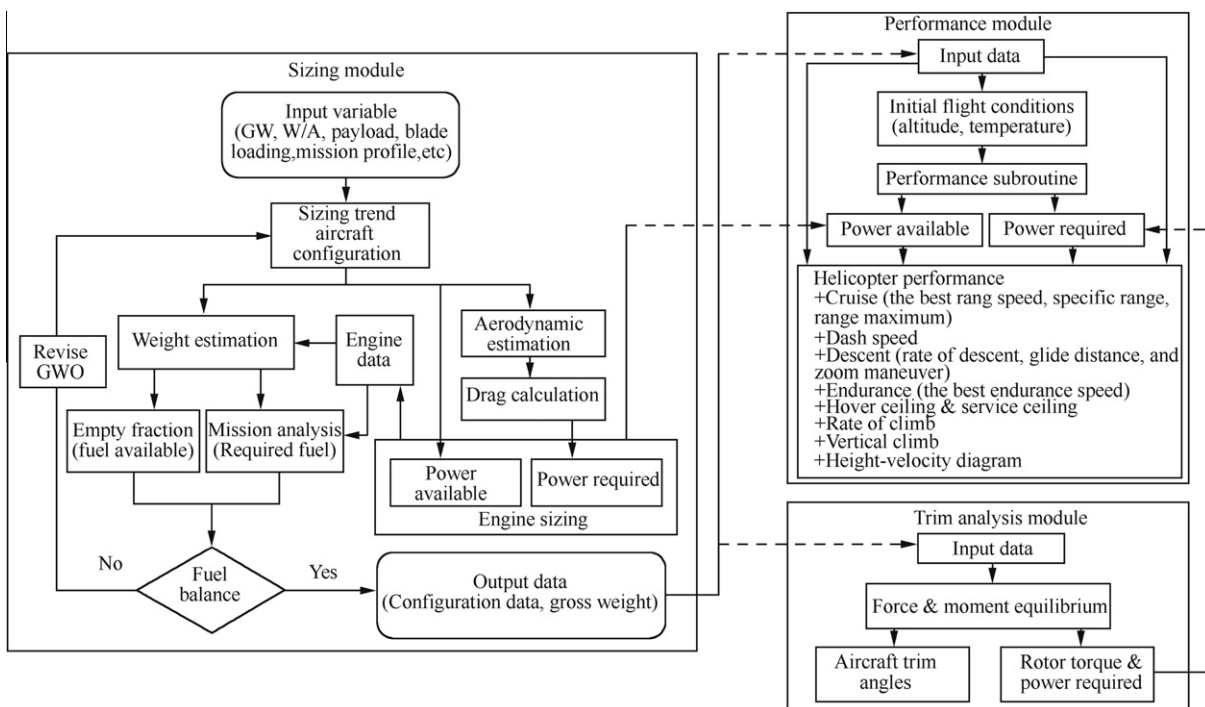


Fig. 5 KHDP program process.



tive function of this study is chosen as the required hover power. Helicopter data are analyzed by the performance code obtained from either the sizing module or user inputs.

The KHDP program process using BET is shown in Fig. 5.

Validated results of each module are shown in Ref. <sup>12</sup> The differences between calculated results and the existing data are within 5% in general, hence acceptable for preliminary design phase.

After achieving the trim condition, meaning that the trim condition is attainable, the required power is evaluated in order to proceed to the next loop of the optimization process. So, a new set of initial data (root chord, the point of taper initiation, taper ratio, pretwist, and  $A_0$  and  $A_4$  of the airfoil distribution function) is generated depending on the optimization algorithm. This loop continues until the convergence condition is satisfied.

The harmonic balance method was used in the trim code module to calculate trim angles' (collective pitch, cyclic pitch, etc.) forces and moments. Required power is then calculated using below equations:

$$PH = PH_{MR} + PH_{TR} \quad (5)$$

$$PH_{MR} = M_{MR} \times \Omega_{MR}/746 \quad (6)$$

$$PH_{TR} = M_{TR} \times \Omega_{TR}/746 \quad (7)$$

where PH is helicopter required hover horsepower,  $PH_{MR}$  main rotor horsepower,  $PH_{TR}$  tail rotor horsepower,  $M_{MR}$  main rotor moment,  $M_{TR}$  tail rotor moment,  $\Omega_{MR}$  rotational frequency of main rotor, and  $\Omega_{TR}$  rotational frequency of tail rotor

### 3. Optimization formulation and method

#### 3.1. Design variables

The design variables are maximum pretwist, taper ratio, point of taper initiation, blade root chord, and  $A_0$  and  $A_4$  of the airfoil distribution function. The blade is rectangular until the station of the point of taper initiation and then tapers linearly to the tip.<sup>14</sup> The twist varies linearly from the root to the tip. NACA0012 was chosen as the baseline airfoil, and  $A_0$  and  $A_4$  are the design variables of the airfoil shape.

#### 3.2. Constraints

The required power in hover must be less than the power available. The trim constraint in hover is implemented by expressing the constraint in terms of the number of trim iterations, iter, and the maximum number of trim iterations allowed,  $iter_{max}$ .

$$iter \leq iter_{max} \quad (8)$$

The other constraint is used to ensure that the blade tip chord does not become too small.

$$c_t \leq c_{tmin} \quad (9)$$

where  $c_t$  is the tip chord and  $c_{tmin}$  the minimum tip chord allowed.

This constraint can be described in terms of the taper ratio range shown in Table 1. The magnitudes of the  $A_0$  and  $A_4$  of the airfoil distribution function are less than 1.

$$g_i = |A_0, A_4| - 1 \leq 0 \quad (10)$$

**Table 1** Design variables and constraints.

Parameter		Lower bound	Upper bound
Design variable	$A_{u0}$	0.05	0.5
	$A_{u4}$	0.05	0.5
	$A_{l0}$	-0.5	-0.05
	$A_{l4}$	-0.5	-0.05
	Tapr	0.2	1
	Potap	0.5	1
	Chord (m)	0.2	0.35
	Twist (°)	-16	-5
Constraint	FM	0.69	1
	ITM	1	15

Where Tapr is taper ratio, Potap position of taper initiation, Chord chord length, FM figure of merit, ITM number of trim iterations;  $A_{u0}$ ,  $A_{u4}$ ,  $A_{l0}$ , and  $A_{l4}$  are coefficients of airfoil shape distribution function.

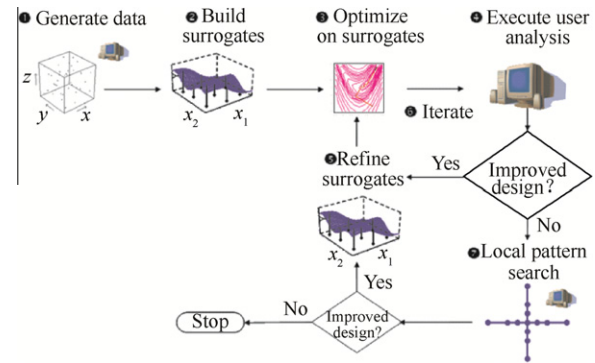
#### 3.3. Objective function and optimization tool

The performance module allows the objective function of the optimization problem to be varied. In this study, the required power in hover was chosen as the objective function.

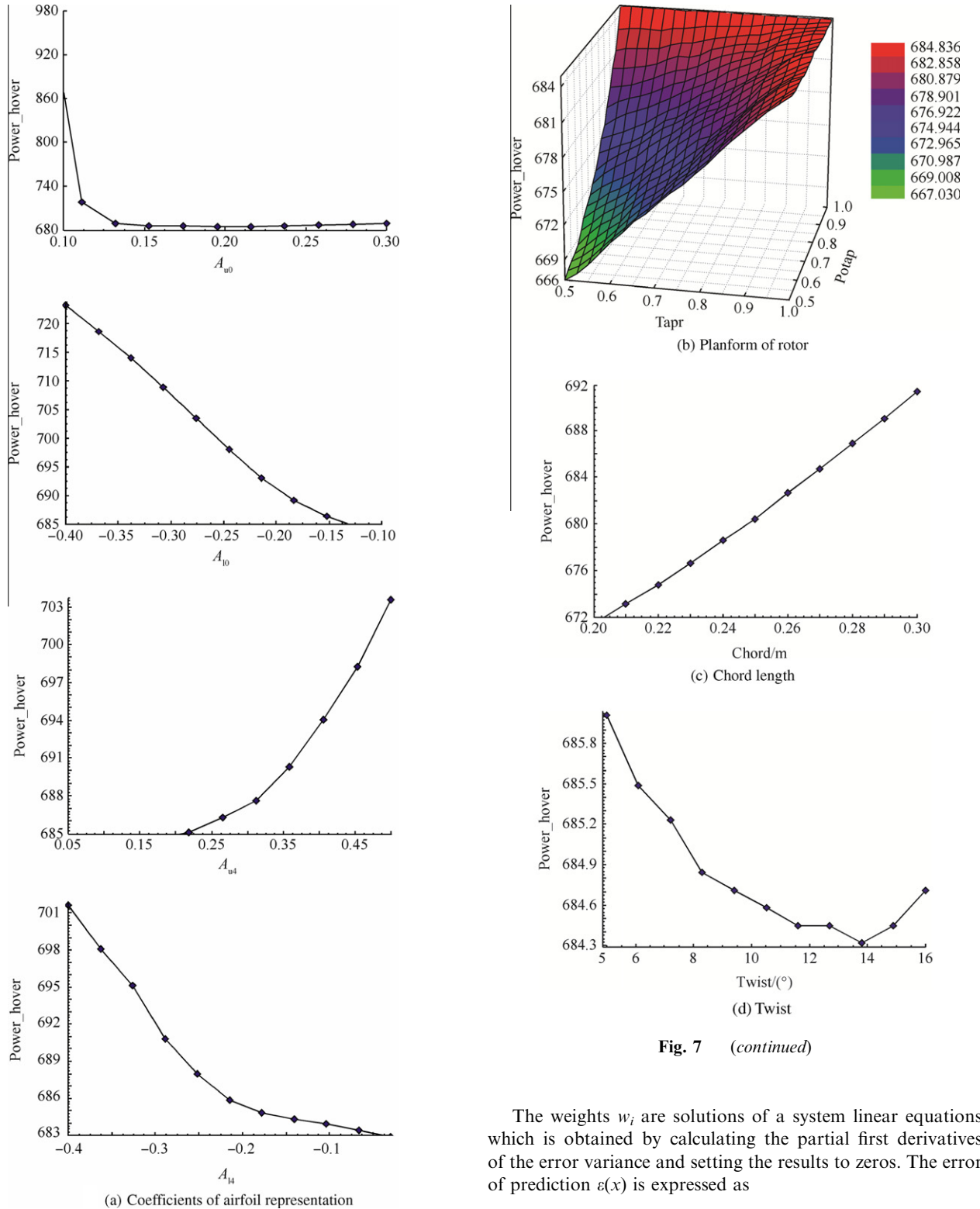
All modules were wrapped in the program modelcenter, which is a powerful tool for automating and integrating design codes. The Design Explorer tool was used to perform the optimization search using modelcenter. Design Explorer's key technologies are the systematic and efficient sampling of the design space using design of experiments (DOE) methods and the intelligent use of "surrogate" models for problem analysis and optimization. The smooth surrogate models serve as substitutes for potentially expensive and "noisy" computer simulations and make global analysis and optimization of complex systems practical.

The surrogate models used by Design Explorer are Kriging interpolation models.<sup>13</sup> To create a surrogate model, Design Explorer executes the analysis code (ModelCenter model) multiple times and stores the results of each run in a table. The input variable values for this series of runs are chosen to efficiently canvas the design space (using an orthogonal array).

The aim of Kriging interpolation is to estimate the value of an unknown function,  $f$ , at a point  $x^*$  using weighted linear



**Fig. 6** Process of using surrogate model in Design Explorer option of ModelCenter.<sup>3</sup>



**Fig. 7** Sensitivity analysis of design variables.

combinations of given the values of the function at some other points,  $x_1, x_2, \dots, x_n$  the predicted value  $\hat{f}(x^*)$  is expressed as:

$$\hat{f}(x^*) = \sum_{i=1}^n w_i(x^*) f(x_i) \quad (11)$$

The weights  $w_i$  are solutions of a system linear equations which is obtained by calculating the partial first derivatives of the error variance and setting the results to zeros. The error of prediction  $\varepsilon(x)$  is expressed as

$$\varepsilon(x) = f(x) - \sum_{i=1}^n w_i(x) f(x_i) \quad (12)$$

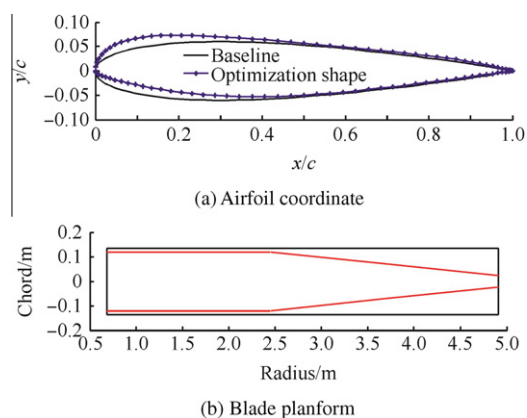
The process of using surrogate model in design explorer tool is shown in Fig. 6. The surrogate models are selectively updated and refined as the optimization process progresses. Global search mechanisms are implemented to avoid local minima. A final pattern search guarantees that the best design found is at least a local minimum.

A gradient based optimization algorithm (sequential quadratic programming) is used in conjunction with the surrogate models to predict the optimum design for the design problem.<sup>3</sup>

The lower and upper bounds of each design variable are summarized in Table 1.

**Table 2** Optimization results.

Parameter		Baseline	Optimization	Improvement
Design variable	$A_{u0}$	0.1718	0.2803	
	$A_{u4}$	0.1671	0.2293	
	$A_{l0}$	-0.1718	-0.0755	
	$A_{l4}$	-0.1671	-0.1256	
	Tapr	1	0.2	
	Potap	1	0.5	
	Chord (m)	0.27	0.24	
Constraint	Twist (°)	-8	-15.3	
	FM	0.72	0.77	6.5%
Objective function	ITM	6	12	
	Power (HP)	687.44	636.22	7.4%



**Fig. 8** Optimum results.

## 4. Results

Fig. 7 shows the sensitivity analysis of the effect of each design variable on the objective function. These analyses reveal that the coefficients obtained by the airfoil distribution function have an important role in the performance of the rotor. Therefore, this study has demonstrated that airfoil shape should be considered as a design variable. This optimization problem is applied to the rotor blade of a Bo 105 LS helicopter.

Table 2 and Fig. 8 shows the optimum results in which the objective function decreases by 7.4% and FM increases by 6.5%.

## 5. Conclusion and future works

This study is performed for the hover case only. We can see that the optimum taper ratio and position of the taper are on the boundary of these design variables. These results match the optimum hovering rotor described in Leishman's textbook which requires a local chord distribution over the blade to be given by

$$c(r) = \frac{c_{\text{tip}}}{r} \quad (13)$$

The local blade chord must vary hyperbolically with span and can be adequately approximated by a linear taper over the outer part of the blade.<sup>4</sup> Therefore, each section of the blade operates at optimum lift-to-drag ratio.

The optimum blade shape has smaller solidity in comparison with the baseline. In this case, the twist decreases from  $-8^\circ$  to  $-15.3^\circ$  in order to compensate lift reduction due to smaller solidity. The optimum blade planform could generate uniform inflow from taper position to tip, hence minimize induced power. The optimum airfoil shape has higher thickness and camber compared to baseline, thereby increasing maximum lift of the airfoil. We can easily see that with the optimum taper, twist and airfoil shape, values of local lift coefficient reduce at the blade root and increase at the tip. This reduces the profile power component, so the rotor can be operated at the same thrust but with an improvement of FM. The optimum results in which the required hover power decreases by 7.4% and FM increases by 6.5% are good values for rotor blade design.

For any airfoil baseline case, the airfoil shape represented by two coefficients for the upper curve,  $A_{u0}$  and  $A_{u4}$ , and two coefficients for the lower curve,  $A_{l0}$  and  $A_{l4}$ , always plays an important role in the effective performance of the rotor. By using the CST method, we can represent the airfoil curve with few coefficients, which is reasonable for the performance of an optimization problem.

A further study on rotor blade design in forward flight and maneuver flight also needs to consider airfoil shape. The requirements are that the airfoil section must not stall in forward flight and the drag divergence Mach number must be avoided. The airfoil analysis performed by 2Kfoil using the panel method with viscous and compressibility correction is suitable for subsonic flow analysis only. An additional effort to create transonic and supersonic flow analysis code needs to be performed for optimum blade design including airfoil shape in forward flight.

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## References

- Walsh JL, Bingham GJ, Riley MF. Optimization method applied to the aerodynamic design of helicopter rotor blades. *J Am Helicopter Soc* 1987;**32**(4):39–44.
- Kulfan BM. Universal parametric geometry representation method. *J Aircraft* 2008;**45**(1):142–58.
- ModelCenter Software. *The process of using surrogate model in Design Explorer option of ModelCenter*; 2009.
- Leishman JG. *Principles of helicopter aerodynamics*. Cambridge: Cambridge University Press; 2006.

5. Azamatov AI, Lee JW, Byun YH, Kim SH. Advanced configuration generation technique for the complex aircraft geometry. In: *2008 IEEE/ASME international conference on advanced intelligent mechatronics (AIM)*; 2008.
6. Drela M, Youngren H. *XFOIL 6.94 user guide*. New York: Cambridge University Press; 2001.
7. Kang HJ, Park HU, Vu NA, Lee J, Kim C, Yu Y, et al. Development of robust design and optimization process for unmanned rotorcraft design. In: *AHS international 65th annual forum & technology display*; 2008. p. 25.
8. HQS U.S. AMC. *AMCP 706-201 Engineering design handbook: helicopter engineering, part 1: preliminary design*; 1974.
9. Simonds RM. A generalized graphical method of minimum gross weight estimation. In: *The National Conference of the Society of Aeronautical Weight Engineers*; 1956. p. 25.
10. Stepniewski WZ. *Rotary-wing aerodynamics*. New York: Dover Publications; 1984.
11. Herda R, Greenfield W. *Performance data report*. Hiller Aircraft Corporation; 1955. Report No: 60-92.
12. Vu NA, Azamatov AI, Lin T, Lwin T, Kang HJ, Lee JW, et al. Development of rotorcraft design and virtual manufacturing framework. In: *Second international forum on rotorcraft multidisciplinary technology*; 2009.
13. Booker AJ. Design and analysis of computer experiments. In: *Seventh AIAA/USAF/NASA/ISSMO symposium on multidisciplinary analysis & optimization*; 1998.
14. Walsh JL, LaMarsh WJ, Adelman HM. Fully integrated aerodynamic/dynamic optimization of helicopter rotor blades. *Math Comput Model* 1993;**18**(3):53–72.

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