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Editorial

Combinatorial approach to image analysis

In recent years image analysis has become a research field of exceptional significance, due to its relevance to real life problems in important societal and governmental sectors, such as medicine, defense, and security. Theoretical research in image analysis relies on a variety of classical and modern mathematical disciplines. As image representation in the computer is discrete, the discrete approach to image analysis appears to be naturally relevant. In relation to this, a new discipline known as *digital geometry* has been developed within the recent decades, with the explicit purpose of providing theoretical foundations for image analysis and other areas of visual computing. *Digital geometry* can be classified as a branch of discrete applied mathematics, studying geometric properties of digital objects (modeled as sets of points with integer coordinates) and providing solution to problems defined on such objects.

Over the past two decades, substantial research has been performed in combinatorial image analysis and digital geometry. The field has a large number of excellent researchers, a massive body of literature, and various scientific conferences. One of the first major conferences in the area was the International Workshop on Combinatorial Image Analysis (IWCIA). The IWCIA series was initiated in 1991 in Paris. Subsequent meetings were held in Ube, Washington, D.C., Lyon, Hiroshima, Madras, Caen, Philadelphia, Palermo, Auckland, Berlin, and Buffalo. The IWCIA workshops provide a forum for researchers throughout the world to present cutting-edge results in combinatorial image analysis, to discuss recent advances and new challenges in this research field, and to promote interaction with researchers from other countries.

The present special issue includes eleven papers. The first one is a *Perspective*, providing an overview of certain current challenges in digital geometry. The other ten are extended versions of papers presented at the IWCIA 2008 that was held in Buffalo, NY, April 7–9, 2008.

Following the call for papers, IWCIA 2008 received 117 submissions from 24 countries from Africa, Asia, Europe, North America, and South America. After a rigorous double-blind process of review by three reviewers per submission, 28 papers were accepted for oral presentation and 10 for poster presentation. The Workshop proceedings were published in Springer's *LNCS* 4958. After the Workshop, authors were invited to submit substantial extensions of their work to the present special issue of *Discrete Applied Mathematics*. The basic selection criteria for invitation included the score that the paper received from the workshop reviewers and its relevance to the scope of the journal. After a further review process by at least two anonymous reviewers per paper, 11 papers were accepted for publication. We believe that as a result of the very rigorous selection process, only papers of the highest quality appear in this special issue.

In the leading article "Some theoretical challenges in digital geometry: A perspective", Tetsuo Asano, Valentin E. Brimkov, and Reneta P. Barneva suggest a number of strategic objectives for theoretical research, with an emphasis on the combinatorial approach in image analysis. Most of the proposed objectives relate to the need of making the theoretical foundations of combinatorial image analysis better integrated within a number of well-established subjects of theoretical computer science and discrete applied mathematics, such as the theory of algorithms and problem complexity, combinatorial optimization and polyhedral combinatorics, integer and linear programming, and computational geometry.

A number of papers address certain problems of image analysis that may also be of interest regarding other branches of discrete applied mathematics. In the first paper, Anvesh Komuravelli, Arnab Sinha, and Arijit Bishnu consider connectivity issues of binary images under different connectivity models. In particular, they define connectivity-preserving local transformations (called "interchanges"), and study the possibilities of transforming a given binary image to another binary image through a sequence of interchanges. Various technical results, such as time complexity bounds of the procedures, are presented in the paper.

In the next paper, Robin Strand and Benedek Nagy define path-based distance functions on n-dimensional generalizations of the face-centered cubic and body-centered cubic grids. These functions, that use both weights and neighborhood sequences, share various properties of the traditional path-based distance functions but are less rotation-dependent. For

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the three-dimensional case, the authors introduce four different error functions. These are used to find the optimal weights and neighborhood sequences that can be used to define the distance functions.

Next, K.G. Subramanian, Rosihan M. Ali, M. Geethalakshmi, and Atulya K. Nagar introduce a new syntactic model, called pure 2D context-free grammar, which is based on the notion of pure context-free string grammar. The authors investigate the picture generative power of the model and prove certain closure properties. They also consider an analogue of the 2D grammar model for the purposes of generating hexagonal picture arrays on triangular grids.

In the last paper of this group, Jérôme Darbon first presents an approach for global optimization of Markovian energies, defined on an arbitrary graph, assuming a linearly ordered label set and ''pairwise interactions'' described by submodular functions. The proposed algorithm maps the original problem into a combinatorial one that is solved by a maximum-flow/s-t minimum-cut technique. Then he uses the results obtained for image processing, and discusses results of experiments.

The next three papers consider classical problems of digital geometry and combinatorial image analysis. Petra Wiederhold and Sandino Morales consider some theoretical questions related to Kovalevsky's thinning method for 2D digital binary images modeled by cell complexes constructed from polygonal tilings. The authors analyze the relation between local and global simplicity of cells, and prove their equivalence under certain conditions. They also investigate the parallel performance of Kovalevsky's thinning method as regards preservation of image topology.

The next paper, by Alexandre Dupas and Guillaume Damiand, presents a region merging algorithm. When a number of connected components of an image are merged together, the topology of the evolving 3D image partition is controlled by Betti number computation, using information represented by the topological map of an image. The efficiency of the proposed procedures is tested experimentally and the impact of the topological control performed is illustrated with practical examples.

In the next paper, Péter Balázs considers the problem of reconstruction of an *hv*-convex discrete set. He first provides a summary of the most important generation methods developed for the subclasses of *hv*-convex discrete sets. Then he presents new generation techniques that complement the existing methods. This makes it possible to design a complete benchmark set for testing the performance of reconstruction algorithms on the class of *hv*-convex discrete sets and its subclasses. Using the benchmark set, the author obtains several statistics on *hv*-convex discrete sets, which can be used in the analysis of reconstruction algorithms.

As mentioned in the Perspective in this special issue, using ideas and results of computational geometry in research in digital geometry may be rather advantageous and is therefore highly desirable. The last three papers illustrate this thesis. In the first one, Kristof Teelen and Peter Veelaert present an uncertainty model for geometric transformations based on polygonal uncertainty regions and transformation polytopes. The authors propose a systematic approach for the computation of regions of interest for features by using the uncertainty model for affine and projective transformations. The focus is on the solution of transformation problems for geometric primitives, especially lines, so that regions of interest can be computed for corresponding geometric features in distinct images.

The next paper, by Emilie Charrier and Lilian Buzer, considers the problem of determining the rational number that best approximates a given real number, provided that the denominator of the approximation is required to belong to a certain interval. The proposed algorithm is based on a new convex hull computation that combines ideas from number theory and computational geometry. The algorithm preserves the optimal time complexity of some well-known convex hull algorithms and is the first one that is ''output sensitive''. More specifically, the convex hull is computed in time linear in the number of its vertices.

In the last paper of this special issue, Henrik Schulz presents an algorithm for polyhedral approximations of certain classes of digital objects in three dimensions. The objects are sets of voxels represented as strongly connected subsets of an abstract cell complex. The proposed algorithm generates the convex hull of a given object and after that modifies it by recursive generation of convex hulls of subsets of the given voxel set or subsets of the background voxels. The result is a polyhedron which separates object voxels from background voxels.

We believe that the papers of this special issue will be of interest to a broad audience, including researchers working in such areas as pattern recognition, shape modeling, computer vision, image processing, computer graphics, and discrete mathematics in general.

We would like to thank all those who contributed to this special issue. First, we are very much obliged to Endre Boros, the Editor-In-Chief of *Discrete Applied Mathematics*, for providing us with the opportunity to edit this special issue. Our thanks go to the authors who submitted their works and made this publication possible. We are grateful to the reviewers for their time and for their reliable and insightful remarks. We also gratefully acknowledge the assistance of Katie D'Agosta from the *DAM* Editorial Office and Andreja Pisnik of Elsevier in the preparation of this issue.

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