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A transmission line model for the calculation of phononic band gaps in perforated mems structures

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Abstract

In this paper an acoustic transmission line model for the calculation of phononic band gaps is presented. We study the propagation of longitudinal waves in thin, rectangular perforated rods. We show that the 1D nature of the rod vibrations allows the modeling of the system by a series of acoustic transmission lines with different characteristic impedance, where the perforated section is modeled as an equivalent filled material with Young's modulus and mass density depending from the size of the holes. Once this model is defined, the derivation of the transmission matrix and transmission coefficient (in matching conditions) of the entire rod is straightforward by using the classical transmission line theory. Finally, the phononic band gap predicted by this model is verified by means of an ANSYS harmonic analysis.

Keywords: MEMS, MEMS modeling, phononic crystals.

1. Motivation

Phononic crystals are a very active field of research, even more as MEMS micromachining technologies have scaled their operating frequencies to a range suitable for radio-frequency and sensing applications [1, 2]. Numerical methods are commonly employed to investigate band structures of phononic crystals: plane wave expansion (PWE), finite difference time domain (FDTD) and the layer multiple scattering method (LMSM) [3, 4]. Those powerful but cumbersome methods are often unnecessary if the acoustic wave has a unique propagation direction. This is the case of a thin rod where

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longitudinal waves are capacitively excited and sensed through air gaps at the ends of the resonator. In this work the longitudinal vibrations of a thin rod are studied. The spatial periodicity of the mechanical properties (mass density ρ and Young's modulus E), which are needed to realize a phononic crystal, are achieved through rectangular perforations on the rod.

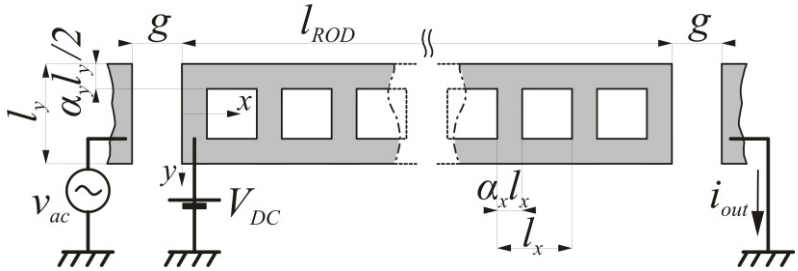

Fig. 1. Schematic structure of the modeled rod along with the termination (driving and sensing) conditions. The dimensions used in the example are $l_y = l_x = 40$ µm, $l_{ROD} = 820$ µm, $\alpha_x = \alpha_y = 0.5$, $g = 1$ µm, $V = 20$ V, $v_{ac} = 1$ V. The rod thickness is 60 µm.

2. Derivation of the model

A simplified model for a MEMS phononic crystal with electrostatic excitation is given by a rod of rectangular section with periodic rectangular holes (Fig. 1). In this section, we investigate the propagation of longitudinal acoustic waves along such a structure. The governing equations of motion for longitudinal waves along x axis of a vibrating thin rod with constant section are:

$$\frac{\partial \sigma_x(x,t)}{\partial x} = \rho \frac{\partial v_x(x,t)}{\partial t}, \qquad \frac{\partial v_x(x,t)}{\partial x} = \frac{1}{E}\frac{\partial \sigma_x(x,t)}{\partial t} \tag{1}$$

where $\sigma_x(x,t)$ is the stress along x and $v_x(x,t)$ the velocity of the section at x. The equations are formally identical to the well-known telegraph equations for a transmission line [5], as long as the stress and the velocity are mapped to the voltage and current along the line, respectively. To preserve consistency with the standard definitions in acoustics (were the hydrostatic pressure is used instead of the stress) the acoustic impedance is defined as:

$$Z_{ac} = -\frac{\sigma}{v}$$

In the analogy, mass density and the inverse of Young's modulus take the place of the inductance and capacitance per unit length of the line respectively. The characteristic acoustic impedance of the equivalent acoustic line is $R_0 = \sqrt{\rho E}$. The frequency domain solution of the equations has a travelling-wave form:

$$\begin{aligned} V_x(x) &= V_x^{+} e^{-ikx} + V_x^{-} e^{ikx} \\ \Sigma_x(x) &= R_0 \left(-V_x^{+} e^{-ikx} + V_x^{-} e^{ikx} \right) \end{aligned} \tag{2}$$

where $k = \omega\sqrt{\rho / E}$ is the wave number. The perforated rod of Fig.1 can be considered as the cascade of several acoustic line segments with (alternatively filled and perforated) constant cross-section. Reasonable models for the equivalent density and Young's modulus of the perforated section are $\alpha_y \rho$, $\alpha_y E$, respectively, where α_y is the filling ratio of this section. It is important to note that the Young's modulus model implicitly assumes that only compressive deformations are allowed in the rod. With this assumptions, the wave number will be constant along the rod, while the characteristic impedance will be R_0 in the filled sections and $\alpha_y R_0$ in the perforated ones. For each segment, and for the interfaces (discontinuities) between adjacent segments, the relevant transmission matrices (connecting the forward and backward velocity waves) can be written [5]. For example, the transmission matrix P_f links the velocity waves at the left side of the filled segment (subscript l) to the ones on the right (subscript r) through the expression:

$$\begin{pmatrix} V_l^+ \\ V_l^- \end{pmatrix} = \overbrace{\begin{pmatrix} e^{-ikl_x\alpha_x} & 0 \\ 0 & e^{ikl_x\alpha_x} \end{pmatrix}}^{P_f} \begin{pmatrix} V_r^+ \\ V_r^- \end{pmatrix} \tag{3}$$

while the matrix M_{fp}, modeling the interface between the filled and perforated sections, has the expression:

$$\begin{pmatrix} V_l^+ \\ V_l^- \end{pmatrix} = \overbrace{\begin{pmatrix} \dfrac{1+\alpha_y}{2\alpha_y} & \dfrac{\alpha_y - 1}{2\alpha_y} \\ \dfrac{\alpha_y - 1}{2\alpha_y} & \dfrac{1+\alpha_y}{2\alpha_y} \end{pmatrix}}^{M_{fp}} \begin{pmatrix} V_r^+ \\ V_r^- \end{pmatrix} \tag{4}$$

In the above formulas, l_x is the period of a cell along x and α_x is the filling ratio of that cell in the same direction (see Fig.1). Similar expressions can be derived for M_{pf} and P_p (with obvious meaning of the symbols). Finally, the transmission matrix T_{ROD} of the full rod will be:

$$T_{ROD} = P_f \left(M_{pf} P_p M_{fp} P_f \right)^n$$

where n is the number of holes in the rod. This model allows the study of rod vibrations with general acoustic matching condition, which is often the case for phononic crystals realized by MEMS micromachining. This is a remarkable advantage of this approach, as compared to such methods as PWE, which assumes an infinite crystal, or FDTD, where generic boundary conditions are possible but computationally expensive. The input and output acoustic terminations of the rod Z_{in}, Z_{out}, can be modeled by simply imposing:

$$-\left.\frac{\Sigma_{in} - \Sigma_x(x)}{V_x(x)}\right|_{x=0} = Z_{in}, \qquad -\left.\frac{\Sigma_x(x)}{V_x(x)}\right|_{x=l_{ROD}} = Z_{out}$$

where Σ_{in} is the pressure induced at the input termination. In the case of capacitive excitation this will be the electrostatic pressure induced by the small signal voltage v_{ac} [6]:

$$\Sigma_{in} = \frac{\varepsilon_0 V_{DC} v_{ac}}{g^2}$$

3. Comparison with FEM simulations

To validate the model, the system depicted in Fig. 1 was simulated with ANSYS. The dimensions of the simulated rod are as follows: $l_x = l_y = 40$ µm, $l_{ROD} = 820$ µm, $g = 1$ µm and thickness 60 µm. The material is assumed to be single crystal silicon with Young's modulus $E = 170$ GPa and Poisson's ratio

ν = 0.064. ANSYS harmonic analysis was used to determine the current at the output electrode under the biasing conditions shown in Fig. 1 at different frequencies.

In the ANSYS simulation, to rule out vibration modes not directly related to longitudinal deformations, each rod segment was forced to translate rigidly along x. As can be observed in Fig. 2, the spectrum of the output current predicted with our model is in good agreement with FEM simulations. More importantly, the model correctly predicts the position of the first frequency gap. While the aforementioned assumption of purely longitudinal waves can be unrealistic for specific rod and perforation geometries, exploration of different hole sizes showed that spurious vibrations are not significant if holes with large horizontal aspect ratios (i.e. with $l_x >> l_y$) are used. Depending on the minimum feature size of the process in use, this choice may constrain the design of the phononic crystal.

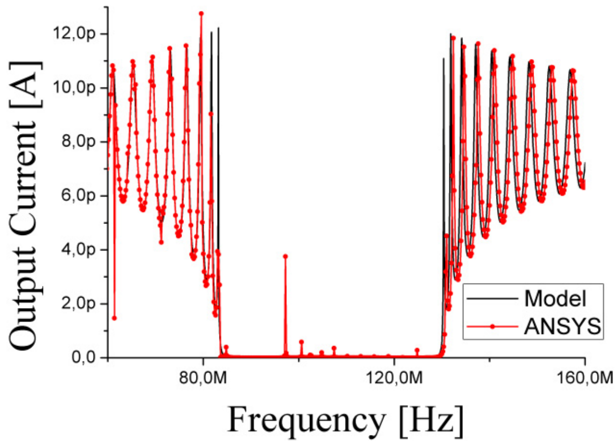

Fig. 2. Frequency response of the perforated rod driven as shown in Fig. 1. Comparison between the proposed model and ANSYS harmonic analysis.

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