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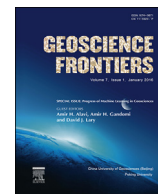


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Research paper

New design equations for estimation of ultimate bearing capacity of shallow foundations resting on rock masses

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ARTICLE INFO

Article history:

Received 4 September 2014

Received in revised form

14 November 2014

Accepted 6 December 2014

Available online 29 December 2014

Keywords:

Rock mass properties

Ultimate bearing capacity

Shallow foundation

Prediction

Evolutionary computation

Linear genetic programming

ABSTRACT

Rock masses are commonly used as the underlying layer of important structures such as bridges, dams and transportation constructions. The success of a foundation design for such structures mainly depends on the accuracy of estimating the bearing capacity of rock beneath them. Several traditional numerical approaches are proposed for the estimation of the bearing capacity of foundations resting on rock masses to avoid performing elaborate and expensive experimental studies. Despite this fact, there still exists a serious need to develop more robust predictive models. This paper proposes new nonlinear prediction models for the ultimate bearing capacity of shallow foundations resting on non-fractured rock masses using a novel evolutionary computational approach, called linear genetic programming. A comprehensive set of rock socket, centrifuge rock socket, plate load and large-scaled footing load test results is used to develop the models. In order to verify the validity of the models, the sensitivity analysis is conducted and discussed. The results indicate that the proposed models accurately characterize the bearing capacity of shallow foundations. The correlation coefficients between the experimental and predicted bearing capacity values are equal to 0.95 and 0.96 for the best LGP models. Moreover, the derived models reach a notably better prediction performance than the traditional equations.

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1. Introduction

Foundations are commonly used as the lowest parts of the civil engineering structures to transmit the applied loads to the underlying soil or rock. According to the properties of the rock mass and its beneath layer, the failure of rocks under applied loads may occur through several mechanisms (Sowers, 1979). Comprehensive descriptions about these failure mechanisms are provided in Canadian Foundation Engineering Manual and the National Cooperative Highway Research Program (NCHRP) reports (Becker, 1996; Paikowsky et al., 2004, 2010; Canadian Geotechnical Society, 2006). It is well-known that the bearing capacity failure of shallow foundations on jointed rock masses depends on the ratio of space between joints (S) to foundation

width (B), joint condition, rock type, and the condition of the underlying layer of rock mass (Bishoni, 1968; Sowers, 1979). The most widely used approaches to determine the bearing capacity of foundations on rocks can be classified into three groups: (1) analytical methods, (2) semi-empirical methods, and (3) in-situ and full-scaled testing methods. The analytical and semi-empirical methods are widely used for the bearing capacity prediction, particularly in the pre-design phases. The analytical methods such as finite element and limit equilibrium methods relate the bearing capacity to the footing geometry and rock properties (Terzaghi, 1946; Bishoni, 1968; Sowers, 1979; Goodman, 1989). The general forms of the analytical models are given in Table 1. The semi-empirical methods often propose a correlation between the bearing capacity and rock mass properties based on the empirical observations and experimental test results (Carter and Kulhawy, 1988; Bowles, 1996; Hoek and Brown, 1997, 1988). Some of the main equations obtained by the empirical approaches are summarized in Table 2.

Generally, the models obtained by the analytical, finite element and empirical approaches have both advantages and disadvantages

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Peer-review under responsibility of China University of Geosciences (Beijing)

Table 1
General forms of equations made by the analytical methods.

Reference	Equation (analytical method)	Factor	
Terzaghi (1946)	$q_{ult} = cN_c + 0.5\gamma BN_\gamma + \gamma DN_q$	$N_c = 2N_\phi^{0.5}(N_\phi + 1)$ $N_\gamma = N_\phi^{0.5}(N_\phi - 1)$ $N_q = N_\phi^2$	$N_\phi = \tan^2\left(45 + \frac{\phi}{2}\right)$
Bishoni (1968)	$q_{ult} = \alpha/cN_{cr}$	For circular footings: $\alpha = 1$ For square footings: $\alpha = 0.85$	$N_{cr} = \frac{2N_\phi}{N_\phi + 1} (\cot \phi) \left(\frac{S}{B}\right) \left(1 - \frac{1}{N_\phi}\right) - N_\phi (\cot \phi) + 2N_\phi$
Sowers (1979)	$q_{ult} = 2c \tan\left(45 + \frac{\phi}{2}\right)$	–	–
Goodman (1989)	For fractured rocks: $q_{ult} = q_u(N_\phi + 1)$ For non-fractured rocks: $q_{ult} = q_u \left(\frac{1}{N_\phi - 1} \left(N_\phi \left(\frac{S}{B}\right)^{(N_\phi - 1)/N_\phi} - 1\right)\right)$	–	$N_\phi = \tan^2\left(45 + \frac{\phi}{2}\right)$

q_{ult} : bearing capacity of shallow foundation on rock; D : depth of foundation below ground surface; c : the cohesion intercepts for the rock mass; ϕ : angle of internal friction for the rock mass; γ : effective unit weight of the rock mass; B : breadth or width of foundation; N_ϕ , N_c , N_q and N_γ : non-dimensional bearing capacity factors as exponential functions of ϕ ; N_c : bearing capacity factor; S : discontinuity spacing; S/B : ratio of joint spacing to foundation width; q_u : unconfined compressive strength of rock.

(Jiao et al., 2012; Chen et al., 2013). For the case of bearing capacity of a shallow foundation on a jointed rock mass, parameters such as the ratio of joint spacing to foundation breadth or loading width, as well as rock mass qualities such as joint conditions (open or closed), rock type and rock mass strength are influencing (Sowers, 1979; Paikowsky et al., 2010). As represented in Table 1, the analytical methods only include the physical and mechanical properties of rock mass and geometry of foundation. Thus, they do not take into account the important role of the rock type and its qualitative mass parameters such as rock quality designation (RQD), rock mass rating (RMR), and geological strength index (GSI). On the other hand, the empirical methods often relate the bearing capacity to qualitative and rock mass classification parameters and do not account for the geometry of the foundations or space between joints (Table 2). The drawbacks of the existing analytical and empirical methods imply the necessity of developing new models correlating the bearing capacity factor to both quantitative and qualitative parameters.

Computational intelligence (CI) techniques are considered as alternatives to traditional methods for tackling real world problems. They automatically learn from data to determine the structure of a prediction model. Artificial neural network (ANN), fuzzy inference system (FIS), adaptive neuro-fuzzy system (ANFIS), and support vector machines (SVM) are well-known branches of CI. These techniques have been successfully employed to solve problems in engineering field (e.g., Das and Basudhar, 2006; Gullu and Ercelebi, 2007; Das and Basudhar, 2008; Gullu and Ercelebi, 2008; Zorlu et al., 2008; Cabalar and Cevik, 2009a,b; Sivapullaiah et al., 2009; Yaghoubi et al., 2009; Al-Anazi and Gates, 2010; Kulatilake et al., 2010; Yaghoubi et al., 2010a,b, 2012; Gullu, 2012, 2013). Besides, these techniques have been used to predict the bearing capacity of shallow foundations resting on soil layers (Soleimanbeigi and Hataf, 2005; Padmini et al., 2008; Kuo et al., 2009; Kalinli et al., 2011). Despite the good performance of ANNs, FIS, ANFIS, SVM and many of the other CI methods, they are

considered black-box models. That is, they are not capable of generating practical prediction equations. In order to cope with the limitations of the existing methods, a robust CI approach, namely genetic programming (GP) has been introduced (Koza, 1992). GP is an evolutionary computational approach. It uses the principle of Darwinian natural selection to generate computer programs for solving a problem. GP has several advantages over the conventional and other similar techniques. A notable feature of GP and its variants is that they can produce prediction equations without a need to pre-define the form of the existing relationship (Çanakcı et al., 2009; Guven et al., 2009; Gandomi et al., 2010; Alavi et al., 2011; Alavi and Gandomi, 2011; Gandomi et al., 2011a; Azamathulla and Zahiri, 2012). This technique has been shown to be a powerful tool for the prediction of the bearing capacity of shallow foundations on soils (Adarsh et al., 2012; Shahnazari and Tutunchian, 2012; Tsai et al., 2012; Pan and Tsai, 2013).

However, application of GP and other CI techniques to the modeling of the bearing capacity of shallow foundations resting on rock masses is conspicuous by its absence. This paper proposes a novel subset of GP, namely linear genetic programming (LGP) to derive precise predictive equations for the ultimate bearing capacity of shallow foundation resting in/on jointed (non-fractured) rock. A comprehensive and reliable set of data including 102 previously published rock socket, centrifuge rock socket, plate load and large-scaled footing load test results is collected to develop the models. The robustness of the proposed models is verified through different validation phases.

2. Evolutionary computation

Evolutionary computation (EC) is a subdivision of CI inspired by the natural evolution. Some of the subsets of EC are evolutionary strategies (ES) and evolutionary programming (EP). These techniques are collectively known as evolutionary algorithms (EAs).

Table 2
General forms of equations made by the empirical methods.

Reference	Equation (empirical method)	Factor
Bowles (1996)	$q_{ult} = q_r \times (ROD)^2$	$m = m_i \exp\left(\frac{GSI-100}{28}\right) s = \exp\left(\frac{GSI-100}{9}\right)$
Hoek and Brown (1997, 1988)	$\sigma_1 = \sigma_3 + (mq_c\sigma_3 + sq_c^2)^{0.5}$	
Carter and Kulhawiy (1988)	$q_{ult} = q_u(m + \sqrt{s})$	

RQD: rock quality designation (rock mass classification); q_r : ultimate strength of rock determined by uniaxial compression test; σ_1 : major principal stress (compressive stresses are taken as positive); σ_3 : minor principal stress; GSI: geological strength index (rock mass classification); m and s : material constants in the Hoek and Brown failure criterion; q_c : uniaxial compression strength of intact rock.

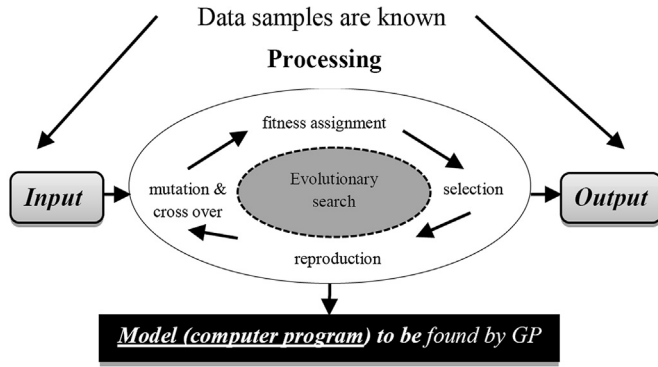


Figure 1. Conceptual scheme of input-process-output (IPO) in GP.

Genetic algorithms (GAs) (Holland, 1975) has been shown to be a suitably robust branch of EAs for dealing with a wide variety of complex civil engineering problems (Simpson and Priest, 1993; Kalantary et al., 2009; Chang et al., 2011; Hung et al., 2012; Liu et al., 2013). GP is a specialization of GA where the encoded solutions (individuals) are computer programs rather than binary strings (Banzhaf et al., 1998). In GP, inputs and corresponding output data samples are known and the main goal is to generate predictive models relating them (Fig. 1) (Brameier and Banzhaf, 2001, 2007; Weise, 2009).

LGP is a new subset of GP in which the programs are evolved in an imperative language (such as C/C++) (Brameier and Banzhaf, 2001, 2007). LGP has a linear structure similar to the DNA molecule in biological genomes. Fig. 2 represents a comparison of different representation of the GP program structure. As shown in this figure, in classical tree-based GP, the data flow is more rigidly determined by the graph structure of the program (Brameier and Banzhaf, 2001; Schmidt and Lipson, 2008; Gandomi et al., 2010, 2011b).

Here are the steps which the LGP system follows for a single run (Brameier and Banzhaf, 2007; Gandomi et al., 2010):

- I. Initializing a population of randomly generated programs and calculating their fitness values.
- II. Running a Tournament. In this step four programs are selected from the population randomly. They are compared based on their fitness. Two programs are then picked as the winners and two as the losers.
- III. Transforming the winner programs. After that, two winner programs are copied and transformed probabilistically into two new programs via crossover and mutation operators.
- IV. Replacing the loser programs in the tournament with the transformed winner programs. The winners of the tournament remain unchanged.

V. Repeating steps II through IV until termination or convergence conditions are satisfied.

Crossover occurs between instruction blocks. During this process, a segment of random position and arbitrary length is selected in each of the two parents and exchanged. If one of the two children would exceed the maximum length, crossover is aborted and restarted with exchanging equally sized segments (Brameier and Banzhaf, 2001; Gandomi et al., 2010). The mutation operation occurs on a single instruction. Two commonly used types of standard LGP mutations are micro and macro mutation. The micro mutation changes an operand or an operator of an instruction. The macro mutation inserts or deletes a random instruction (Brameier and Banzhaf, 2001; Gandomi et al., 2011b).

It is well-known that the LGP system can run several orders of magnitude faster than comparable interpreting systems. The enhanced speed of the linear variants of LGP permits conducting many runs in realistic timeframes (Oltean and Grosan, 2003; Francone and Deschaine, 2004; Gandomi et al., 2010, 2011b).

3. Numerical simulation of bearing capacity

In order to reach reliable estimations of the bearing capacity of shallow foundations on rock mass, the impact of several parameters should be incorporated into the model development. The general forms of the existing prediction equations, represented in Tables 1 and 2, indicate that the ultimate bearing capacity of shallow foundations on rock mass mainly depends on the foundation width and properties of the rock beneath it. The present study takes into account the effects of both quantitative and qualitative parameters to predict the bearing capacity. Referring to the form of the existing models, two prediction models (LGP 1 and LGP 2) are developed using the LGP technique. Consequently, the proposed models for the prediction of the ultimate bearing capacity (q_{ult}) are considered to be the functions of the following parameters:

$$q_{ult,LGP1} = f\left(RMR, q_u, \frac{S}{B}, \phi\right) \quad (1)$$

$$q_{ult,LGP2} = f\left(m, s, q_u, \frac{S}{B}, \phi\right) \quad (2)$$

where,

- RMR: Rock mass rating (rock mass classification),
- m : Material constant in the Hoek and Brown failure criterion,
- s : Material constant in the Hoek and Brown failure criterion,
- q_u (ksf): Unconfined compressive strength of rock,

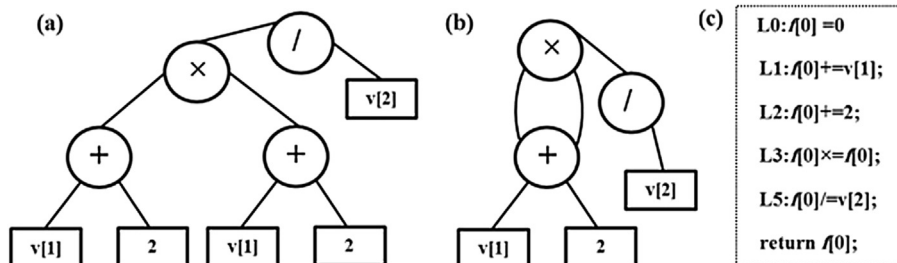


Figure 2. A comparison of a program structure evolved by (a) tree-based, (b) graph-based GP, and (c) linear GP for the expression of $y = f(v[i]) = (v[1] + 2)^2/v[2]$.

Table 3
Descriptive statistics of parameters in database used to develop LGP-based models.

Statistical index	Variable						
	RMR	<i>m</i>	<i>s</i>	<i>q_u</i> (ksf)	<i>S/B</i>	ϕ (°)	<i>q_{ult}</i> (ksf)
Mean	62.69	1.13	0.02	90.42	5.75	30.94	210.45
Standard deviation	24.37	1.12	0.03	170.73	7.33	4.93	268.23
Coefficient of variation	0.39	0.99	1.80	1.89	1.28	0.16	1.27
Sample variance	593.76	1.26	0.00	29147.04	53.67	24.28	71948.10
Kurtosis	-0.43	-0.75	0.18	17.42	4.26	0.39	11.46
Skewness	-0.69	0.86	1.47	3.81	2.11	0.12	2.96
Minimum	15.00	0.03	0.000003	5.00	0.35	20.00	5.22
Maximum	100.00	3.43	0.08	1148.70	36.69	45.00	1578.95

S/B: Ratio of joint spacing to foundation width (equivalent diameter),
 ϕ (°): Angle of internal friction for the rock mass.

3.1. Experimental database

A comprehensive database including 102 elaborate experimental data from different studies is gathered to develop the LGP-based models (Ward and Burland, 1968; Burland and Lord, 1970; Lake and Simons, 1970; Webb, 1976; Wilson, 1976; Mallard, 1977; Goeke and Hustad, 1979; Pells and Turner, 1979, 1980; Spanovich and Garvin, 1979; Thorne, 1980; Williams,

1980; Jubenville and Hepworth, 1981; Glos and Briggs, 1983; Baker, 1985; Hummert and Cooling, 1988; Radhakrishnan and Leung, 1989; Leung and Ko, 1993; Maleki and Hollberg, 1995; Nitta et al., 1995; Carrubba, 1997; Lord, 1997; Abu-Hejleh and Attwooll, 2005; McVay et al., 2006). The database contains the results of 49 rock socket tests (6 centrifuge rock socket tests), 40 plate load tests and 13 load tests on scaled footings. The database includes the results of experiments on circle and square footings of different sizes tested on various types of masses such as sandstone, claystone, shale, chalk, and basalt. The *q_{ult}* values of rock mass is initially obtained or interpreted from load–displacement curves proposed by Hirany and Kulhawy (1988). Different parts of the employed database have been used by other researchers to develop prediction models for *q_{ult}* (AASHTO, 2007; Paikowsky et al., 2010). The descriptive statistics of the experimental data are given in Table 3. Fig. 3 presents the frequency histograms of the parameters used for the model development.

3.2. Data classification

In order to avoid overfitting, the available data sets are randomly classified into three subsets: (1) training (learning), (2) validation (check), and (3) test subsets. The training set is used to fit the models and the validation set is used to estimate the prediction error for model selection. Finally, the test set is employed for the evaluation of the generalization ability of the final chosen model. The training, validation and test data are usually taken as 50–70%, 15–25% and 15–25% of all data, respectively (Shahin and Jaksa,

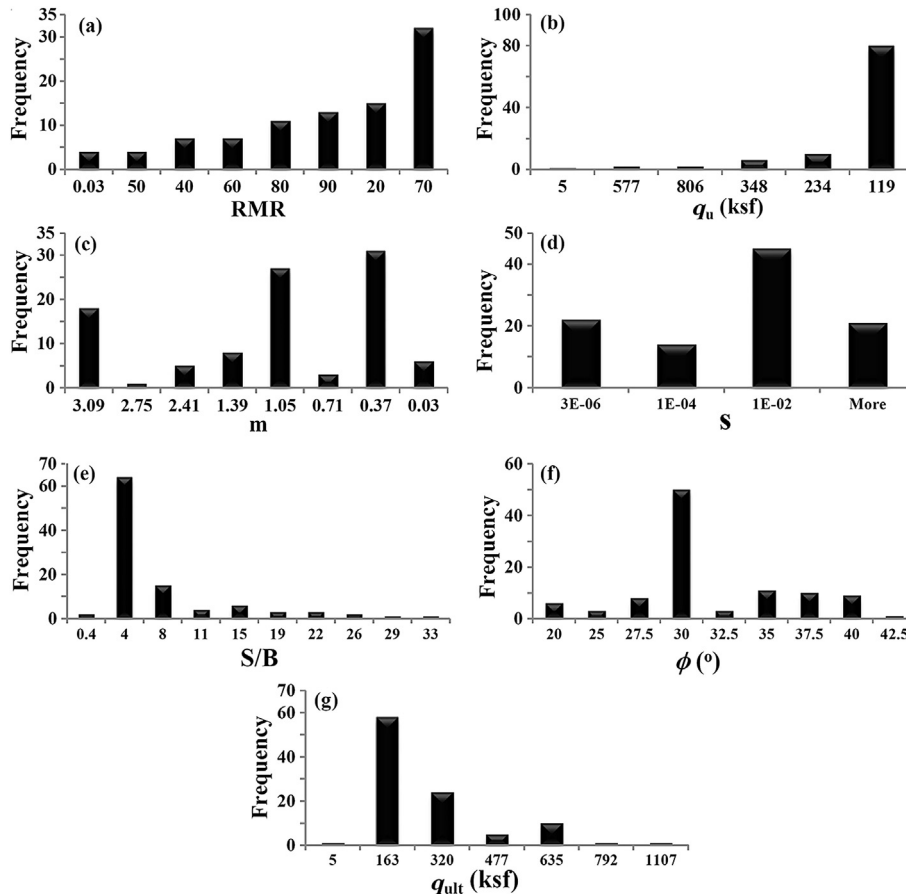


Figure 3. Frequency histograms of the parameters.

Table 4
Statistical criteria used for the evaluation of models.

Performance index	Mathematical definition
Correlation coefficient (R)	$R = \frac{\sum_{i=1}^n (h_i - \bar{h}_i)(t_i - \bar{t}_i)}{\sqrt{\sum_{i=1}^n (h_i - \bar{h}_i)^2 \sum_{i=1}^n (t_i - \bar{t}_i)^2}}$
Root mean square error (RMSE)	$RMSE = \sqrt{\frac{\sum_{i=1}^n (h_i - t_i)^2}{n}}$
Mean absolute error (MAE)	$MAE = \left[\frac{1}{n} \sum_{i=1}^n h_i - t_i \right]$

h_i : The actual outputs for the i th output; t_i : The calculated outputs for the i th output; n : The number of samples; \bar{h}_i : The average of the actual outputs; \bar{t}_i : The average of the calculated outputs.

2005; Alavi et al., 2011). In the present study, 80% of the data sets are taken for the training and validation processes (61 data vectors for the training process and 21 data sets as the validation data). The remaining 20% of the data sets are used for the testing of the obtained models.

4. Development of LGP-based models

Several runs are performed with different combination of the input parameters to obtain the best models for the prediction of q_{ult} . The LGP parameters are changed for each run. The parameters are selected based on both some previously suggested

values (Francone, 1998–2004; Baykasoglu et al., 2008; Alavi and Gandomi, 2011; Alavi et al., 2011) and making several preliminary runs to check the overall performance of the algorithm. The modeling process is controlled via evolutionary parameters such as population size, probability of crossover, probability of mutation and selecting arithmetic operators and mathematical functions. The success of the LGP algorithm usually depends on increasing the initial and maximum program size parameters. Thus, three optimal population sizes of 500, 1000 and 2000 are used for the prediction. Two levels are considered for each probability of crossover and mutation (0.5 and 0.95). During the process, the complexity of evolved functions increases. Hence, to prevent decreasing the speed of algorithms, two optimal values (64, 128) are considered for the maximum program size as trade-offs between the running time and the complexity of the solutions. Two values (10, 20) are set for the number of demes. It is worth mentioning that demes are semi-isolated subpopulations that evolution proceeds faster in them in comparison to a single population of equal size (Brameier and Banzhaf, 2007). Different arithmetic operators and mathematical functions are utilized to formulate the LGP-based models. The termination criterion for each run is considered based on the number of generations that run has gone without improving. The number of generations without improvement is set to 1000. There are $3 \times 2 \times 2 \times 2 = 24$ different combinations of the parameters. All

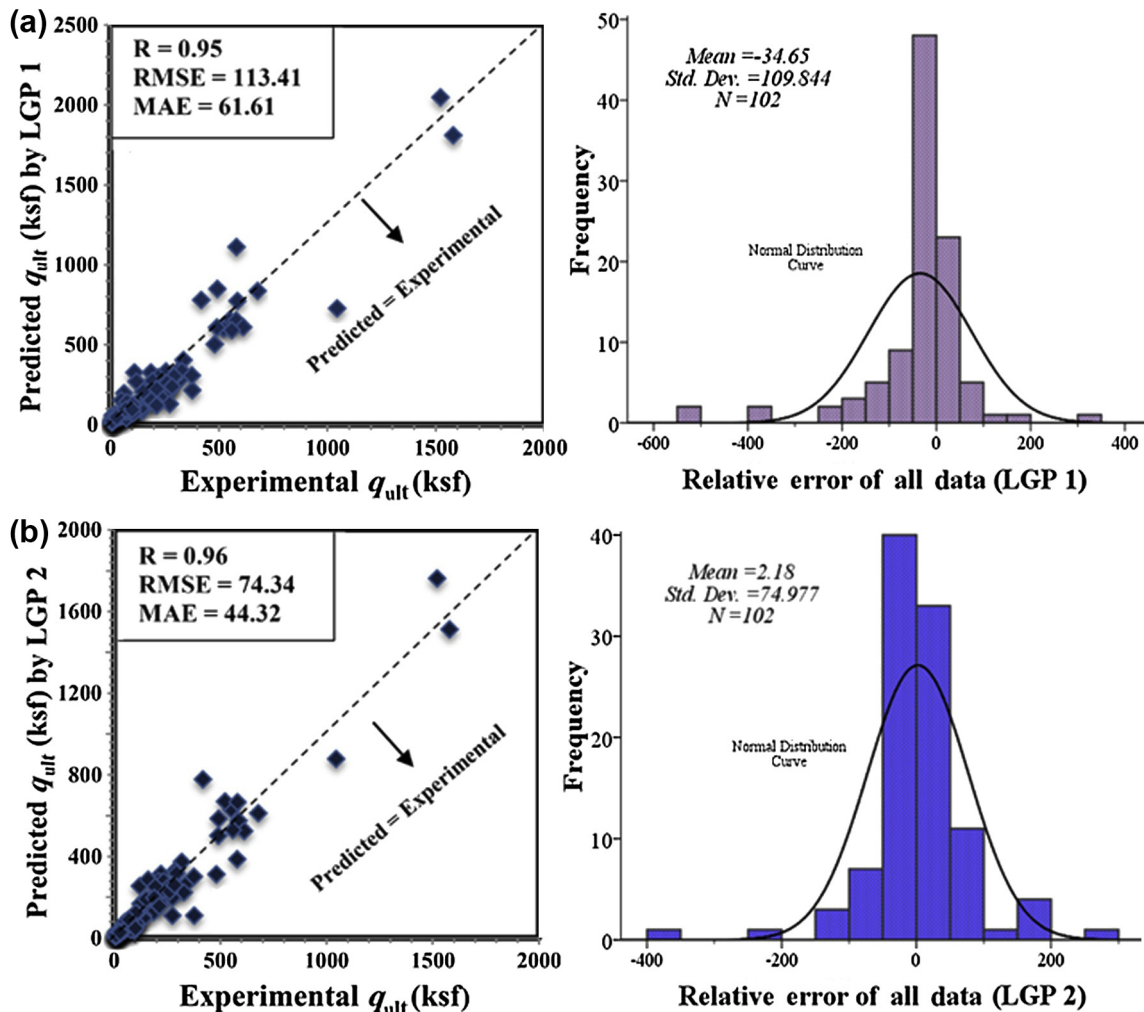


Figure 4. Experimental versus predicted q_{ult} values and frequencies of relative error using the LGP models.

Table 5
Values of statistical criteria for the LGP-based models based on data classification.

Model	Statistical index	Data			
		All	Training	Validation	Testing
LGP 1	R	0.95	0.95	0.96	0.99
	RMSE	113.41	122.56	76.60	115.14
	MAE	61.61	70.97	42.10	53.00
LGP 2	R	0.96	0.97	0.96	0.98
	RMSE	74.64	56.81	119.60	63.72
	MAE	44.32	35.89	68.64	45.62

of these combinations were tested and 10 replications for each combination were carried out. This makes 240 runs for each of the combinations of the predictor variables. To evaluate the fitness of the evolved programs, the average of the squared errors is used. The Discipulus computer program is used to implement the LGP algorithm (Conrads et al., 2004). The parameters used to evaluate the performance of the models are shown in Table 4.

The best LGP programs obtained at the end of the training process are given in C++ in Appendix A. These programs can be

run in any C environment. The resulting code may be linked to the optimizer and compiled or it may be called from the optimization routines (Deschaine, 2000). In order to facilitate the use of the derived codes via hand calculations, they are simplified and converted into functional representations by successive replacements of variables (Oltean and Grosan, 2003). The optimal LGP-based formulations of q_{ult} are as follows with different parameters, namely LGP 1 (Eq. (3)) and LGP 2 (Eq. (4)):

$$q_{ult, LGP1} = 0.45(RMR + 2q_u) + 0.25 \frac{S}{B} (0.35q_u + \phi + (0.5q_u(0.2\phi - 6)^2)) \quad (3)$$

$$q_{ult, LGP2} = \frac{S}{B} + 1.7 \left(\sqrt{\frac{S}{B} q_u \sqrt{mq_u \left(2\frac{S}{B} - \phi + \left(\phi \frac{B}{S} \left(2s\frac{S}{B} + 0.25 \right)^2 \right) \right)}} \right) \quad (4)$$

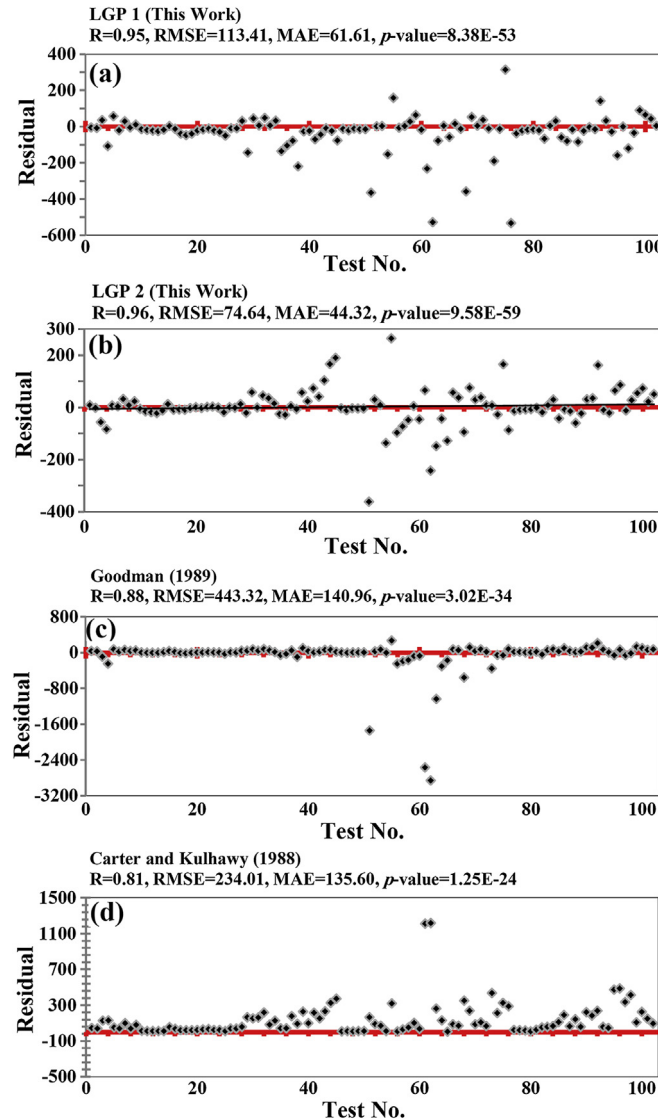


Figure 5. Residual plots of the standard error between observed and predicted q_{ult} values for different models.

Fig. 4 shows a comparison and the relative error between the measured and predicted q_{ult} values by the LGP models for the entire data.

5. Results and discussion

5.1. Performance analysis and validation

Different statistical indices should be considered to evaluate the performance of a numerical correlation. Smith (1986) proposed the following criteria for judging the performance of a model based on several observations:

- if a model gives $|R| > 0.8$, a strong correlation exists and the model is suitable,

In addition, the error values should be considered in all cases (Alavi et al., 2011). The performance measures for the training, validation, testing, and all data are summarized in Table 5. It can be observed from Fig. 4 and Table 5 that the LGP models, with $R > 0.8$ and low RMSE and MAE values, are able to predict the target values with an acceptable degree of accuracy. As it is, the model (LGP 2, Eq. (4)) developed using m , s , q_u , S/B , and ϕ has a better performance that derived using RMR, s , q_u , S/B , and ϕ .

In addition to the performance indices described, the values of the residuals (standard error between the observed and predicted values) are visualized in Fig. 5. In order to check whether the calculated R really exists between the observed and predicted data sets or not, the p -values at 5% significance level were also obtained and presented in this figure. Furthermore, this figure presents a comparative study between the results obtained by proposed models and those provided by the well-known models of Carter and Kulhavy (1988) and Goodman (1989). As can be observed from Fig. 5, the proposed formulas notably outperform the existing models. Besides, it can be seen that the p -values for the LGP models are approximately equal to 0. This indicates that there is a sound correlation between the observed and predicted data sets. It is worth mentioning that most of the existing models are derived based on traditional statistical analyses (e.g. regression analysis). The major limitation of this type of analysis is that the structures of the models are designated after controlling only few equations established in advance. On the other hand, the best equations generated by the LGP technique are determined after controlling numerous linear and nonlinear preliminary models. For instance, the proposed design equation (LGP 1) is selected among a total of 560,892,763 programs evolved and evaluated by the LGP method during the conducted 240 runs.

5.2. Sensitivity analysis

As discussed before, the effect of all considered parameters (i.e., RMR, s , m , q_u , S/B , and ϕ) on q_{ult} is well understood. Herein, a sensitivity analysis is conducted to provide a more in-depth understanding of the contribution of these important parameters to the prediction of q_{ult} . For the sensitivity analysis, the frequency values of the input parameters are obtained. A frequency value equal to 100% for an input indicates that this input variable has been appeared in 100% of the best thirty programs evolved by LGP (Francone, 1998–2004; Alavi et al., 2011; Gandomi et al., 2011a,b). Herein, the average and maximum impact values are also determined. The average and maximum impact values, respectively, show the average and the maximum effect of removing all instances of that input from the thirty best programs of the project. The greater the value, the more impact removal had. The sensitivity analysis results are summarized in Fig. 6. As expected (Paikowsky et al., 2004, 2010), Fig. 6 indicates that both LGP-based models

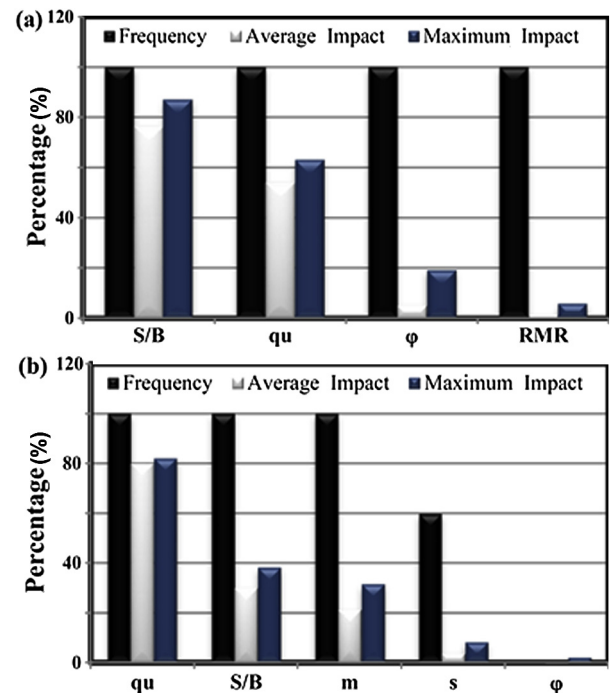


Figure 6. Contributions of the predictor variable in the LGP analysis.

are more sensitive to q_u and S/B compared to other variables. The good agreement between of the results of the LGP sensitivity analysis and those reported by various researchers (Carter and Kulhavy, 1988; Goodman, 1989; Paikowsky et al., 2004, 2010) guarantees the robustness and applicability of proposed models.

6. Conclusion

This paper aimed at developing new nonlinear LGP-based models for the estimation of the q_{ult} of shallow foundations on non-fractured rock masses. A comprehensive database collected from the literature is used for the model development. The optimal LGP-based models are selected after several assessment procedures. The validation of the models is verified with different criteria. The parametric and sensitivity analyses are conducted to evaluate the robustness of the models. The results indicate that the proposed models provide precise estimations of q_{ult} . The R values of LGP 1 model on the training, validation and testing data are equal to 0.95, 0.96 and 0.99, respectively. The corresponding values for LGP 2 model are equal to 0.97, 0.96 and 0.98, respectively. Moreover, the p -values at 5% significance level are approximately equal to 0 for both of the models. Considering the better performance of the second model (LGP 2, Eq. (4)), m seems to be a more efficient parameter than RMR for the prediction of q_{ult} . Both of the derived models have notably better performances than the existing traditional models. In addition, the LGP models simultaneously incorporate the effect of both the quantitative and qualitative parameters. Unlike other intelligent methods such as ANN, FIS and ANFIS, LGP method provides simplified equations that can be readily used for the design purposed via hand calculating. Furthermore, the results of the LGP analysis provide transparent programs, in this case C++ codes, for further use in the bearing capacity prediction, as well as optimization purposes.

Appendix A. Supplementary data

Supplementary data related to this article can be found online at <http://dx.doi.org/10.1016/j.gsf.2014.12.005>.

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