Rate of Phase Difference Change Estimation in Single Airborne Passive Locating System


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Abstract

As an important parameter in the single airborne passive locating system, the rate of phase difference change contains range information of the radio emitter. Taking single carrier sine pulse signals as an example, this article illustrates the principle of passive location through measurement of rates of phase difference change and analyzes the structure of measurement errors. On the basis of the Cramér-Rao lower bound (CRLB), an algorithm associated with time-chips is proposed to determine the rates of phase difference change. In the measurement of the rates of phase difference change, phase discrimination in the frequency domain outperforms that in the time domain when signal noise rate (SNR) is lower. Multi-chip processing can significantly reduce variance of the measurement of rates of phase difference change. Simulations demonstrate the validity and accuracy of the proposed algorithm. The simulations carried out on the typical single airborne passive location have proved its adaptability to dynamic measurements. The proposed algorithm to determine the rates of phase difference change proves simple and easy to implement with less computation workload.

Keywords: location; rate of phase difference change; Cramér-Rao lower bound; phase discrimination; multi-chip processing

1. Introduction

Passive location is a technique that locates the target emitter through receiving its radio wave without emitting any radio signals during operation. It is characterized by long effective distance, electromagnetic silence, and undetectable possibility. It has long been an indispensable part of an integrated air defense system and long distance airborne warning system either on land or on sea. Meanwhile, it is a key means to develop marine, aeronautic, astronomic, detection, tracking, and geographical sciences and technologies.

It is required to acquire parameters carrying the location information of the target emitter from the received signals by means of the kinematics-based passive locating and tracking algorithm[1]. Ref.[2] put forward a method to locate a single station through arriving time and direction of a signal as well as its changing rate. Ref.[3] presented a method on the ground of Doppler rate of change. Refs.[4-7] studied passive locating techniques using rates of phase difference change and analyzed the accuracy in determining the rates of phase difference change. All the above methods focused the attention on the full utilization of the information involved in radio signals about the position of the target emitter so as to quickly and accurately locate the emitter itself. This makes accurate acquisition of parameters one of the key techniques in passive locating system.

Ref.[8] suggested two methods: One is to measure the integer ambiguity during a fixed length of time, and the other is to measure the phase difference of dual-channel signals and obtain the rate of phase difference change through further frequency estimation. The former needs a time length of about 10 s and its accuracy is quite poor. The latter requires high consistency among the dual-channel signals and there exists phase ambiguity. Ref.[5] pointed out that the narrow width of the pulse and the extra-low value of the rate of phase difference change are the main reasons for the
difficulty in measurement, and he suggested to use orthogonal phase shift to transform the received signal into the complex signal and then determine it by maximum likelihood (ML). Although the variance of estimation reaches the Cramér-Rao lower bound (CRLB), it is not practicable to implement owing to the large computational workload. After comparing the existing methods to measure the rates of phase difference change, Ref.[9] pointed out that the basic assumption they were based on is too ideal to be applied in engineering implementation. This article, after analyzing CRLB of rates of phase difference change, develops a multi-chip method based on multiple observations with full exploitation of phase coherence. This method proves to have high accuracy and less computational complexity as well as practical feasibility.

2. Principles of Single Airborne Passive Location

2.1. Principles of single airborne passive location based on rates of phase difference change

When there is a relative motion existing between observation platforms, i.e. the airborne vehicle and the target emitter, the angles of a received signal to be measured are steadily changing variables, which comprise the information about the range from the emitter to the airborne receiver. To simplify the exposition of the principles, the sine law can be used in 2D plane.

Assume that the position of a target emitter is fixed, the airborne observer moves in a straight line at a constant speed, $v$, and the azimuth of the emitted radio signals $\beta$. After a length of time, $dt$, the measured azimuth is $\beta + dt \beta$, and the distance between the emitter and the airborne observer is $R$ (see Fig.1).

From the sine law, the following can be achieved[8]

$$ R = \frac{v dt}{\sin \beta} = \frac{vd t}{\sin d \beta} \tag{1} $$

Eq.(1) can be rewritten as

$$ R = \frac{\sin \beta}{\sin d \beta / dt} = \frac{\sin \beta}{d \beta / dt} \cdot \frac{\sin \beta}{\beta} \tag{2} $$

From Eq.(2), it can be understood that the distance $R$ can be obtained given the speed of the airborne observer $v$, the azimuth of the emitted radio signals $\beta$, and the rate of azimuth change $\dot{\beta}$. Thus, with the distance $R$ and the azimuth $\beta$ known, the location of the target emitter is realized. Here, $\beta$ is the key parameter.

Several methods are available to acquire azimuth $\beta$, the typical one is called the instant locating method. It is a through measurement of the phase difference of the received signal using an interferometer. In this case, the measurement of azimuth is converted into determination of phase differences, and the rate of azimuth change into the rate of phase difference change.

In Fig.2, $E_a$ and $E_b$ are two antennas of the airborne interferometer, $\phi(t)$ is the phase difference of the received signal, then

$$ \phi(t) = \omega_0 \Delta t = \frac{2\pi d}{c} f_T \cos \beta(t) \tag{3} $$

where $\omega_0$ is the angular frequency of the signal, $\Delta t$ the time difference of the signal arriving at both antennas, $d$ the length of the base line of the interferometer, $c$ the light speed, $f_T$ the signal frequency, and $\beta(t)$ the azimuth of the radio wave, it is a time-varying parameter.

$$ \dot{\phi}(t) = \frac{d\phi(t)}{dt} \tag{4} $$

$$ \dot{\beta}(t) = \frac{d\beta(t)}{dt} \tag{5} $$

Let $k = -\frac{2\pi d}{c}$, then Eq.(3) is simplified into

$$ \dot{\phi}(t) = k f_T \sin \beta(t) \cdot \dot{\beta}(t) \tag{6} $$

Obviously, Eq.(5) can easily be transformed into

$$ \dot{\beta}(t) = \frac{\dot{\phi}(t)}{kf_T \sin \beta(t)} \tag{7} $$

Substituting Eq.(6) into Eq.(2), we can obtain

$$ R = \frac{k f_T \sin \beta(t)}{\dot{\phi}(t)} \tag{7} $$

From Eq.(7), it can be concluded that given the informative parameters of the moving airborne observer, the passive location of the target emitter can be accomplished with the measured values of $\beta$, $\phi$, and $f_T$. 

Fig.1 Rate of angle change in terms of geometrics.

Fig.2 From azimuth difference to phase difference.
2.2. Effects of the errors of rates of phase difference change on the range errors

Eq.(7) shows the relationship between $R$ and $\dot{\phi}$. By differentiating Eq.(7) and dividing the result by $R$, we can obtain

$$\frac{\Delta R}{R} = \frac{\Delta \nu}{\nu} + \frac{\Delta f_R}{f_R} + \frac{2 \cos \beta}{\sin \beta} \Delta \beta - \frac{\Delta (\dot{\phi})}{\dot{\phi}} \tag{8}$$

It can be seen from Eq.(8) that the relative error of $R$ is composed of relative errors of velocity, frequency, azimuth, and rate of phase difference change. Generally, the relative errors of frequency and velocity can be removed because it is easy to fix their accurate values. The error of angle measurement is a matter of several milliradians ($0.2^\circ$). In an angle measurement, an error of $0.5^\circ$ will result in a relative error of 0.02 when the azimuth is $45^\circ$. When the observer is 100 km away from the emitter at a relative radial speed of about 200 km/s with a signal frequency of 1 GHz, the phase difference is about 0.4 rad/s. This is liable to be polluted by noise and even an error of 0.004 rad/s will lead to a range error of $10\% R$, which is quite small because an error of 0.004 rad/s will cause a range error of $10\% R$. Consequently, it is clear that the relative error of rates of phase difference change commands an overwhelming position in the whole range error of $R$, which makes the high-precision measurement of $\dot{\phi}$ decisive in passive location.

2.3. Cramér-Rao bound of rates of phase difference change

The rate of phase difference change is, in fact, the frequency of phase difference. Therefore, the determination of the rates of phase difference change can be done with the Cramér-Rao bound of frequency.$^{10}$ The Cramér-Rao bound of single carrier complex signal phase difference$^{11}$ is defined as

$$\text{var}(\dot{\phi}) = \text{var}(\omega) \geq \frac{6}{\text{SNR} \cdot T^2 N (N^2 - 1)} \approx \frac{6}{\text{SNR} \cdot T^2 N} \tag{9}$$

where $T = NT_r$ is the whole observation time, $N$ the sampling number, $T_r$ the sampling interval, and SNR the signal noise ratio. CRLB is obtained when equal mark is applied in Eq.(9). It is seen that the variance of $\dot{\phi}$ changes inversely with SNR, sampling number, and observation time $T$. Especially, $T$ exerts a square exponential influence on the variance of $\dot{\phi}$. Thus, the longer the observation time is, the smaller the variance of $\dot{\phi}$ is and the higher the measuring accuracy is. In practice, this means increasing the observation time each time helps in obtaining a higher accurate rate of phase difference change. This is because in an observation period, the $\phi$ (as well as the phase difference) varies so little that it can be supposed to be constant. Therefore, it may well assume the phase difference in a relatively long period able to mitigate the noise effects. Taking into account the speed limit of the airborne carrier observer platform, it may well be a good solution to combine several pulses together to obtain a more accurate determination of rates of phase difference change.

3. Methods to Measure Rates of Phase Difference Change

3.1. Signal model

Supposing that the signal from the target emitter is a train of sine coherent pulses with a constant frequency defined as

$$s(t) = A_t \cos(2\pi f_0 t + \theta_0) \sum_{n=0}^{\infty} p(t - t_0 - k T_s) \tag{10}$$

where $A_t$ is the pulse amplitude supposed constant, $\theta_0$ the initial phase, $t_0$ the initial time, $T_s$ the pulse repetition interval (PRI) of the rectangular pulse train, and

$$p(t) = \begin{cases} 1 & 0 < t \leq \tau \\ 0 & \text{Others} \end{cases} \tag{11}$$

where pulse width $\tau \ll T_s$.

As signals propagate in the air, the common portion of the propagation delay can be reckoned into the change of initial phase. For example, the single pulse signals received from the two respective antennas of the interferometer after propagation delay are

$$s_1(t) = A_r \cos(2\pi f_0 t + \theta'_r) + u_1(t) \tag{12}$$

$$s_2(t) = A_r \cos[2\pi f_0 (t - \Delta t) + \theta'_r] + u_2(t) \tag{13}$$

where $A_r$ is the received signal amplitude, $\Delta t$ the time difference between the signals arriving at two antennas of the interferometer, $\theta'_r$ the change of the signal initial phase, and $u_1$, $u_2$ indicate Gaussian white noise. After frequency transformation and sampling, the two signals change into intermediate frequency (IF) signals expressed by

$$z_1(n) = A_r \cdot \cos(\omega_n T_s + \phi_0) + v_1(n T_s) \tag{14}$$

$$z_2(n) = A_r \cdot \cos(\omega_n T_s + 2\pi f_0 \Delta t + \phi_0) + v_2(n T_s) \tag{15}$$

where $\omega$ is the intermediate angular frequency, $\phi_0$ the initial phase after down frequency transformation, $2\pi f_0 \Delta t$ the phase difference between the two signals, $n T_s$ the present sampling time, and $v_1(n T_s)$ and $v_2(n T_s)$ are Gaussian random sequences with a zero mean and variance of $\sigma^2$.

3.2. Methods to measure phase difference

(1) Phase discrimination in the time domain

As the duration of a pulse is very short, it may be
well supposed that the phase difference of a pulse remains unchanged. The algorithm of phase discrimination for a single carrier pulse signal in the time domain is introduced as follows.

By transforming the signal received from the two antennas of the interferometer, the acquired analytic results are

\[ s_1(t) = A \cdot \exp\{j(2\pi ft + \phi_0)\} \]  
\[ s_2(t) = A \cdot \exp\{j(2\pi f(t - \Delta t) + \phi_0)\} \]

To perform correlation computation in the time domain, after multiplying \( s_1(t) \) by \( s_2(t) \) with conjugate transformation and averaging the result, we can achieve

\[ \Phi = E[s_1(t)s_2^*(t)] = A^2 \exp(j2\pi f \Delta t) \]

Conducting averaging phase difference in one pulse leads to significant restraint of white noise if the phase difference remains the same in one pulse.

2) Phase discrimination in the frequency domain

It is well known that the convolution computation in the time domain is tantamount to multiplication in the frequency domain, and convolution is actually the same as the correlation computation, therefore, phase difference can be obtained by calculating the correlation spectroscopy in the frequency domain.

Firstly, the two received signals are transformed into those in the frequency domain by way of Fourier transformation:

\[ s_1(t) \rightarrow F \rightarrow S(\omega) \]  
\[ s_2(t) \rightarrow F \rightarrow S(\omega) \cdot \exp(-j2\pi f \Delta t) \]

The computation of the correlation spectrum is

\[ Y(\omega) = S(\omega) \cdot \left[ S(\omega) \cdot \exp(-j2\pi f \Delta t) \right]^* = \left| S(\omega) \right|^2 \cdot \exp(j2\pi f \Delta t) \]

The position of the peak in the correlation spectrum is the carrier frequency and its phase is just the phase difference of the two signals:

\[ \phi = 2\pi f \Delta t = \text{angle}\left( Y(\omega) \right|_{\omega = 2\pi f} \)\]

where \( \text{angle}(\bullet) \) is the function to acquire angle. Using both methods in Section 3.2 and Section 3.3, a phase difference can be obtained from a pair of pulses, and a train of phase differences can be obtained from two trains of coherent pulses.

3.3. Time-chip-based processing method to determine rates of phase difference change

1) Single time-chip processing

From the above discussion, it is known that as each pair of pulses yields one phase difference, a train of phase differences can be obtained in one observation, which is termed a time-chip.

Since phase ambiguity is inevitable, the phase difference is also similar. Since the ambiguity is limited to be in one time chip, only an augment of \( n\pi \) times is needed, where \( n \) is an integer.

The rate of phase difference change can be obtained through determination of the slope of the train of phase difference with linear fitting after removal of ambiguity.

\[ \phi(n) = a + bn \]

where \( b \) is the estimate of rate of phase difference change \( \dot{\phi} \).

In Fig.3, the real line represents the phase differences before removal of ambiguity, while the broken line indicates those after its removal.

Fig.3 A train of phase differences in one time-chip.

(2) Multi-chip processing method

In practices, the hardware capability inclusive of storage capacity imposes limitation upon the length of a time-chip. According to the conclusion from the analysis of CRLB in Section 2.3, it is not easy to make an accurate determination of rates of phase difference change during a short observation time or at lower SNRs. Eq.(9) shows that the variance of measurement changes inversely with the square of the signal duration. In order to obtain a highly accurate determination, two separate observations can be combined to prolong the observation time (see Fig.4).

Fig.4 Multi-chip processing method.

A train of phase differences is obtained by correlation computation of two corresponding trains of pulse signals.

\[ \Delta \phi(i) = \phi_{N+1}(i) - \phi_N(i) \]

where \( i \) is an integer which depicts the pulse index in the train. Dividing Eq.(24) by the chip interval and averaging the results, the rate of phase difference change can be found by

\[ \dot{\phi} = \frac{\sum_i \Delta \phi(i)}{\Delta t(i)} \]

Thus, an estimation of the rate of phase difference change is obtained by two adjacent time-chips. This is
very effectual especially in the case of passive locating system because the phase difference varies very slightly in a time-chip. However, it should be noted that this is true merely under the assumption that the rate of phase difference change remains constant in two adjacent chips, which will cause some deviation in the measurement. In other words, the measured rate of phase difference change is no longer in an exact correspondence to the time point. In practices, the time can be supposed to be the middle of two time-chips, which will be demonstrated to be viable later in the ensuing simulation. In addition, application of linear interpolation can further reduce the deviation.

4. Numerical Results and Discussions

In the following, simulation as well as analysis of the proposed algorithms will be carried out by taking single-carrier pulse signal as an instance.

4.1. Simulation on phase difference measurement

This simulation is meant to verify the phase difference measurement methods and compare the discriminating performances in the frequency and the time domains. Table 1 lists the parameter setting.

<table>
<thead>
<tr>
<th>Table 1 Simulation parameters</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Pulse width /μs</td>
</tr>
<tr>
<td>Intermediate frequency/Hz</td>
</tr>
<tr>
<td>Sampling frequency/Hz</td>
</tr>
<tr>
<td>Ideal phase difference/°</td>
</tr>
<tr>
<td>SNR/°</td>
</tr>
<tr>
<td>Monte Carlo number</td>
</tr>
</tbody>
</table>

The simulation results are presented in Fig.5 and Fig.6, in which “Method 1” refers to the phase difference measurement with phase discrimination in the time domain and “Method 2” refers to that in the frequency domain.

From Fig.5 and Fig.6, it is clear that Method 2 outperforms Method 1 when SNR is lower. For instance, in Fig.6, when SNR = -10 dB, the relative error of Method 2 reaches 30%, while that of Method 1 exceeds 100%, which indicates that the estimation is of no avail.

Both methods have similar performances when the SNR is above zero. Their relative errors are lower than 10% and their averaged variances are lower than ~20 dB. In particular, when the SNR equals 20 dB, their relative errors attain 1% and the mean variances are lower than ~40 dB, an equivalent of root mean square (RMS) lower than 0.01 rad.

It can be concluded from the simulation that phase discrimination of Method 2 is superior to that of Method 1 at lower SNR.

4.2. Simulation on measurements of rates of phase difference change

This simulation aims to verify the proposed algorithm to determine the rates of phase difference change. Fig.7 and Fig.8 depict the comparison of the results acquired by the two methods. Notice: herein ‘Method 1’ refers to the measurement of rates of phase difference change with phase discrimination in the time domain while “Method 2” refers to that in the frequency domain. In simulation, the parameter setting is the same as in Section 4.1 with application of single chip processing.

Fig.7 shows an analogue among measurement variance curves of rates of phase difference change acquired by both methods comparing to CRLB. This demonstrates the efficacy of the two algorithms. Fig.7 shows the results from the two methods having a measurement variance of about 5 dB higher than CRLB. This is because the average rate of phase difference change in one pulse sacrifices the accuracy to some extent for a better SNR gain, and this proves practical and acceptable in implementation.

The curves in Fig.8 show that the relative errors from both methods reach 10% when the SNR is 10 dB, whereas they are below 1% when the SNR is 30 dB.
Still, it can be concluded that Method 1 performs better at low SNR.

Fig.7 Variances of rates of phase difference change.

Fig.8 Relative errors of rates of phase difference change.

4.3. Multi-chip-based processing

Assume that the radio emitter lies at \( E (0, 0) \) and the airborne observation platform flies from \( A (-100 \text{ km}, 100 \text{ km}) \) to \( B (100 \text{ km}, 100 \text{ km}) \) along \( x \) axis at a constant speed of 200 m/s (see Fig.9).

Fig.9 Simulated condition.

Table 2 lists the parameter setting. The simulation takes only the first 100 chips into consideration. Figs.10-13 illustrate the results from the simulation. It is clear from Figs.10-13 that the measurement variance decreases as the chip spacing increases in processing. Here, chip spacing indicates the interval of two chips combined in processing. Table 3 lists the RMS of Figs.10-13, which accords with the analysis in Section 2.3.

As seen in Table 3, the RMS of rates of phase difference change decreases as the process spacing increases. The multi-chip processing proves effectual in determination of rates of phase difference change. As a result, the deviation caused by multi-chip processing in

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Carrier frequency/GHz</td>
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</tr>
<tr>
<td>Intermediate frequency/MHz</td>
<td>20</td>
</tr>
<tr>
<td>Sampling frequency/MHz</td>
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</tr>
<tr>
<td>Pulse width/( \mu )s</td>
<td>10</td>
</tr>
<tr>
<td>Ideal phase difference/rad</td>
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<tr>
<td>Pulse repetition intervals/ms</td>
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</tr>
<tr>
<td>SNR/dB</td>
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</tr>
<tr>
<td>Pulse number per chip</td>
<td>100</td>
</tr>
<tr>
<td>Chip spacing/s</td>
<td>1</td>
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Table 3 RMS of rate of phase difference change

<table>
<thead>
<tr>
<th>Chip spacing</th>
<th>0.1 s</th>
<th>1 s</th>
<th>2 s</th>
<th>3 s</th>
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<tbody>
<tr>
<td>RMS</td>
<td>0.01490</td>
<td>0.00080</td>
<td>0.00050</td>
<td>0.00025</td>
</tr>
</tbody>
</table>

Fig.10 Rates of phase difference change by one single chip processing.

Fig.11 Rates of phase difference change by two adjacent chip processing.
Section 3.3 can be ignored when chip spacing is at a lower level. It has been proved that multi-observation is efficacious especially when each observing interval is very short. It can also overcome the problem caused by the limited storage of airborne equipments. In addition, the accuracy can be improved further with linear interpolation.

5. Conclusions

Passive location can be realized with the help of rates of phase difference change and angle information of the emitted signal by a target emitter. This article centers attention on the measurement of rates of phase difference change of sine pulse signals. Comparison of the results from the method of discrimination in the frequency domain with those from the method in the time domain shows that the former outperforms the latter thanks to the noise mitigation of Fourier transforms. The simulation proves the efficacy of the proposed algorithm based on multi-chips. Under the condition of limited speed of airborne vehicle, the increase of spacing between two co-processing chips can reduce the variance of rates of phase difference change. Besides, the proposed method is easy to use without performing complex calculation.

References


Biography:

Wang Junhu  Born in 1974, he received B.S. from Academy of Equipment Command and Technology in 1998 and M.S. from National University of Defense Technology in 2004. His main research interests lie in passive location and wireless communication techniques.  
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