Chordwise bending vibration analysis of functionally graded beams
with concentrated mass

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Abstract

The natural frequencies of a rotating functionally graded cantilever beam with concentrated mass are studied in this paper. The beam made of a functionally graded material (FGM) consisting of metal and ceramic is considered for the study. The material properties of the FGM beam symmetrically vary continuously in thickness direction from core at mid section to the outer surfaces according to a power-law form. The equations of motion are derived from a modeling method which employs Rayleigh-Ritz method to estimate the natural frequencies of the beam. Dirac delta function is used to model the concentrated mass in to the system. The influence of the material variation, tip mass and its location on the natural frequencies of vibration of the functionally graded beam is investigated.

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Keywords: Functionally graded beam; Rotating beam; Chordwise vibration; Concentrated mass; Natural frequency.

1. Introduction

Functionally graded material is a type of materials whose thermo mechanical properties have continuous and smooth spatial variation due to continuous change in morphology, composition, and crystal structure in one or more suitable directions. The concept of FGMs is originated in Japan in 1984 during space-plane project to develop heat-resistant materials. In these materials, due to smooth and continuous variation in material properties, noticeable advantages over homogeneous and layered materials i.e., better fatigue life, no stress concentration, lower thermal stresses, attenuation of stress waves etc., can be attained. FGMs are considered as one of the
strategic materials in aero space, automobile, aircraft, defense industries and recently in biomedical and electronics sectors. Since, FGMs are used in prominent applications in various sectors, the dynamic behavior is important.

<table>
<thead>
<tr>
<th>Nomenclature</th>
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<tbody>
<tr>
<td>$\vec{a}^p$</td>
</tr>
<tr>
<td>$A$</td>
</tr>
<tr>
<td>$b$</td>
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<tr>
<td>$E_{(s)}$</td>
</tr>
<tr>
<td>$h$</td>
</tr>
<tr>
<td>$i, j, k$</td>
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<tr>
<td>$f_{11}^E$</td>
</tr>
<tr>
<td>$f_{11}^p$</td>
</tr>
<tr>
<td>$f_{22}^{Ez}$</td>
</tr>
<tr>
<td>$L$</td>
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<tr>
<td>$n$</td>
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<tr>
<td>$P$</td>
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<tr>
<td>$\rho_{(c)}$</td>
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<tr>
<td>$\rho_{(m)}$</td>
</tr>
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<td>$\rho_{(c)}$</td>
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<tr>
<td>$q_{1i}$, $q_{2i}$</td>
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<td>$r$</td>
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<tr>
<td>$v^p$</td>
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<td>$x$</td>
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<tr>
<td>$\gamma$</td>
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<tr>
<td>$\delta$</td>
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<tr>
<td>$\Theta$</td>
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<tr>
<td>$\mu_1, \mu_2$</td>
</tr>
<tr>
<td>$\tau$</td>
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<tr>
<td>$\phi_{ji}, \phi_{2j}$</td>
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<td>$(\cdot')$</td>
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Hoa [1] investigated frequency of rotating uniform beam with mass located at tip. A third order polynomial was used for estimating the lateral displacements. Results show that, tip mass decreases (dishearten) the frequencies at lower angular speeds and increases at higher speeds. Hamilton principle was used to formulate the equations of motion for a rotating beam with mass at the tip and results were compared with the results obtained by various methods in [2]. Shifu et al.[3] and Xiao et al.[4] developed a non-linear dynamic model and its linearization characteristic equations of a cantilever beam with tip mass in the centrifugal field by using the general Hamilton variational principle. Yaman [5] investigated theoretically the dynamic behavior of cantilever beam which is partially covered by damping and constraining layers with a concentrated mass at the free end and found that the resonant frequencies and loss factors are strongly dependent on geometrical and physical properties of the constrained layers and mass ratio. Yoo et al. [6] presented free vibration analysis of a homogeneous rotating beam. Piovan, Sampaio [7]developed a nonlinear beam model to study the influence of graded properties on the damping
effect and geometric stiffening of a rotating beam. Structures like rotating beams are often used in many engineering applications like turbine blades, helicopter wings, etc. These structures with tip mass are having greater significance in many engineering applications to improve the performance of the components. Tip mass helps to increase the airflow, to modify the vibration frequency of the components, to increase the flexing motion of the wind turbine blade and helicopter rotor, auto cooling as in case of turbine blades, airplane wings, missile fans and like other two dimensional structures. The degree of (greater or lesser) importance of dynamic behavior (spanwise and chordwise) depends on the nature of the transverse loads, geometry of the component and boundary conditions.

The objective of this paper is to derive the governing equation of motion using Lagrange’s equation for chordwise vibration of a rotating functionally graded beam with concentrated mass and investigating the effect of power law index, concentrated mass, its location and hub radius ratio on the chordwise bending natural frequencies of functionally graded rotating beams.

2. Functionally graded beam

Consider a functionally graded beam with length L, width b and total thickness h and composed of a metallic core and ceramic surfaces as shown in Figure 1. The graded material properties vary symmetrical along thickness direction from core towards surface according to power law:

\[ \varphi (z) = \varphi (m) + \left[ \varphi (c) - \varphi (m) \right] \left( \frac{2x}{h} \right)^n \]  

where \( m \) is a variable which have values greater than or equal to zero and the variation in properties of the beam depends on its magnitude. Structure is constructed with functionally graded material with ceramic rich at top and bottom surfaces (at \( z = +h/2 \) and \( -h/2 \)) with protecting metallic core (at \( z = 0 \)).

2. Equation of motion

For the problem considered in this study, the equations of motion are obtained under the following assumptions. The material properties vary only along the thickness direction according to power law, the neutral and centroidal axes in the cross section of the rotating beam coincide so that effects due to eccentricity, torsion are not considered and cross section of the beam is uniform along its length. Shear and rotary inertia effects of the beam are neglected due to slender shape of the beam.

Figure 2 shows the deformation of the neutral axis of a beam fixed to a rigid hub rotating about the axis z. No external force acts on the FG beam and the beam is attached to a rigid hub which rotates with constant angular speed. A concentrated mass, m is located at an arbitrary position of the neutral axis of the beam at a distance d from the rigid hub as shown. The rotation of the beam is characterized by means of a prescribed rotation \( \Omega (t) \) around the z-axis. The position of a generic point on the neutral axis of the FG beam before deformation located at
changes to \( P \) after deformation and its elastic deformation is denoted as \( \dot{d} \) that has three components in three dimensional spaces. Conventionally the ordinary differential equations of motion are derived by approximating the two Cartesian variables, \( u \) and \( v \). In the present work, a hybrid set of Cartesian variable \( v \) and a non Cartesian variables are approximated by spatial functions and corresponding coordinates are employed to derive the equations of motion.

![Configuration of the functionally graded rotating beam.](image)

3.1 Approximation of deformation variables

By employing the Rayleigh-Ritz assumed mode method, the deformation variables are approximated as

\[
s(x,t) = \sum_{j=1}^{\mu_1} \phi_{1j}(x)q_{1j}(t)
\]

\[
v(x,t) = \sum_{j=1}^{\mu_2} \phi_{2j}(x)q_{2j}(t)
\]

In the above equations, \( \phi_{1j} \) and \( \phi_{2j} \) are the assumed modal functions for \( s \) and \( v \) respectively. Any compact set of functions which satisfy the essential boundary conditions of the cantilever beam can be used as the test functions. The \( q_{ij} \)s are the generalized coordinates and \( \mu_1 \) and \( \mu_2 \) are the number of assumed modes used for \( s \) and \( v \) respectively. The total number of modes, \( \mu \), equal to the sum of individual modes i.e., \( \mu = \mu_1 + \mu_2 \).

The geometric relation between the arc length stretch \( s \) and Cartesian variables \( u \) and \( v \) given in [6] as

\[
s = u + \frac{1}{2} \int_0^s \left| \left( \frac{dv}{ds} \right)^2 \right| d\sigma
\]

\[
u = s - \frac{1}{2} \int_0^s \left| \left( \frac{dv}{ds} \right)^2 \right| d\sigma
\]

Where a symbol with a prime (') represents the partial derivative of the symbol with respect to the integral domain variable.

3.2 Kinetic energy of the system

The velocity of a generic point \( P \) can be obtained as

\[
\vec{v}^p = \vec{v}^o + \frac{A\vec{d}_p}{dt} + \vec{\omega}^A \times \vec{p}
\]

Where \( \vec{v}^o \) is the velocity of point O that is a reference point identifying a point fixed in the rigid frame A; \( \vec{\omega}^o \) vector \( \vec{P} \) in the reference frame A and the terms \( \vec{P}, \vec{v}^o \) and \( \vec{\omega}^A \) can be expressed as follows

\[
\vec{p} = (x+u)\hat{i} + v\hat{j};
\]

\[
\vec{v}^o = r\Omega\hat{j};
\]

\[
\vec{\omega}^A = \Omega\hat{k};
\]
\( \dot{v}^p = (\dot{u} - \Omega \dot{\omega}) + [\dot{v} + \Omega (r + x + u)] J \)  
(10)

Where \( i, j \) and \( \hat{k} \) are orthogonal unit vectors fixed in \( A \), and \( r \) is the distance from the axis of rotation to point \( O \) (i.e., radius of the rigid frame) and \( \Omega \) is the angular speed of the rigid frame. Using the Eq. (6), the kinetic energy of the rotating beam is derived as

\[
T = \frac{1}{2} \int_{V} J_1^p \ddot{v} dv
\]
(11)

Where

\[
J_1^p = \int_{A} \rho(z) dA
\]
(12)

In which, \( A \) is the cross section, \( J_1^p \) and \( \rho(z) \) are the mass density per unit length and mass density of the functionally graded beam respectively, \( V \) is the volume.

3.3 Strain energy of the system

Based on the assumptions given in section 3, the total elastic strain energy of a functionally graded beam can be written as

\[
U = \frac{1}{2} J_{11} \int L \left( \frac{d\sigma_{ii}}{dx} \right)^2 dx + \frac{1}{2} J_{22} \int_{22} \left( \frac{d^2 \sigma_{zz}}{dx^2} \right)^2 dx
\]

\[ \quad (i = 1, 2, \ldots, \mu) \]  
(13)

Where

\[
J_{11} = \int_{A} E(z) dA
\]
(14)

and \( J_{22} = \int_{A} E(z) y^2 dA \)
(15)

3.4 Equation of motion

Using the Eqs. (2) and (3) in to Eqs. (11) and (13), the using Lagrange’s equation for free vibration of distributed parameter system can be obtained as

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = 0 \]

\[ \quad i = 1, 2, 3, \ldots, \mu \]  
(16)

The linearized equations of motion can be obtained as follows

\[
\sum_{j=1}^{\mu} \left[ J_{11j}^L \varphi_{1j} \dot{\varphi}_{1j} dx \right] \ddot{q}_{1j} - 2\Omega \left[ J_{11j}^p \varphi_{1j2j} \dot{\varphi}_{1j2j} dx \right] \dot{q}_{1j} - \Omega^2 \left[ J_{11j}^p \varphi_{1j1j} dx \right] q_{1j}
\]

\[ + \left( J_{22j}^L \varphi_{1j} \dot{\varphi}_{1j} dx \right) q_{1j} = \Omega^2 \left[ J_{22j}^p \varphi_{1j1j} dx \right] q_{1j} \]

\[ \quad + \Omega^2 \left[ J_{22j}^L \varphi_{1j} \dot{\varphi}_{1j} dx \right] q_{1j} = \Omega^2 \left[ J_{22j}^p \varphi_{1j1j} dx \right] q_{1j} \]

\[ \quad + \Omega^2 \left[ J_{22j}^L \varphi_{1j} \dot{\varphi}_{1j} dx \right] q_{1j} = \Omega^2 \left[ J_{22j}^p \varphi_{1j1j} dx \right] q_{1j} \]

\[ \quad i = 1, 2, \ldots, \mu \]  
(17)

Where a symbol with double prime ("") represents the second derivative of the symbol with respect to the integral domain variable. Dirac’s delta function was considered to express the mass per unit length of the beam for an
arbitrary location of the concentrated mass of the beam.
\[ \rho^*_e(x) = \rho_e(x) + m\delta(x-d) \]  \hspace{1cm} (19)

Where \( \rho^*_e(x) \) and \( \rho_e(x) \) are modified mass per unit length and mass per unit length of the functionally graded beam respectively.

3.5 Dimensionless transformation

For the analysis, the equations in dimensionless form may be obtained by substituting Eq.(19) in to Eq.(17) and Eq.(18) and introducing following dimensionless variables in the equations.

\[ \tau = \frac{t}{T}, \]  \hspace{1cm} (20)
\[ \xi = \frac{x}{L}, \]  \hspace{1cm} (21)
\[ \theta_j = \frac{q_j}{L}, \]  \hspace{1cm} (22)
\[ \delta = \frac{r}{L}, \]  \hspace{1cm} (23)
\[ \gamma = T\Omega, \]  \hspace{1cm} (24)
\[ \beta = \frac{d}{L}, \]  \hspace{1cm} (25)
\[ \alpha = -\frac{m}{\rho_e(x)L} \]  \hspace{1cm} (26)

Where \( \tau, \delta, \alpha, \beta \) and \( \gamma \) refers to dimensionless time, hub radius ratio, concentrated mass ratio, concentrated mass location ratio and dimension less angular speed respectively.

4. Analysis of chordwise bending natural frequencies

The Eq.(18) governs the chordwise bending vibration of the functionally graded rotating beam which is coupled with the Eq. (17) . With the assumption that the first stretching natural frequency of an Euler beam far separated from the first natural frequency, the coupling terms involved in Eq. (18) are assumed to be negligible and ignored.

The equation can be modified as

\[ \sum_{j=1}^{\mu} \left[ J_{11}^p \int \phi_2 \phi_j dx + m \phi_2 \phi_j dx \right] \dot{q}_j - \Omega^2 \left[ J_{11}^p \int \phi_2 \phi_2 dx + m \phi_2 \phi_2 dx \right] q_j \]
\[ + \Omega^2 \left[ r J_{11}^p \int (L-x) \phi_2 \phi_2 dx + J_{11}^p \int \frac{1}{2} (L^2 - x^2) \phi_2 \phi_2 dx + m(r+d) \int \phi_2 \phi_2 dx \right] q_j = 0 \]  \hspace{1cm} (27)

The Eq. (27) involves the parameters \( L, \Omega, x \) and \( E(z), \) \( \rho(z), \) which are the properties may vary arbitrarily along the transverse direction of the beam. After introducing the dimensionless variable from Eqs. (20-26) in Eq. (27), the equation becomes

\[ \sum_{j=1}^{\mu} \left[ \int_0^L \psi_{ai} \psi_{bj} d\xi + \alpha \psi_{ai} (\beta) \psi_{bj} (\beta) \right] \ddot{q}_j + \left[ \int_0^L \psi_{ai} \psi_{ai} '' d\xi \right] \dot{q}_j - \gamma^2 \left[ \int_0^L \psi_{ai} \psi_{bj} d\xi + \alpha \psi_{ai} (\beta) \psi_{bj} (\beta) \right] \theta_j \]
\[ + \gamma^2 \left[ \int_0^L \delta (1-\xi^2) \psi_{ai} \psi_{aj} d\xi + \int_0^L (1-\xi^2) \psi_{ai} \psi_{aj} d\xi + \alpha (\beta + \delta) \psi_{ai} \psi_{aj} d\xi \right] \theta_j = 0 \]  \hspace{1cm} (28)
Where

$$T = \left( \frac{J^{p}L^{k}}{J^{p}L^{k}} \right)^{\frac{1}{2}} \tag{29}$$

Eq. (28) can be written as

$$\sum_{j=1}^{N} \left[ M^{22}_{j} \theta_{j} + K^{B}_{j} \theta_{j} + \gamma^{2} \left( K^{G2}_{j} - M^{22}_{j} \right) \theta_{j} \right] = 0 \tag{30}$$

$$K^{B}_{ij} = \frac{1}{\delta} \left( 1 - \zeta^{2} \right) \psi_{ai} \psi_{bj} d\zeta + \frac{1}{2} \left( 1 - \zeta^{2} \right) \psi_{ai} \psi_{bj} d\zeta + \alpha(\beta + \delta) \int_{0}^{\beta} \psi_{ai} \psi_{bj} d\zeta \tag{33}$$

Where \( \psi_{ai} \) is a function of \( \zeta \) has the same functional value of \( x \).

From Eq. (30), an eigenvalue problem can be derived by assuming that \( \theta' \)'s are harmonic functions of \( \tau \) expressed as

$$\theta = e^{i\omega \tau} \Theta \tag{34}$$

Where \( j \) is the imaginary number, \( \omega \) is the ratio of the chordwise bending natural frequency to the reference frequency, and \( \Theta \) is a constant column matrix characterizing the deflection shape for synchronous motion and this yields

$$\omega^{2} M \Theta = K^{C} \Theta \tag{35}$$

Where \( M \) is mass matrix and \( K^{C} \) stiffness matrix which consists of elements are defined as

$$M_{ij} = M^{22}_{ij} \tag{36}$$

$$K^{C}_{ij} = K^{B2}_{ij} + \gamma^{2} \left( K^{G2}_{ij} - M^{22}_{ij} \right) \tag{37}$$

5. Numerical results and discussion

Table 1. Properties of metallic (Steel) and ceramic (Alumina) materials.

<table>
<thead>
<tr>
<th>Properties of materials</th>
<th>Steel</th>
<th>Alumina (Al₂O₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus E (Gpa)</td>
<td>214.0</td>
<td>390.0</td>
</tr>
<tr>
<td>Material density ( \rho ) (kg/m³)</td>
<td>7800.0</td>
<td>3200.0</td>
</tr>
</tbody>
</table>

Table 2. Comparison of natural frequencies of a metallic (Steel) cantilever beam (Hz).

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>96.9</td>
<td>96.9</td>
<td>97.0</td>
</tr>
<tr>
<td>607.3</td>
<td>607.6</td>
<td>610.0</td>
</tr>
<tr>
<td>1700.4</td>
<td>1699.0</td>
<td>1693.0</td>
</tr>
</tbody>
</table>

Initially, the accuracy of the present modeling method is validated by considering two examples as follows. In Table 2, the first three chordwise natural frequencies of the functionally graded beam without concentrated mass, are presented as a first example by using the present modeling method and are compared with the works of Piovan and Sampio [7] on FGM rotating beam without concentrated mass, that provides analytical solution of a classic model and experimental data. The metallic beam with power law index, \( n \to \infty \) (i.e., steel) having geometrical dimensions breadth = 22.12 mm, height = 2.66 mm and length = 152.40 mm is modeled with the material properties given in the table 1. At zero rotational speed, with clamped-free end (clamped at \( x = 0 \) and free at \( x = L \)) boundary conditions, the chordwise bending frequencies are calculated with ten assumed modes to obtain the three lowest natural frequencies.
Fig 3. Chordwise natural frequency variation of non rotating FG beam without concentrated mass. As a second example, a functionally graded non rotating beam without concentrated mass with dimensions length $L = 1000$ mm, breadth $= 20$ mm and height $= 10$ mm is considered for the analysis. Steel is considered as metallic constituent and Alumina as ceramic constituent whose mechanical properties are given in table 1. In Figure 3, the variation of the lowest three chordwise bending natural frequencies of a functionally graded beam with respect to variation in power law index, $n$ is presented. It has been observed that the three frequencies decrease with an increase in power law index up to a critical $n$ value, after which the frequencies are relatively un-effected by increase in $n$ value. The results of these examples are comparable with experimental results presented in the work of Piovan and Sampio [7] and are observed to be within 0.5 percent error. From the above examples it may be concluded that the present modeling method is appropriate for further evaluation. Keeping this in view rigorous analysis has been carried out as detailed below. For further analysis the beam parameters considered are dimensions length $L = 1000$ mm, breadth $= 20$ mm and height $= 10$ mm as in example two.

Table 3. Comparison of the first chordwise bending natural frequencies at $n = 0, 1, 2$ and $\delta = 0.0, 0.5, 2.0$

<table>
<thead>
<tr>
<th>$N$ (rps)</th>
<th>$\delta$</th>
<th>First chordwise bending natural frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>35.45</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>35.49</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>35.63</td>
</tr>
<tr>
<td>25</td>
<td>0.0</td>
<td>37.07</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>43.16</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>57.61</td>
</tr>
<tr>
<td>50</td>
<td>0.0</td>
<td>41.07</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>60.08</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>96.59</td>
</tr>
</tbody>
</table>

In Table 3, the fundamental chordwise bending natural frequencies obtained using present modeling method at various values of power law index and hub radius ratio are presented.

Figure 4 shows the variation in fundamental chordwise bending natural frequencies for different magnitudes of concentrated mass at different locations of the functionally graded rotating cantilever beam. In general it has been observed that the first chordwise bending natural frequencies initially increase with an increase in mass ratio $\alpha$. There after decreasing trend has been observed for all values of power law index. It is pertinent to add that the difference in frequencies with an increase in mass ratio increases with an increase in power law index. However, it may be noted that the initial trends change at higher values of location of concentrated mass, in that, with an increase in concentrated mass there is a reduction in frequencies resulting crossover of frequencies. At some stage,
the frequency of beam containing heavier concentrated mass is lower than that of lighter concentrated mass. From the above one can infer that as the beam composition changes from ceramic to metal the effect of concentrated mass on frequencies decreases. In second and third frequencies the variation in frequencies with respect to location of concentrated mass has been observed to be wavy (Figure 4).

Figure 5 shows the effect of locator of concentrated mass on chordwise bending natural frequencies with respect to angular speed. It has been observed that, the rate of increase in chordwise bending natural frequencies with respect to angular speed is different for different locations of the concentrated mass in different modes. The order of rate of increase of frequency for different locations of concentrated mass is in the order $0.3 > 0.0 > 0.6 > 0.9$ for $1^{st}$ natural frequency. While for the second frequency the order is $0.9 > 0.0 > 0.6 > 0.3$ and for the third frequency the order is $0.9 > 0.6 > 0.0 > 0.3$. These observed relations between chordwise bending natural frequency and angular speed are related that observed in dependence of chordwise bending natural frequency on the location of concentrated mass as Figure 4.

The influence of location of concentrated mass on the relation between chordwise bending natural frequency and angular speed is presented in Figure 6. In general, it is observed that, the chordwise bending natural frequencies decreases as the location of the mass shift towards free end. However, this effect is marginal when the mass is located nearer to the hub. It may also be noted that at higher angular speeds the frequency remains nearly same when the concentrated mass is near to the hub. However, towards free end the frequencies are observed to be on the decreasing trend. It is pertinent to add that, as the location of concentrated mass is shifting towards free end, the effect of gradient in the property of material namely power law index, is has influence on the frequency, in that, as location of concentrated mass shifting towards free end, the band width in frequencies decreases.

Fig. 4. Effect of power law index and concentrated mass location on first three chordwise bending natural frequency.

Fig. 5. Effect of concentrated mass location on first three chordwise bending natural frequency.
6. Conclusion

The chordwise bending natural frequencies of a rotating FGM beams with concentrated mass are investigated using an approximate solution Rayleigh-Ritz method. The variable studied were location of concentrated mass, its magnitude, power law index, hub radius and angular speed. The results show that for a stationary beam, chordwise bending natural frequencies decrease with an increase in power law index up to a critical value after which frequencies relatively un-effected. The magnitude of the concentrated mass has been found to have an influence on the chordwise bending natural frequencies depending on the location of the mass. It has been observed that the frequencies are effected by the variation in the composition of the beam. The relation between chordwise bending frequencies and angular speed is dependent on the location of the concentrated mass. The power law index has been found to have an influence on the relation between chordwise bending versus angular speed depending on the location of concentrated mass.

References