Efficient Traffic Modelling and Dynamic Control of an Urban Region.

Dora L. Borg a,*, Kenneth Scerri a

*Department of Systems and Control Engineering, University of Malta, Msida, MSD2080, Malta

Abstract

Traffic congestion in densely populated urban areas negatively impacts our standard of living. To mitigate these problems, a computationally efficient traffic model together with a hierarchical control strategy for traffic light timings is presented in this paper. While competing macroscopic models witness an exponential increase in computational complexities with every added junction, the proposed model adds only a linear computational demand per junction for comparable accuracy. The proposed model also accurately describes the block-back of upstream junctions due to the overflow of the queues from neighbouring urban intersections. A hierarchical control strategy based on this model is developed and tested in this paper. The comparative advantages of the hierarchical controller over decentralised control are highlighted through an example. Results will show that with minimal computational power, communication requirements and infrastructure investment, the hierarchical control strategy manages to minimise the effect of junction block-back by minimising the queues on critical road sections. The consistency of the results obtained is highlighted through a Monte Carlo run.

1. Introduction

Continuous migration towards major cities around the world has brought an escalation in the number of inhabitants of urban areas. In this urban environment, mobility demands are exceeding the infrastructure capacity leading to increased traffic congestion. Expansion of the current road infrastructure is a possible...
solution, yet not always viable due to land-use restrictions and financial limitations. An alternative solution to the urban traffic congestion problem is a more efficient use of the current infrastructure through the adoption of dynamic and intelligent control strategies adaptable to prevailing traffic conditions. Such control strategies seek to increase the throughput of the urban traffic network by alleviating congestion at traffic bottlenecks often localized at the network intersections.

Dynamic control schemes applied to urban traffic control can be categorized as either centralized or decentralized strategies. A centralised control strategy seeks to find a global solution to optimise the traffic flow over the complete urban network thus obtaining a significant reduction in congestion and improved travel times. Nevertheless, centralised solutions require costly infrastructural investments for communication with the centralized controller as well as significant computational power at this central control unit. In a decentralised approach, an optimal solution is computed in real-time at each junction; a cheaper solution but with the significant setback that the local solutions do not provide a global optimum. In an effort to amalgamate both these ideas a hierarchical control strategy is being proposed, developed and tested in this work. The proposed methodology splits the complex control problem into multiple autonomous levels thus alleviating computational requirements while still approaching a globally optimum solution through the use of a centralised supervisory controller.

A number of dynamic control systems have already been implemented in various cities and have shown promising results. The two most popular implementations are SCOOT (Hunt, Bretherton and Royle, 1982) and the SCATS (Lowrie, 1982), which have been adopted in numerous cities around the world including Shanghai and Beijing. Both systems are fed with real-time information about current traffic conditions and apply changes to the split times, offsets and cycle times accordingly. The whole network is optimised centrally using a performance index. SCOOT and SCATS mostly focus on local level control by considering individual intersections or a small number of neighbouring intersections, thus their performance tends to deteriorate in heavy traffic (Papageorgiou et al., 2003).

A number of model-based control strategies have also been implemented including OPAC (Gartner, 1983), PRODYN (Farges, Henry and Tuffal, 1984) and RHODES (Sen and Head, 1997). Such systems can predict future traffic behaviour and apply appropriate control actions based on the dynamics of their models. The last decade also saw the introduction of the TUC system (Diakaki, Papageorgiou and Aboudolas, 2002), specifically aimed at urban traffic networks. The TUC system is based on a cost effective offline design of a feedback regulator aimed at controlling the traffic signals based on real-time measurements of traffic conditions. Although it has been successfully implemented in Greece and the UK, such an approach cannot handle constrains and hence requires a further sub-optimal tuning procedure.

Most model-based control strategies are developed on macroscopic models due to their low computational demands and control possibilities. Over the years, various macroscopic models have been introduced. In 1963, Gazis and Potts (1963) introduced the store and forward model on which the TUC system is based. This model represents the urban network as a graph of links and junctions, where each link is modelled using conservation theory. A model proposed by Kashani and Saridis (1983) and later extended by Van den Berg et al. (2003), models different traffic scenarios by updating a discrete-time model in small simulation time steps. Daganzo (1994) proposed the cell transmission model in 1994. This model is based on the kinematic wave equation, with average traffic flows and densities modelled on separate sections of each link. Although robust, this model is hampered by high computational requirements and thus might face some difficulties for real-time implementations. Recently Pecherkova, Dunik and Flidr (2008), proposed a novel computationally efficient model for an urban micro-region, which describes each junction through the dynamic evolution of the queue length, vehicle intensity and vehicle occupancy of the links leading to the junction.

The work in this paper expands on the model of an urban region as proposed by Pecherkova et al. and also provides a novel hierarchical control strategy to optimize the traffic flow within the region. The
computational efficiency of the proposed model is improved resulting in a linear increase in the model dimension with every added junction, as opposed to the exponential increase of the original model. To further capture the true dynamics of traffic at urban intersections, a novel macroscopic model describing junction block-back is developed. The system switches between these two models based on the queue lengths of the interlinks. A hierarchical control algorithm based on model predictive control is also implemented. To avoid block-back due to excessive queues at neighbouring junctions, individual cost functions are updated centrally to approach a global optimum solution. The comparative advantages of the proposed scheme over decentralised control are highlighted in this paper through an example.

This paper is divided into 4 sections. Section 1 provided a brief motivation, an overview of competing implementations and the main aims of this work. Section 2 presents the computationally efficient model of an urban region being proposed, together with the novel model that captures junction block-back. The development of the hierarchical controller proposed is also presented in this section. In Section 3, an example based on two adjacent intersections is given, together with a discussion of the results obtained based on the proposed methodology and a competing decentralized approach. Finally, Section 4 identifies the main results and draws some concluding remarks.

2. Modelling and Hierarchical Control of Urban Traffic Networks

The state space model of an urban region proposed in this work is based on the model introduced by Pecherkova et al. (2008). Traffic flow is represented using non-linear dynamics and each link of the controlled region is represented by the following quantities:

- Queue length ($\zeta_k$) – the number of cars waiting on each link to pass through the intersection at the start of each cycle in unit vehicles [uv]
- Input intensity ($\gamma_{l_k}$) – the rate of incoming unit vehicles on each link per cycle [uv/c]
- Occupancy ($\phi_k$) – the portion of time the detector is occupied by a vehicle [%]

with $k$ being the index of each cycle. The duration of the full traffic light cycle can be set to any reasonable value for each junction in the network.

2.1. Modelling traffic dynamics at a single junction

The dynamics of the queue length on each link can be described by the principle of conservation (Homolova and Nagy, 2005), where the queue in the next cycle ($\zeta_{k+1}$) depends on the previous queue ($\zeta_k$), the input intensity ($\gamma_{l_k}$) and the outgoing intensity ($\gamma_{S_k}$) from the link through the relationship:

$$\zeta_{k+1} = \zeta_k + \gamma_{l_k} - \gamma_{S_k}(\zeta_k, \gamma_{l_k}, \mu_k)\mu_k + w_{1_k}$$  \hspace{1cm} (1)

where the outgoing intensity is given by the non-linear relationship:

$$\gamma_{S_k}(\zeta_k, \gamma_{l_k}, \mu_k) = S - S e^{-\frac{\zeta_k + \gamma_{l_k}}{S\mu_k}}$$  \hspace{1cm} (2)

with $S$ being the saturation flow determined by the physical properties of the intersection, $\mu_k$ being the ratio of green light at each link per cycle and $w_{1_k}$ being a white zero-mean Gaussian noise process describing the random variations from the mean behaviour.

The input intensity to the junction is modelled as a Markovian process with known mean and standard deviation which is given by:
The occupancy at each cycle period is defined as a random process with the mean given through a linear relationship between the previous occupancy and the previous queue length and known standard deviation given by:

$$y_{k+1} = y_k + w_{2k}$$  \hspace{1cm} (3)

Based on the junction dynamics given by equations (1) to (4), the dynamic equation of a state space model representing each link at a junction is given by:

$$\begin{bmatrix} \xi_{k+1} \\ y_{k+1} \\ \phi_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ \kappa & 0 & \beta \end{bmatrix} \begin{bmatrix} \xi_k \\ y_k \\ \phi_k \end{bmatrix} + \begin{bmatrix} \kappa \xi_k \\ -y_k \\ 0 \end{bmatrix} \mu_k + \begin{bmatrix} w_{1k} \\ w_{2k} \\ w_{3k} \end{bmatrix}$$  \hspace{1cm} (5)

Such dynamics may be expanded based on the number of arms of an intersection as shown in (Pecherkova et al., 2008). Also, the measurement equation represents the type and number of detectors implemented on the links of the controlled intersection. In this work, only one loop detector at the end of each road leading into an intersection will be assumed so as to minimise the infrastructural costs of the proposed implementation. Thus, the measurement equation takes the form:

$$\begin{bmatrix} y_k \\ \phi_k \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \xi_k \\ y_k \\ \phi_k \end{bmatrix} + v_k$$  \hspace{1cm} (6)

where $v_k$ represents zero mean, Gaussian measurement noise.

2.2. Extending the model to multiple junctions

With every added junction, the original work proposed to append extra states to the states space dynamics of equation (5). Such a methodology results in a quadratic increase in the model complexity with every added junction. To overcome this problem, a novel modification is proposed in this work to reduce the computational demand while fully preserve the model accuracy. Thus, each junction in the urban region is modelled by a separate state space model with the input intensities of the interlinks connecting adjacent junctions modelled as inputs to the state space model. To feed these inputs, the measurement equation is modified to include the output intensities of the roads connecting the junctions. This reduces the dimensionality of the model with a linear increase in the computational demand with every added junction. This methodology is illustrated based on an urban region with two signal-controlled T-junctions depicted in Figure 1.
The region illustrated in Figure 1 is assumed to be equipped with a strategic detector on the input links placed a few meters away from the stop line to provide measurements of the input intensity and the occupancy and an output detector on the output links to give the output intensity. The interlinks (L3, L4) connecting the neighbouring junctions are equipped with both detectors. Note that, such a sensor arrangement is not a requirement for this implementation. Either the input or the output sensors can be removed if the junction turning ratios are assumed to be known. Nevertheless, the cost saving obtained from such an implementation is partially off-set by the added computational effort required to estimate the intensities which are not measured and a possible reduction in accuracy due to inaccurate or varying turning ratios.

Based on the proposed implementation, the state dynamics of junction A are given by:

\[
\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{u}_A + \mathbf{w}_A
\]

where \( \mathbf{x}_A = [\zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \gamma_{l_1} \gamma_{l_2} \phi_{k_1} \phi_{k_2} \phi_{k_3}]^T, \mathbf{u}_A = [\mu_{k_1} \mu_{k_2} \mu_{k_3} \gamma_{l_3}]^T \)

and \( \mathbf{w}_A = [w_{k_1} \ldots w_{k_8}]^T \).

The measurement equation now also includes the output intensity at each link of the junction which is dependent on the turning ratio \( \alpha_{ij} \) of the outgoing intensities going from the \( i \)th arm to the \( j \)th arm. Therefore the measurement equation for junction A is given by:

\[
\gamma_i - \gamma_o = \mathbf{z}_i - \mathbf{A}_i \mathbf{x}_i
\]

where \( \mathbf{z}_i = [\zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \gamma_{l_1} \gamma_{l_2} \phi_{k_1} \phi_{k_2} \phi_{k_3}]^T, \mathbf{A}_i = [\mu_{k_1} \mu_{k_2} \mu_{k_3} \gamma_{l_3}]^T \)

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where \[
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
\(u_{A_k} + v_{A_k}\) (8)

The same modeling procedure is used for junction B where the state equation is given by:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\kappa_4 & 0 & 0 & 0 & 0 & \beta_4 & 0 & 0 \\
0 & \kappa_5 & 0 & 0 & 0 & 0 & \beta_5 & 0 \\
0 & 0 & \kappa_6 & 0 & 0 & 0 & 0 & \beta_6 \\
\end{bmatrix}
\begin{bmatrix}
\gamma S_4 & 0 & 0 & 1 \\
0 & -\gamma S_6 & 0 & 0 \\
0 & 0 & -\gamma S_6 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_{B_k} + u_{B_k} + v_{B_k} \\
\end{bmatrix}
\]
\( (9)\)

where \(x_{B_k} = [\xi_{k_4} \xi_{k_5} \xi_{k_6} \gamma_{l_4} \phi_{k_4} \phi_{k_5} \phi_{k_6}]^T, u_{A_k} = [\mu_{k_4} \mu_{k_5} \mu_{k_6} \gamma_{l_4}]^T\) and \(\omega_B = [w_{k_1} ... w_{k_8}]^T\).

The measurement equation is given by:

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\alpha_{54} \gamma S_5 & \alpha_{64} \gamma S_6 & 0 \\
\alpha_{45} \gamma S_4 & 0 & \alpha_{65} \gamma S_6 & 0 \\
\alpha_{46} \gamma S_4 & \alpha_{56} \gamma S_6 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_{B_k} + u_{B_k} + v_{B_k} \\
\end{bmatrix}
\]
\( (10)\)

where \(y_{B_k} = [\gamma_{k_4} \gamma_{k_5} \gamma_{k_6} \phi_{k_4} \phi_{k_5} \phi_{k_6} \gamma_{l_4} \gamma_{l_5} \gamma_{l_6}]^T\) and \(\nu_{B_k} = [v_{k_1} ... v_{k_8}]^T\).
2.3. Modelling junction block-back

To obtain an accurate model of the traffic flow through an urban region, the inclusion of junction block-back dynamics is crucial. Junction block-back occurs when the vehicle queue on a link connecting neighbouring junctions reaches its maximum capacity. Thus, an otherwise unsaturated junction starts to exhibit reduced traffic flows due to junction blocking as a result of the queue build-up from its neighbour.

In this work we propose to model this behaviour based on the observation that when a junction is approaching block-back, the outgoing intensity from the input links of the junction are no longer dependent on the saturation value of the links but are limited by the buffer space available in the interlink. Thus, considering the urban region depicted in Figure 1, the condition for block-back of junction A is given by:

\[
\zeta_{k_4} + (\alpha_{13} y_{s_1} \mu_{k_1}) + (\alpha_{23} y_{s_2} \mu_{k_2}) - (y_{s_4} \mu_{k_4}) > \zeta_{k_4 \text{max}}
\]  

(11)

where \( \zeta_{k_4 \text{max}} \) is the maximum queue possible in L4. If this condition is satisfied, the outgoing intensities of L1 and L2 are therefore given by:

\[
y_{s_1} = a_{14} (\zeta_{4 \text{max}} - \zeta_4) + \alpha_{13} y_{s_1} \mu_1
\]

(12)

\[
y_{s_2} = a_{24} (\zeta_{4 \text{max}} - \zeta_4) + \alpha_{23} y_{s_2} \mu_2
\]

(13)

where \( a_{14} \) and \( a_{24} \) are the ratios of cars from L1 and L2 respectively that travel to L4.

2.4. Hierarchical control of an urban micro-region

Given the state space model of a single junction presented in Section 2.1, various well known control strategies may be implemented. In this work, a Model Predictive Controller (MPC) as presented in (Borg and Scerri, 2014) will be adopted. The control framework of an MPC contains a model-based prediction and an online optimization based on a constrained receding horizon (Camacho and Bordons, 2004). One advantage of MPC is that the model-based prediction allows the controller to take decisions based on future predicted traffic and hence myopic decisions are avoided. A receding horizon of just a few time-steps is usually sufficient, thus minimizing significantly the computational burden and making the controller idea for a real-time implementation. In this work, an MPC will be adopted with the aim to minimize the cost function given by:

\[
J = \sum_{i=k+1}^{k+N} x_i^T R_k x_i + \sum_{j=k}^{k+N-1} \mu_j^T Q_k \mu_j
\]

s.t  \( \zeta_{k}, y_{i_k} > 0 \)

\( 0 < \phi_k < 100 \)

\( \mu_{\text{min}} < \mu_k < \mu_{\text{max}} \)

\( \sum_{i=1}^{n} \mu_i = 1 \)  

(14)
where $N$ is the number of cycles used by the controller to reach the desired state value and $R$ and $Q$ are the weighting matrices on the states and the inputs respectively.

The main setback of MPC is that as the number of controlled intersections increases, the optimization procedure of the MPC becomes too complex to be solved in real time (Papageorgiou et al., 2003). To mitigate this problem, a hierarchical control strategy is developed. At the lower control level, MPC controllers with a cost function given by (14) are used to control independently each junction in the urban region. To avoid block-back, the queue lengths of the interlinks are monitored at a higher, centralized control level. In the case of large queues on these links, the higher level controller changes MPC weights ($R$ and $Q$) of the lower level controllers to eliminate or alleviate the effect of block-back.

Please note that only $\gamma_{lk}$ and $\phi_k$ can be measured directly from the loop detector. Therefore, since direct measurement of $\zeta_k$ cannot be obtained, a Kalman filter (Anderson and Moore, 2012) is used to estimate the unmeasured states. The Kalman filter is not required if full knowledge of the states can be obtained from the detectors.

3. Example

To demonstrate and analyse the performance of the hierarchical control strategy being proposed, it was tested on the urban region depicted in Figure 1. The simulation time at every junction is taken as the traffic light cycle time of 180s. The minimum green time for a link at each cycle time is specified to be 15s while the maximum green time is 150s. The parameters of the two junctions considered are specified in Tables 1 and 2, where the saturation values are determined based on the physical properties of the junction while the $\kappa$, $\mu$, and $\alpha$ can be obtain through linear regression based on collected data (Pecherkova et al., 2008). The cost function (14) will be used to minimise the queue lengths with all control parameters also given in Tables 1 and 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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Table 1: Parameters of Junction A

<table>
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<tr>
<th>Parameter</th>
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</table>

Table 2: Parameters of Junction B
To quantify the effectiveness of the proposed hierarchical controller and compare the results obtained with a more conventional implementation, a decentralised control strategy is also implemented. The controller applied on each junction in the decentralised control strategy is the MPC controller proposed in (Borg and Scerri, 2014) with the same cost function used for the hierarchical control strategy. A set of typical intensities was generated as inputs to the links leading to the region as shown in Figure 2.
Figure 2: Input intensities on the input links to the urban region

The resulting queue build-up on all the links for the simulation of the urban network using the decentralised control scheme and the hierarchical control strategy are shown in Figures 3 and 4, respectively.

Figure 3: Queue build-up on all links using the decentralised control scheme
Comparing the queue lengths of Figure 3 and Figure 4, significant improvements can be noted for the network making use of the hierarchical control strategy, especially for the queues on links 1 and 2. When utilizing a decentralized strategy, in order to cater for the large queues on links 5 and 6, the controller reduces significantly the green time for link 4, which cause this link to reach its maximum capacity within a few cycles. As a result, traffic through the other junction is also blocked leading to large queues on links 1 and 2. On the other hand, the hierarchical control strategy manages to alleviate the queue build-ups on the interlinks as shown in Figures 4c and 4d. Consequently, the traffic flow through junction A is not impeded, resulting in significantly lower queues on links 1 and 2.

The low queues on the interlinks when using the hierarchical controller may also have a further added advantage which, although not visible in microscopic simulations, could have significant implications in practice. This controller manages to maintain a significant buffer for traffic going towards links 3 and 4. Thus any traffic travelling between these junctions will at no point overflow into the intersection and thus it will not impede any vehicle travelling in any other direction.

To test the robustness of the hierarchal controller and its sensitivity to the model parameters, the saturation values of the links at each junction were changed in the simulation model to the values in Table 3, giving the queue build-up results shown in Figure 5.
Table 3: Saturation values

<table>
<thead>
<tr>
<th>Saturation</th>
<th>Value [uv/c]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>90</td>
</tr>
<tr>
<td>$S_2$</td>
<td>30</td>
</tr>
<tr>
<td>$S_3$</td>
<td>35</td>
</tr>
<tr>
<td>$S_4$</td>
<td>40</td>
</tr>
<tr>
<td>$S_5$</td>
<td>50</td>
</tr>
<tr>
<td>$S_6$</td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 5: Queue build-up on all links with new saturation values on the links of the urban region.

With the same input intensities, the hierarchical controller still managed to minimise the queues and prevent the occurrence of junction block-back. This shows that the performance of the proposed hierarchical controller is independent from the model parameters.

To analyse the sensitivity of the proposed hierarchical strategy to the controller design, the controller cost function parameters given in (14) were changed to, $[R_{int1}...R_{int6}] = [1]$. The resulting queues are depicted in Figure 6.
Comparing the results of Figure 6 with the results in Figure 4, minimal differences in the queue length can be noted. Thus the performance of the hierarchical control strategy seems mostly insensitive to variations in the controller and model parameters.

Finally, to verify the repeatability and consistency of the improvements obtained by the hierarchical controller over the decentralised control scheme, a Monte Carlo run of 100 iterations was performed for each implementation. The histograms of Figures 7 and 8 depict the resulting queues after 100 time-steps for each realisation. From the histograms it can be noted that the hierarchical controller consistently outperformed the decentralised controller with significantly lower queues on both the interlinks and on links 1 and 2. This highlights the consistent ability of proposed controller to avoid the blocking of junction A due to the overflow of queues from junction B.
Figure 7: Histograms representing the results of 100 realisations using the decentralised control scheme.

Figure 8: Histograms representing the results of 100 realisations using the hierarchal control strategy.
4. Conclusions

The increased traffic congestion witnessed in urban areas can be mitigated through the use of better traffic control strategies. Results in this paper demonstrate that based on a computationally efficient macroscopic model capturing both normal traffic conditions and upstream junction block-back scenarios, the hierarchical control strategy developed was able to significantly reduce the traffic congestion in an urban areas. Results also highlight the excellent repeatability of the results and the robustness of the proposed hierarchal system to model and controller parameter variations. Future work will focus on the adoption of different control strategies using the proposed macroscopic model with particular attention given to the creation of green waves as well as the joint estimation in real-time of the control action and model parameters.

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