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Transport Planning Realized Through the Optimization Methods

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Abstract

The passenger traffic is the most sensitive field in terms of the passengers´ perception of the transport line layout. Every drawback will show soon and be transferred into the whole range of the provided services. The aim in the passenger traffic is to remove any obstacles that prevent the passengers from obtaining the continuous, available and high quality transport and to develop such activities to meet the requirements of the passengers as much as possible.

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Keywords: operational analysis methods, solving optimization problems, public transport, assignment problem, linear programming;

1. Introduction

When evaluating the quality of the provided services, the maximum speed of a particular employed connection is not decisive for the passengers, but the total time it takes for the passenger to transfer for the initial point to the destination point. Also the connection regularity, frequency, travelling comfort and, last but not least, the price of transportation are the aspects that matter the most to the passengers. Thus, the considerable attention shall be paid to the transport line layout, its adjustments or modifications.

Theoretical activities which completely cover the issue of the transport line project supported by the mathematical methods are pretty interesting not only from the academic point of view. There are even some organizations and centres which deal with this subject in order to find a solution of the given problems using the methods of operational analysis. Even the transport companies themselves use such methods to find the solution for their problems.

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At present, the other up-to-date approaches in mathematical methods start to appear in numerous studies or scientific theses. It is primarily based on the growing demands on the information technology. The systems promoting the decision-making and the other simulation methods may be used in many processes in the area of multi-criteria evaluation of alternatives; however, many of these highly demanding processes cannot be effectively applied due to the complexity of the considered subject.

Here are brief examples which have been dealt with in the world so far. The transport company of New Zealand providing the public transport services applied the operational research methods to use its motor vehicle fleet more efficiently. While keeping the provided service capacity, this company managed to reduce the number of operating vehicles by up to 35% to save a considerable amount of financial means. Similar examples can be found at most of Western European transport companies, such as in Zurich, Bern, etc. Canada solving the implementation of the optimum transfer of passengers using the linear programming. Also some research institutes or universities deal with certain aspects of the transport line project and the coordination of such lines, such as University of Žilina, Czech Technical University of Prague, Jan Perner Transport Faculty of University of Pardubice and Technical College in Dresden, Germany.

2. Mathematical model of the problem and its solution

To propose link lines, the method of linear programming was chosen, specifically the algorithm to solve the transport problem used for solving the assignment problem. This method was chosen in order to save costs which would have been much higher if other models had been chosen to solve the same problem e.g. VISEVA, VISUM (Modal Split) [10].

2.1 Mathematical model of the assignment problem

Optimization of transport links in public transport can be formulated and solved as an assignment in which individual districts are regarded both as sources and destinations. The rates are sums of the identified source and destination traffic intensities between any two districts. In order to avoid assigning the same two districts, so called prohibitive rates were chosen on the main diagonal (see *Table 2*). The criterion of optimality is the maximization of the total number of passengers transported without transfer. The mathematical model of the problem has the following form [12 - 16]: To maximize:

$$f = \sum_{j=1}^{26} \sum_{i=1}^{26} c_{ij} x_{ij} \tag{1}$$

$x_{ij} = 1$, if the i-th transport district is assigned to the j-th transport district
$x_{ij} = 0$, in the opposite case,

In case of restrictions:

$$\sum_{j=1}^{26} x_{ij} = 1; \quad i = 1,2,..26; \quad \sum_{i=1}^{26} x_{ij} = 1; \quad j = 1,2,..26 \tag{2}$$

Classical task assignment is characterized by minimization. However, in our case it is necessary to maximize traffic flows. In this case we only need to replace the objective function f with the function [12, 17, 18]:

$$-f = \sum_{j=1}^{26} \sum_{i=1}^{26} \left(-c_{ij}\right) x_{ij} \tag{3}$$

The assignment problem is usually solved by the so called Hungarian method (see e.g. [3, 11]). Due to software options the considered assignment problem was solved as a transportation problem using the Dumkosa program (an add-in XLA, created by the Department of Operational and Systems Analysis at PEF ČZU in Prague). As follows from the theory of linear programming, the solution of an assignment problem obtained by methods for solving the transport problem is greatly degenerated. Completing the number of occupied boxes to the number required for a non-degenerated solution is usually done using a negligibly small amount of EPS. The symbol ALT in some fields means that by filling this field we would obtain the equivalent optimal solution with the same value of the objective function (see *Tables* 3-6). Using the Dumkosa program, first grade transport flows were obtained - the strongest links between hubs (see *Table 3*). The algorithm, however, suggests such connections so as at a given moment the value of the objective function (the sum of all transported passengers without transfer) is at its maximum [10 - 16].

The procedure was then repeated 4 times. The procedure was always the same but further prohibitive rates were put into the places of assigning rates. As a result, 4 transport flows were obtained (see *Tables 3 - 6*) which are marked for simplicity's sake. It is necessary to point out that the first grade is the most significant.

It is also very important to realize that the obtained grades of transport flows are not comparable given the importance of the individual districts. For this reason, it is necessary, in the case of a large district (estate), to consider every transport grade as significant whereas in the case of a weak transport district (e.g. Havlickova Kolonie) it is possible to consider only the first and possibly the second grade transport flows. The proposed allocation of districts represents the best direct links according to the selected mathematical model [8 – 10]. A situation where individual transport flows come together or coincide is very crucial in this context. A typical case of this occurs in districts A and S (Table 1, 2).

Table 1 - Adjusted matrix - the sum of the source and destination traffic intensities.

Adjusted matrix – sums of source and destination transport intensities - Values obtained by research

Total data for 23rd November (5.30 – 11.30) and 24th November (11.30 – 17.30)

Number	Σ	z A	z B	z C	z D	z E	z F	z G	z H	z I	z J	z K	z L	z M	z N	z O	z P	z Q	z R	z S	z T	z U	z V	z W	z X	z Y	ze Z
Σ	60699	10880	776	4213	1178	1787	858	146	821	3143	4680	215	721	6386	6003	512	818	1257	2033	2858	3750	1286	2097	407	1915	854	1105
do A	7735	1221	75	390	109	20	41	10	50	228	619	2	10	1937	998	149	89	24	207	168	292	167	268	4	298	328	31
do B	540	120	22	16	1	9	2	1	0	10	175	0	2	24	46	1	8	9	9	17	20	0	6	0	7	34	1
do C	5110	1299	46	12	175	125	133	26	46	320	351	55	245	219	68	6	26	166	362	365	400	69	212	192	113	48	31
do D	938	60	2	162	79	27	5	0	3	29	106	0	3	6	248	2	11	13	7	22	77	5	25	1	27	11	7
do E	1022	45	7	94	19	47	23	0	3	152	88	1	2	249	86	3	13	5	49	31	26	43	9	1	20	2	4
do F	1089	158	1	278	6	16	19	2	6	11	41	0	0	173	168	0	11	19	16	7	25	2	52	0	57	13	8
do G	381	47	3	42	2	21	23	4	2	13	40	2	14	3	74	0	2	5	20	11	9	2	17	0	11	7	7
do H	987	50	3	113	2	11	8	0	24	146	72	0	4	146	150	1	7	9	5	64	16	0	7	0	146	1	2
do I	3472	538	11	152	28	260	36	1	158	6	269	21	64	308	233	2	83	115	119	202	352	184	53	7	111	12	147
do J	6677	1356	317	692	120	232	57	22	63	289	460	0	158	423	644	7	86	146	397	407	322	73	109	105	74	63	55
do K	469	18	0	38	2	1	0	2	27	33	3	11	0	42	131	3	3	53	7	18	23	1	4	0	46	0	3
do L	810	21	0	318	3	20	2	2	29	40	75	0	50	68	75	0	3	52	2	29	6	1	10	0	4	0	0
do M	6522	2165	27	145	38	266	160	1	140	282	276	44	85	112	245	10	25	205	162	355	964	275	286	2	156	69	27
do N	6703	1505	23	162	288	285	150	60	96	395	487	43	30	272	32	220	135	175	145	411	388	249	386	68	123	106	469
do O	720	147	1	12	4	14	17	0	5	9	41	0	6	41	171	7	13	0	16	15	52	36	50	0	53	2	8
do P	829	98	7	7	11	81	12	0	7	136	48	0	0	3	178	0	3	58	23	10	53	3	17	0	27	3	44
do Q	1120	150	6	37	17	7	5	1	39	151	49	6	3	115	151	1	75	34	36	56	22	7	15	1	88	4	44
do R	1821	185	131	256	15	43	15	3	6	80	307	0	1	167	183	12	21	27	51	145	38	49	22	0	24	33	7
do S	2570	173	13	496	4	8	10	4	5	141	439	0	4	228	450	3	16	19	140	120	91	10	77	11	78	16	14
do T	3612	484	19	248	92	59	28	0	34	216	305	9	11	847	314	14	39	10	84	115	142	52	138	4	222	92	34
do U	1518	236	4	79	12	189	13	0	10	109	86	0	4	267	111	22	7	18	50	45	95	22	35	0	97	0	7
do V	1167	139	3	151	29	0	37	5	5	40	30	2	2	167	208	16	24	19	13	38	51	0	137	0	37	3	11
do W	384	5	0	195	0	8	3	1	0	1	48	0	0	2	56	1	4	4	3	26	4	0	7	6	6	0	4
do X	1532	204	8	67	27	1	20	0	53	82	93	19	19	241	147	27	39	5	22	110	177	23	60	4	24	3	57
do Y	1610	425	47	49	80	20	29	0	2	26	151	0	2	315	102	5	16	5	81	51	95	10	85	0	7	0	7
do Z	1361	31	0	2	15	17	10	1	8	198	21	0	2	11	734	0	59	62	7	20	10	3	10	1	59	4	76

The proposed direct transport links represent the best direct link lines of public transport in České Budějovice while respecting the criterion of optimality where the maximum number of passenger's travel are direct and without transferring. For those transferring, it is proposed to minimize waiting times based on transport theory - limiting parallel lines following the same stretches to different destinations. Furthermore, suitable transfer terminals can be read out of the model (*Table 3 - 6*) [8 – 10].

Table 2 - Adjusted table - the sum of the source and destination traffic intensities - prohibitive rates on the main diagonal.

Adjusted matrix – sums of source and destination transport intensities – prohibitive rates on the main diagonal

Values obtained by research

Total data for 23rd November (5.30 – 11.30) and 24th November (11.30 – 17.30)

Number		z A	z B	z C	z D	z E	z F	z G	z H	z I	z J	z K	z L	z M	z N	z O	z P	z Q	z R	z S	z T	z U	z V	z W	z X	z Y	ze Z
do	A	-1000	75	390	109	20	41	10	50	228	619	2	10	1937	998	149	89	24	207	168	292	167	268	4	298	328	31
do	B	120	-1000	16	1	9	2	1	0	10	175	0	2	24	46	1	8	9	9	17	20	0	6	0	7	34	1
do	C	1299	46	-1000	175	125	133	26	46	320	351	55	245	219	68	6	26	166	362	365	400	69	212	192	113	48	31
do	D	60	2	162	-1000	27	5	0	3	29	106	0	3	6	248	2	11	13	7	22	77	5	25	1	27	11	7
do	E	45	7	94	19	-1000	23	0	3	152	88	1	2	249	86	3	13	5	49	31	26	43	9	1	20	2	4
do	F	158	1	278	6	16	-1000	2	6	11	41	0	0	173	168	0	11	19	16	7	25	2	52	0	57	13	8
do	G	47	3	42	2	21	23	-1000	2	13	40	2	14	3	74	0	2	5	20	11	9	2	17	0	11	7	7
do	H	50	3	113	2	11	8	0	-1000	146	72	0	4	146	150	1	7	9	5	64	16	0	7	0	146	1	2
do	I	538	11	152	28	260	36	1	158	-1000	269	21	64	308	233	2	83	115	119	202	352	184	53	7	111	12	147
do	J	1356	317	692	120	232	57	22	63	289	-1000	0	158	423	644	7	86	146	397	407	322	73	109	105	74	63	55
do	K	18	0	38	2	1	0	2	27	33	3	-1000	0	42	131	3	3	53	7	18	23	1	4	0	46	0	3
do	L	21	0	318	3	20	2	2	29	40	75	0	-1000	68	75	0	3	52	2	29	6	1	10	0	4	0	0
do	M	2165	27	145	38	266	160	1	140	282	276	44	85	-1000	245	10	25	205	162	355	964	275	286	2	156	69	27
do	N	1505	23	162	288	285	150	60	96	395	487	43	30	272	-1000	220	135	175	145	411	388	249	386	68	123	106	469
do	O	147	1	12	4	14	17	0	5	9	41	0	6	41	171	-1000	13	0	16	15	52	36	50	0	53	2	8
do	P	98	7	7	11	81	12	0	7	136	48	0	0	3	178	0	-1000	58	23	10	53	3	17	0	27	3	44
do	Q	150	6	37	17	7	5	1	39	151	49	6	3	115	151	1	75	-1000	36	56	22	7	15	1	88	4	44
do	R	185	131	256	15	43	15	3	6	80	307	0	1	167	183	12	21	27	-1000	145	38	49	22	0	24	33	7
do	S	173	13	496	4	8	10	4	5	141	439	0	4	228	450	3	16	19	140	-1000	91	10	77	11	78	16	14
do	T	484	19	248	92	59	28	0	34	216	305	9	11	847	314	14	39	10	84	115	-1000	52	138	4	222	92	34
do	U	236	4	79	12	189	13	0	10	109	86	0	4	267	111	22	7	18	50	45	95	-1000	35	0	97	0	7
do	V	139	3	151	29	0	37	5	5	40	30	2	2	167	208	16	24	19	13	38	51	0	-1000	0	37	3	11
do	W	5	0	195	0	8	3	1	0	1	48	0	0	2	56	1	4	4	3	26	4	0	7	-1000	6	0	4
do	X	204	8	67	27	1	20	0	53	82	93	19	19	241	147	27	39	5	22	110	177	23	60	4	-1000	3	57
do	Y	425	47	49	80	20	29	0	2	26	151	0	2	315	102	5	16	5	81	51	95	10	85	0	7	-1000	7
do	Z	31	0	2	15	17	10	1	8	198	21	0	2	11	734	0	59	62	7	20	10	3	10	1	59	4	-1000

Table 3 - Optimal solution of the transport model - First grade directional transport flows

Optimal solution of a transport model - Directional transport streams of first degree outside of main diagonal

Optimal value of purpose function is 16402

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
A	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
B	0	0	0	EPS	0	0	0	0	0	0	EPS	0	0	0	EPS	0	0	EPS	0	0	0	0	1	0	0	0	1
C	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	1	0	1
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	1	0	0	0	0	1
G	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
H	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	EPS	0	0	0	0	EPS	1
J	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	1	0	0	0	0	0	0	0	0	1
K	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	EPS	0	0	0	0	0	0	0	0	0	1
L	0	0	EPS	0	0	0	1	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	1
M	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	1
N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
O	0	0	0	ALT-	0	0	0	0	0	0	1	EPS	0	0	0	0	0	0	0	0	EPS	0	0	EPS	0	0	1
P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
Q	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
R	0	EPS	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
T	0	0	0	0	0	0	0	0	1	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	EPS	0	0	1
U	0	0	0	0	1	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	1
V	0	0	0	0	0	1	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	1
W	0	1	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
X	0	0	0	0	0	0	0	1	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	1
Y	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Z	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

Table 4 - Optimal solution of the transport model – Second grade directional transport flows.

Optimal solution of a transport model - Directional transport streams of second degree outside of main diagonal

Optimal value of purpose function is 13376

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
A	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	EPS	0	1
C	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
D	0	0	0	0	0	EPS	0	ALT-	0	0	0	0	0	0	ALT-	EPS	0	0	0	0	0	EPS	1	0	0	0	1
E	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
F	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	1
G	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
H	0	0	0	0	0	0	0	0	EPS	0	1	EPS	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	1
I	0	0	0	0	1	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
J	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
K	0	0	0	0	0	0	0	ALT-	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
L	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
M	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
N	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	1
O	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	EPS	0	EPS	0	0	1
P	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
Q	0	0	0	0	0	0	0	1	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
R	0	1	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	1
S	0	0	0	0	0	0	0	0	0	1	0	0	0	EPS	0	0	0	EPS	0	0	0	0	0	0	0	0	1
T	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	EPS	0	0	1
U	0	0	0	0	0	0	0	0	0	0	0	0	0	0	ALT-	0	0	0	0	0	0	0	0	1	0	0	1
V	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
W	0	0	0	1	0	0	EPS	EPS	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	1
X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	EPS	EPS	0	0	0	0	0	1
Y	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
Z	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

Table 5 - Optimal solution of the transport model – Third grade directional transport flows.

Optimal solution of a transport model - Directional transport streams of third degree outside of main diagonal

Optimal value of purpose function is 10160

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
A	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
C	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
D	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	1
E	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
F	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	EPS	0	1
G	0	EPS	0	EPS	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
H	0	0	0	0	0	0	0	0	EPS	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
J	1	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
K	0	0	0	EPS	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
L	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
M	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
O	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
P	0	0	0	EPS	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Q	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	1
R	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	EPS	0	0	0	EPS	0	1
S	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	EPS	0	0	0	0	0	0	0	0	1
T	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
U	0	0	0	0	0	0	0	0	0	0	0	0	1	0	EPS	0	0	EPS	0	0	0	0	0	0	0	0	1
V	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
W	0	0	0	0	0	EPS	1	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	EPS	0	0	0	0	0	1
Y	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Z	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	1
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

Table 6 Optimal solution of the transport model – Fourth grade directional transport steams.

Optimal solution of a transport model - Directional transport streams of fourth degree outside of main diagonal

Optimal value of purpose function is 8856

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
A	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
B	0	0	0	0	0	0	1	0	0	0	0	EPS	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	1
C	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	1
D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	1	0	0	0	0	1
E	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	EPS	EPS	0	1
G	0	ALT-	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
H	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	1
I	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	1
J	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
K	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
L	0	1	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	1
M	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	EPS	0	0	0	0	1
N	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	1
O	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
P	0	EPS	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Q	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
R	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
S	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	1	0	0	0	0	0	EPS	0	0	1
T	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
U	0	0	0	0	0	EPS	0	0	EPS	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
V	0	0	0	1	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	EPS	0	0	0	0	0	0	1
W	0	0	0	0	0	0	0	0	0	0	EPS	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	EPS	1
Y	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
Z	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1

According to the above-mentioned mathematical algorithm 4 transport flow grades were identified [8 -10]. Figure 1 shows the total number of passengers according to transport district in České Budějovice.

2.2 Determination of the significance of the districts

To connect the various proposed transport links to specific lines of urban and suburban transport, it is necessary to determine the significance of the district. It specifically determines how the decision-maker should interconnect the direct links between individual districts proposed by the mathematical model and how many links are needed to serve the districts. The significance of the districts is determined by the sum of source and destination flows (see *Table 7*). The obtained value is the number of passengers who travelled *from, to* or *in* the district [8 – 10].

Table 7 - Number of passengers from, to and in the district.

A	B	C	D	E	F	G	H	I	J	K	L	M
18615	1316	4213	2116	2809	1947	527	1808	6615	11357	684	1531	12908

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
12706	1232	1647	2377	3854	5428	7362	2804	3264	791	3447	2464	2466

Subsequently, the individual transport districts were divided into 5 classes according to their significance, Tab. 8.

Table 8 - Determination of the significance of transport districts

A	M	N	J	T	I	S	C	R	X	V	E	U
18615	12908	12706	11357	7362	6615	5428	4213	3854	3447	3264	2809	2804

Z	Y	Q	D	F	H	P	L	B	O	W	K	G
2466	2464	2377	2116	1947	1808	1647	1531	1316	1232	791	684	527

Subsequently, the decision maker must consider the overall stretch intensities between individual districts. Then it is possible to gradually knit together the various proposed direct links and suggest link routing [8, 10, 17].

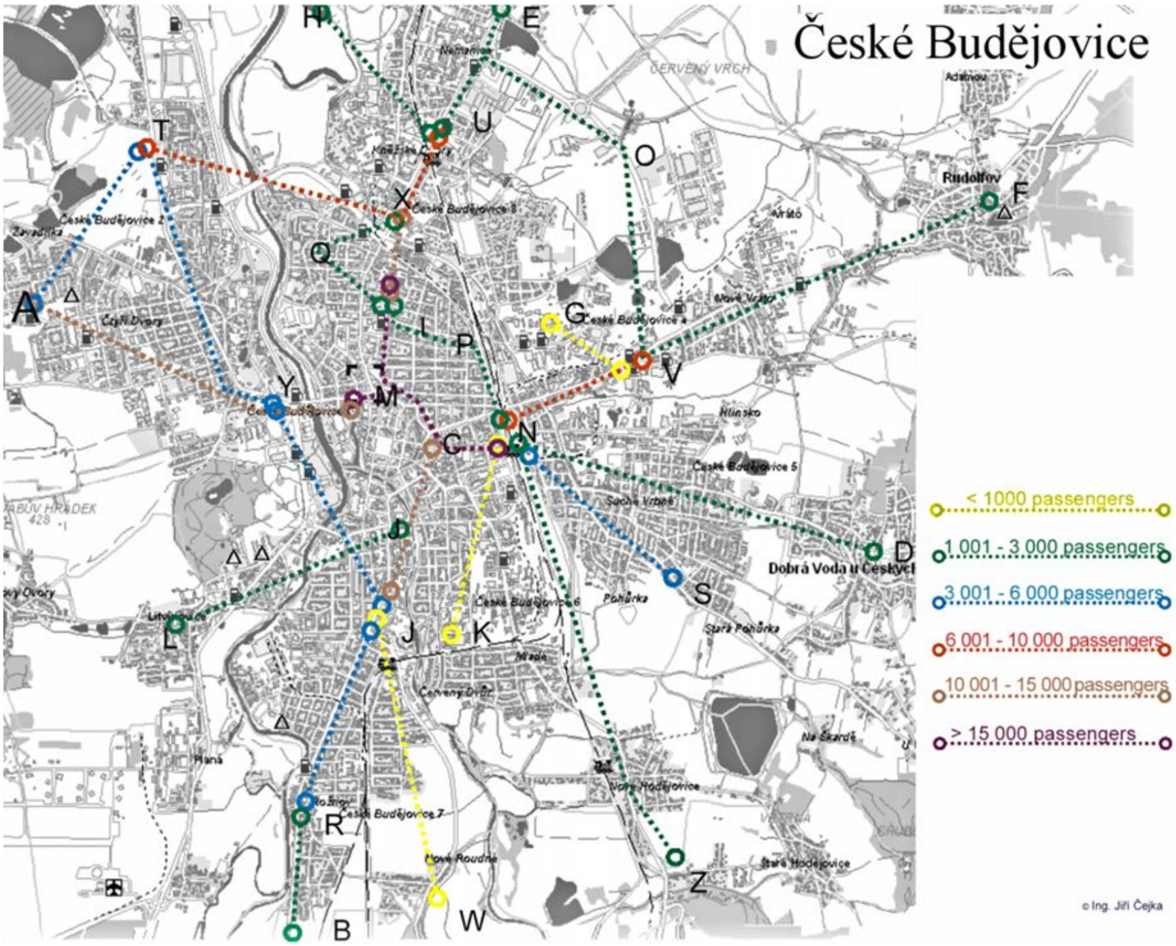

Figure 1 - Total numbers of passengers between districts.

2.3 Comprehensive proposal for the routing of links of urban and suburban transport lines

Based on the results of the mathematical model the distribution of direct transport links was determined. It was necessary to move from the stage when the individual flows are proposed using a mathematical model to the stage where from a number of proposed options such options must be selected, which follow on from the transport significance of a district (stated by the decision maker), from the stretch intensity between districts and from basic transport principles (especially the classical theory of transport) [8, 9, 11].

This procedure also requires the use of common sense, experience and the knowledge of the history of the transport system. It is therefore necessary to also apply the personal approach of the decision maker. It should be noted that as part of the decision making process it is also necessary to take into account the system of trolleybus transport, especially the existing trolleybus network [9, 11].

3. Validation of the model on a specific case solution in practice

The last transport survey to be conducted was in 2000 and on the basis of its findings the Transport Company of České Budějovice (DPCB a.s.) developed a comprehensive new transport system in the city. To the present day no one has been dealt in detail with the problem of routing links (research carried out by the University of Technology and Economics in České Budějovice, 2012) [8 – 10]. Within the framework of the research project (2012) based on the traffic survey and mathematical solution of the problem, several conclusions were drawn (situation in 2000 vs. 2012):

3.1 Model confirmed the correctness of the current routing of some lines [8, 9]

a) Line 1: Haklovy Dvory – Máj – E. Rošického – Výstaviště – Poliklinika Sever – Senovážné náměstí – Nádraží – Vráto – Rudolfov 1 – Rudolfov 2. (see line B)
The correct routing of this line was confirmed, especially the stretch from Máj to Vráto. This result confirmed and significantly supported the fact that this section of the line will be routed as a separate line and will become a separate trolleybus line in the not too distant future. At present, projects are being prepared in order to finance the construction of the trolleybus line with EU funds. In Vráto a new transfer terminal will be created and transport services to and from Rudolfov will be provided in the form of follow-on bus transport.

b) Line 3: Máj (A: Barcala) – Šumava – Výstaviště – Poliklinika Sever – Senovážné náměstí – Nádraží, (see line A). This line was implemented on the basis of a proposal. It was decided to route it in a radial line and to adjust the timetable in line with the strength of the transport flow to four minute intervals at peak times.

c) Line 7 Máj (A. Barcala) – Dubenská – Výstaviště – Poliklinika Jih – Náměstí Bratří Čapků – Včelná – Boršov nad Vltavou – Zahorčice – Kroclov – Jamné (see line C)
The correctness of the routing of this line was confirmed for the basic stretch Máj - Nám. Bratří Čapků. The other connections are dependent on the requirements of the other communities. Half of the connections end at the Nám. Bratří Čapků stop. This tangent line has great significance for the entire transport system of the city. On the basis of this proposal, introducing a shortened line is an option that is being given due consideration. As a result, the transport service to Boršov would need to be solved differently. The shortened line is being prepared for the possible introduction of a trolleybus link in the future.

d) Line 11 Staré Hodějovice – Nové Hodějovice – Mladé – Nádraží – Palackého náměstí – Družba IGY – Pražské sídliště (see line N). The routing of this line was confirmed over the entirety of the route.

e) Line 14 Vltava – Výměník – Výstaviště – Poliklinika Sever – Senovážné náměstí – Poliklinika Jih – Nemocnice – Papírenská (see line O)

The routing of this line was confirmed over the entirety of the route. It should be noted that the change of where the line ends will be implemented once the trolleybus link to České Vrbné is completed. The model also approved the correctness of the planned construction of trolleybus lines to České Vrbné.

f) Line 16 Husova kolonie – Nádraží – Senovážné náměstí – Livínovice – Šindlovy Dvory – Mokré (see line K)
The routing of this line was confirmed over the entirety of the route.

A. The model confirmed the current routing of some lines in the busiest stretches [8, 9]

a) Line 2 The model confirmed the routing of the line over the stretch Borek – Nemanice – Družba IGY – Mariánské náměstí – Poliklinika Sever - Nádraží. The remainder of the route from Nádraží via Poliklinika Jih to Rožnov was not supported by the model (see line F).

b) Line 8. The model confirmed the routing of the line over the stretch Máj (A. Barcala) – E. Rošického – Vltava střed – Strakonická MÖBELIX (see Retail Zone Pražská) – Hřbitov – Okružní Točna (see line E). For the rest of the route to the station the model recommended a connecting bus with an extension to Havlíčkova kolonie (see line L).

c) Line 9. The model confirmed routing of the line over the stretch Vltava – Strakonická MÖBELIX (see Retail Zone Pražská) – Družba IGY – Mariánské náměstí – Poliklinika Sever – Senovážné náměstí – Nádraží (see line Q). The continuation of the line onto Suché Vrbné was not supported by the model.

d) Line 17. The model confirmed routing of the line over the stretch Máj (A. Barcala) – E. Rošického – Vltava střed - Strakonická MÖBELIX (see Retail Zone Pražská) – Družba IGY – Mariánské náměstí – Nádraží only with a diversion of the line into Pekárenská street towards Nádraží and further onto Poliklinika Jih and to the end stop in Papírenská street. (The line ran with a diversion in 2003 only in one direction, the opposite direction was serviced by Line 9 on the route Vltava – Nádraží.)

B. The model found incorrect routing of lines [8, 9]

a) The original Line 3 which ran from Suché Vrbné – Nádraží – Senovážné náměstí – Poliklinika Jih – Papírenská, was cancelled and has not been replaced by any other line. The mathematical model proved that the routing of the line corresponded with one of the strongest transport relations in town. It is therefore undeniable that the cancellation of this line was a fatal error and there is a poor transport relationship with this area.

b) The original Line 15 which ran from České Vrbné – Vltava – Výstaviště – Senovážné náměstí – Poliklinika Jih – Nemocnice – Papírenská was changed to a complicated route that does not match any important transport relation. The line travels around 90% of the city on routes where it is possible to replace the majority of transport relations with another more frequented connection. The line has a round trip time of over 100 minutes (one way journey takes 49 minutes). The routing of this line is appropriate for a small town but not for a city with over 100,000 inhabitants.

c) The original Line 19, which ran the circular route Máj – Výstaviště – Poliklinika Sever - Nádraží was abolished and replaced with a diametrical route Máj – Výstaviště – Poliklinika Sever – Nádraží – Suché Vrbné (as Line 3). This change distorted the balance in the transport relation Nádraží – Máj. The termination of all even connections at the Nádraží bus stop led to several problems. The largest built-up area of Suché Vrbné got an inadequate connection with 20 minute intervals; after 52 years, the direct connection between Suché Vrbné and Rožnov was cancelled and the problem as cited in a) was created. On top of this, due to frequent traffic congestion it often happened that the prolonged connection from Suché Vrbné was so late that it caught up with the shortened line which ran during the middle of the intervals on the long line. The result of this was that the intended 5-minute interval became in reality a 10-minute interval which resulted in two busses following each other. On the basis of the mathematical model the original connection was renewed in such a way that Line 3 was shortened and Line 19 returned to its original route.

From the aforementioned facts, it is clear that the results of the traffic survey (2012) with the support of mathematical methods promoted positive changes in the routing of some public transport lines in České Budějovice and its surroundings.

4. Assessing the benefits of the solutions for transport practice and scientific knowledge - (result)

The methodical solution applied in the research of the transport model proposed with the support of mathematical methods represents a modern approach to designing routes of lines in České Budějovice and can be applied to any city public transport system. The implemented changes represent a significant positive contribution to the overall public transport system. Compared to 2000, the number of passengers carried increased by at least 30% (on the route Máj – Nádraží by more than 40%). The system is gradually becoming more "user friendly" due to the introduction of routes that passengers use [8 - 10]. Another benefit is the expansion of the trolleybus network in České Budějovice. A new trolleybus line to České Vrbné was completed in 2007. Traffic surveys have helped to identify the busiest routes and helped to put forward proposals for a new nighttime urban transport system. The most stretched routes are Máj – Nádraží, Vltava – Nádraží, Suché Vrbné – Senovážné náměstí, Nádraží – Rožnov.

From the point of view of scientific knowledge, the study is an example of creative research applied to a purely practical problem with the use of exact mathematical methods. Furthermore, research has confirmed that for a successful solution to any problem it is necessary to know its substance as well as modern methods for its solution [10 - 17].

5. Conclusion

The theoretical and practical part of the article shows that the use of mathematical methods is not necessarily only a theoretical problem, usable only while studying at university or purely for academic interest.

The benefits cited in the previous chapter are quite significant and it is undeniable that research brings a new insight not only into some mathematical methods, but also into the possible marketing or economic aspects when designing or modifying transport models, not only in public transport sector.

In the economic analysis of the proposed model it was found (by simulated timetables, backbone line with peak intervals of 10 min or 15 min thresholds and evening traffic from 20 to 30 min, other lines according to requirements of the relevant municipalities) that the proposed operation in vehicle-kilometres, whilst maintaining overall public transport, is about 5% cheaper than the current model. Passengers also benefit from mutual inter-routing of lines on shared sections and guaranteed continuity in the early morning and evening traffic at the proposed transfer points. A public transport model prepared in this way can be integrated into the ITS without a problem.

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