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Modeling and Control of Networked Systems with Different Loop Topologies

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Abstract

In this paper, modeling and control of networked control system with different loop topology structures are considered. The indeterminate time-delay is known to the controller node by using time-stamping and introducing buffers for different topology structure NCSs. The uniform discrete state-space model is established by introducing state augmentation for both short and long time-delay. Then, the controller is designed by augmented state pole-placement algorithm.

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Keywords: Network induced time-delay, Topology structure, Modeling, Augmented state, Pole-placement

1. Introduction

The feedback control systems wherein the control loops are closed through a real-time network are called networked control systems (NCSs). The nature of NCS is that information, such as reference input, plant output and control signals is exchanged among control system component (sensors, controller, actuators, etc) by communication channel.

One of the primary effects and major control challenges in NCSs is the presence of uncertain network-induced delays stemming from the very fact of utilizing a common communication channel for closing the loop as well as additional functionality required for physical signal coding and communication processing. It is well known that the occurrence of time delay degrades the system dynamic performance and is a

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source of potential instability; therefore it must be taking into account in the analysis and design of NCS and some considerable attentions have been directed to these issues[1]-[4], [7].

In this paper, we consider the modeling and control problem of networked systems with different loop topologies. The remainder of this paper is structured as follows: In Section II, timing problem in NCS is introduced briefly and the representative topological structures of NCS are summarized. In Section III, the standard discrete state-space model is established by introducing state augmentation for both short and long time-delay. Augmented state pole-placement algorithm is presented in Section IV and conclusions in Section V.

2. Problem Description

Timing problems are more apparent when control loops are closed over a communication network. NCS has three nodes, each dedicated to the sensor, controller and actuator, respectively. During data exchange among these nodes connected to the shared medium, network induced delays occur between the source node and destination node. There are essentially three kinds of delays in the system, as shown in Fig.1.

(1) Communication delay between the sensor and the controller τ_{sc}

(2) Computational delay in the controller τ_c

(3) Communication delay between the controller and the actuator τ_{ca}

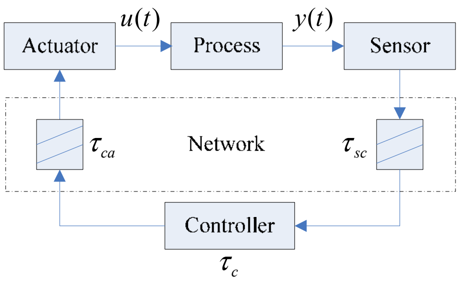


Fig. 1. NCS with induced time delay

Generally speaking, computational time of controller can be included in communication delay between controller and actuator. Therefore, the above three delays can be abbreviated as τ_{sc} and τ_{ca}.

Network induced delay has different characteristics depending on the network hardware and software, and the delay is typically varying due to varying network load, network protocols adopted, scheduling policies in the network. The simplest model of the network delay is to model it as being constant for all transfers in the communication network. However, the features of networked induced time-delays could be constant, bounded, or even random, depending on the network protocols adopted [3][5-6].

In this paper, two classes of delay termed as short time-delay and long time-delay are considered.

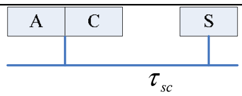
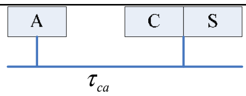
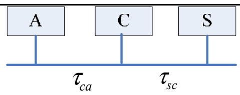
Definition 1: The time-delay τ is called short time-delay if $\tau \in [0, \alpha]$ and $\alpha \le T$.

Definition 2: The time-delay τ is called long time-delay if $\tau \in [0, \alpha]$ and $\alpha > T$.

where τ is network induced time-delay, T is constant sampling period.

In order to get the model of a networked system, we need to consider the specific control loop topology first. According to the distribution of the communication channel among the system component, the representative topological structure of NCS can be summarized as follows:

Table 1. NCS with different loop topologies (A: Actuator C: Controller S: Sensor)

	*Topological structure 1* $\tau=\tau_{sc}$
	*Topological structure 2* $\tau=\tau_{ca}$
	*Topological structure 3* $\tau=\tau_{sc}+\tau_{ca}$

In *Topological structure 1*, the controller and the actuator are combined together into a single node and the sensors transmit the plant information to the controller through network. Since this delay occurs before the control signal generated, indeterminate time-delay can be known to the controller node by use of time-stamping[2]. Although the delay is indeterminate, it can be known at the beginning of each controller execution. Marked all transferred signals with the time they were generated, by comparing the "timestamp" with the internal clock of the controller the time-delay can be calculated. Hence, the time-delay can be expressed by $\tau=\tau_{sc}$.

In the second case, the sensor and the controller are combined together and the controller transmits the control signal to the actuator through network. The system is characterized by the controller to actuator delay τ_{ca} . Unfortunately, since this delay takes place after the controller execution, it can not be known at the beginning.

Topological structure 3 is the more general and realistic NCS structure, that is, all information in NCS is exchanged among system component by communication network The system is characterized by the sensor to controller delay τ_{sc} and controller to actuator delay τ_{ca} . Similarly, the former can be known at the beginning of controller execution and the latter can not.

An approach to overcome the problem of uncertain delay in *Topological structure 2* and *3* is to introduced buffers after the varying communication delays to make the system with certain delay[8]. By introducing buffers longer than the worst case delay, all delays are known before the control tasks execution. Then, the time-delay in *Topological structure 2* and *3* can be expressed by $\tau=\tau_{ca}$ and $\tau=\tau_{sc}+\tau_{ca}$, respectively.

Fig.2 shows the equivalent diagram of NCS with determinate delays.

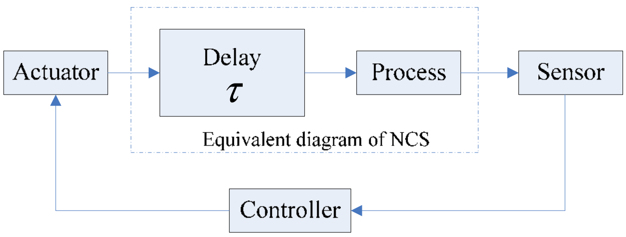


Fig. 2. Equivalent diagram of NCS with determinate delays

3. Modeling of NCS with Delay

Once the time-delay is obtained by the controller, the continuous-time state-space model of the plant can be described by the following standard form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t-\tau) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where the state $x(t) \in \Re^n$, the control $u(t) \in \Re^m$, the output $y(t) \in \Re^r$ and the constant matrices A, B and C.

The general solution of (1) is:

$$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-\lambda)}Bu(\lambda-\tau)d\lambda \tag{2}$$

Let $t_0 = kT$ and $t = kT + T$, then (2) can be rewrite as:

$$x(kT+T) = e^{AT}x(kT) + \int_{kT}^{kT+T} e^{A(kT+T-\lambda)}Bu(\lambda-\tau)d\lambda$$

Let $\xi = kT + T - \lambda$ and substitute to above equation, we obtain:

$$x(kT+T) = e^{AT}x(kT) + \int_{kT}^{kT+T} e^{A\xi}Bu(kT+T-\xi-\tau)d\xi \tag{3}$$

Write the time-delay as :

$$\tau = lT - mT \tag{4}$$

where $l \in Z^+$ and $0 \le m < 1$. It is obvious that short time-delay and long time-delay can be expressed as $l = 1$ and $l > 1$, respectively. Substitute (4) to (3), we have:

$$x(kT+T) = e^{AT}x(kT) + \int_{0}^{T} e^{A\xi}Bu(kT+T-lT+mT-\xi)d\xi$$

If we sketch a segment of the time axis near $t = kT - lT$, as shown in Fig.3, we can break the integral in the above equation into two parts as follows:

$$x(kT+T) = e^{AT}x(kT) + \int_{0}^{mT} e^{A\xi}Bd\xi \cdot u(kT+T-lT) + \int_{mT}^{T} e^{A\xi}Bd\xi \cdot u(kT-lT)$$

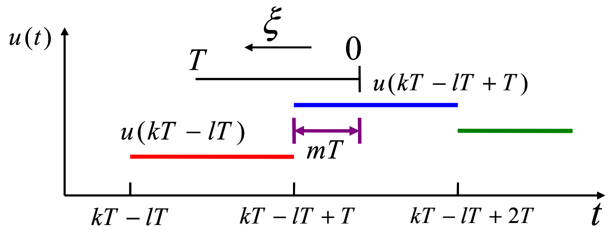


Fig. 3. Time-delay system with piecewise input

Define $\Phi = e^{AT}$, $\Gamma_1 = \int_{mT}^{T} e^{A\xi} B d\xi$, $\Gamma_2 = \int_{0}^{mT} e^{A\xi} B d\xi$, we have:

$$x(kT+T) = \Phi x(kT) + \Gamma_1 u(kT - lT) + \Gamma_2 u(kT + T - lT)$$

3.1. Short time-delay

First, we consider the case of short time-delay. Here $l = 1$ and $\tau = lT - mT < T$, the system can be rewrite as:

$$x(k+1) = \Phi x(k) + \Gamma_1 u(k-1) + \Gamma_2 u(k) \quad (5)$$

Define a new state $x_{n+1}(k+1) = u(k-1)$, construct the augmented state as $X(k) = [x(k) \quad x_{n+1}(k)]^T$, we have thus an increased dimension of the state and the system equations are:

$$X(k+1) = \begin{bmatrix} \Phi & \Gamma_1 \\ 0 & 0 \end{bmatrix} X(k) + \begin{bmatrix} \Gamma_2 \\ 1 \end{bmatrix} u(k)$$
$$y(k) = [C \quad 0] X(k) \quad (6)$$

3.2. Long time-delay

The second case is long time-delay. Consider $l > 1$ and the system are:

$$x(k+1) = \Phi x(k) + \Gamma_1 u(k-l) + \Gamma_2 u(k-l+1)$$

Define l new state variables $x_{n+2}(k) = u(k-l+1)$, $x_{n+2}(k) = u(k-l+1)$, $\cdots$, $x_{n+l}(k) = u(k-1)$, construct the augmented state as $X(k) = [x(k) \quad x_{n+1}(k) \quad x_{n+2}(k) \quad \cdots \quad x_{n+l}(k)]^T$, the system equations are:

$$X(k+1) = \begin{bmatrix} \Phi & \Gamma_1 & \Gamma_2 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} X(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(k)$$
$$y(k) = [C \quad 0 \quad \cdots \quad 0] X(k) \quad (7)$$

4. Pole-placement Algorithm

By introducing augmented state, we have obtained the standard discrete state-space model of the NCS for both short and long time-delay, as shown in Eq.(6) and (7). Denote the standard form as:

$$X(k+1) = \Lambda X(k) + \Gamma u(k)$$
$$y(k) = \Theta X(k) \quad (8)$$

Assume that system (8) is controllable, then by using state feedback control law $u = -KX$, the close-loop system can be described by:

$$X(k+1)=(\Lambda-\Gamma K)X(k) \tag{9}$$

Therefore the z -transform of (9) is:

$$(zI-\Lambda+\Gamma K)X(z)=0$$

and the characteristic equation of the system with hypothetical control law is:

$$\left|zI-\Lambda+\Gamma K\right|=0 \tag{10}$$

The control law consists of finding K so that the roots of (10), that is, the poles of the close-loop system, are in the desired locations. But for (8), traditional pole-placement algorithm must be modified since the dimension of the system with delay is increased due to the introduction of the augmented state. It can be seen from (6) and (7) that the actual dimension, denote as $p(p>n)$, of the system depending on the time-delay. For short time-delay, $p=n+1$ and long time-delay $p=n+l$.

For (8), there will be p actual eigenvalues and n desired eigenvalues. Hence, when use pole-placement algorithm, one can select a pair of dominant pole and let other poles far from the dominant pole. The modified pole-placement algorithm can be summarized in a flow chart given in Fig.4.

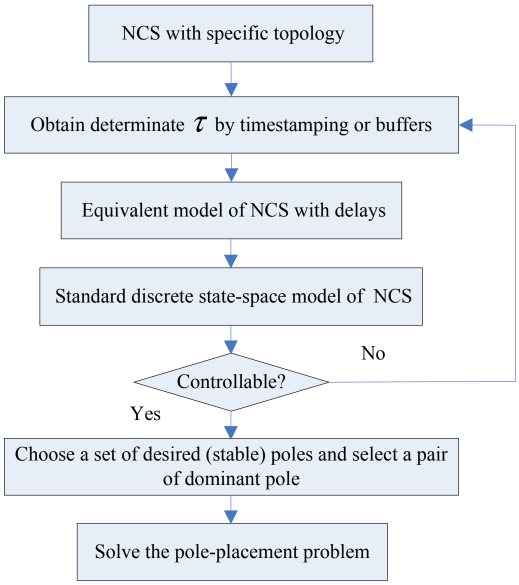


Fig. 4. Pole-placement algorithm

5. Conclusions

In this paper, the problem of modeling and control for three significant classes of NCS were presented. We first use the timestamping and buffers to obtain the time-delay. Then, the standard discrete state equation is transformed from indeterminate model of networked control system by introducing state augmentation for both short and long time-delay. In the end, the controller is designed by augmented state pole-placement algorithm.

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