Decomposition of $K_{13}$ into a torus graph and a graph imbedded in the Klein bottle

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In [2], Ringel gives a decomposition of $K_{13}$ into two toroidal graphs. The second of us raised a question if $K_{13}$ may be decomposed into a toroidal graph and a graph imbedded in the Klein bottle (these surfaces have the same Euler
We have obtained, independently, the following two solutions which differ at least on the number of the vertices of degree 3.

As for the decomposability of $K_{13}$ into two graphs imbedded in the Klein bottle, the opinions of the authors are slightly divergent: the problem seems hopeless to both of us, but the first author conjectures that the decomposition does exist.

Observe that, in contrast to this failure, a quite similar representation of $K_{13}$ as a 2-pire map on the Klein bottle was found recently [1]. Finally, note that a representation of $K_{13}$ as a 2-pire map on the torus was also obtained by Ringel [2].

**References**
