ELSEVIER

Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia Engineering 56 (2013) 157 – 162

Procedia Engineering

www.elsevier.com/locate/procedia

5th BSME International Conference on Thermal Engineering

# Double diffusive mixed convection in a channel with a circular heater

A.K. Azad[a], M.J.H. Munshi[b], M.M. Rahman[c]*

[a]*Department of Business Administration, International Islamic University, Chittagong, Bangladesh*
[b]*Department of Mathematics, Hamdard University Bangladesh, Gozaria, Munshiganj, Bangladesh*
[c]*Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh*

**Abstract**

Double diffusive mixed convection in an open channel with a circular heater on the bottom wall has been investigated in this paper. Constant temperatures and concentrations are considered along the semi-circle and the lateral walls of the channel are adiabatic. Galerkin weighted residual finite element method have been used to solve the governing equations. Calculations were performed for Rayleigh numbers and Lewis number. Reynolds and Prandtl numbers are fixed as 100 and 0.7 for whole study, respectively. Various characteristics such as streamlines, isotherms, isoconcentration and heat and mass transfer rate in terms of the average Nusselt number and Sherwood numbers are presented for the aforesaid parameters. It is found that, average Nusselt number at the heat source decreases and overall mass transfer rate in terms of average Sherwood number increases with the rising of Lewis number. In addition, Rayleigh numbers have also significant effects on the heat and mass transfer process.



*Keywords*: Channel flow; finite element method; circular cavity; double diffusive mixed convection.

**Nomenclature**

| | | | |
|---|---|---|---|
| *Br* | buoyancy ratio | $T_i$ | inlet flow temperature |
| *c* | mass concentration (kg m$^{-3}$) | $(u, v)$ | velocity components (ms$^{-1}$) |
| $c_h$ | high mass concentration | $(U, V)$ | dimensionless velocity component |
| $c_i$ | Inlet mass flow concentration | *w* | height of the channel |
| *C* | dimensionless mass concentration | $(x, y)$ | dimensional coordinates (m) |
| *D* | mass diffusivity (m$^2$s$^{-1}$) | $(X, Y)$ | dimensionless coordinates |
| *g* | gravitational acceleration (ms$^{-2}$) | *Greek symbols* | |
| *k* | fluid conductivity (Wm$^{-1}$K$^{-1}$) | $\alpha$ | thermal diffusivity (m$^2$s$^{-1}$) |
| *L* | length of cavity (m) | $\beta_T$ | coefficient of thermal expansion (K$^{-1}$) |
| *Le* | Lewis number | $\beta_c$ | coefficient of mass expansion (m$^3$kg$^{-1}$) |
| *Nu* | average Nusselt number | $\mu$ | dynamic viscosity (kg m$^{-1}$s$^{-1}$) |
| *p* | dimensional pressure ($Nm^{-2}$) | $\nu$ | kinematic viscosity (m$^2$s$^{-1}$) |
| *P* | non-dimensional pressure | $\theta$ | non-dimensional temperature |
| *Pr* | Prandtl number | $\rho$ | Density (kgm$^{-3}$) |
| *Re* | Reynolds number | $\rho^*$ | dimensionless density |
| *Ri* | Richardson number | $\psi$ | streamfunction |
| *Sh* | average Sherwood number | *subscripts* | |

* Corresponding author. Tel.: +8801552556549; fax: +880-2-8613046.
*E-mail address:* m71ramath@gmail.com


doi:10.1016/j.proeng.2013.03.102

| | | | |
|---|---|---|---|
| $T$ | temperature (K) | i | inlet state |
| $T_h$ | hot wall temperature | | |

## 1. Introduction

Mixed convection is that type of heat transfer in which there is a noteworthy interaction between free and forced convection. Mixed convective heat transfer in open cavities has long been studied and received increases attention due to its application of practical interest, such as nuclear reactors, solar receiver, thermal storage and open cavity packaging of semiconductors. Studies associated with mixed convection in open cavities have received increasing consideration. Pavlovic and Penot, 1991, performed an experimental investigation of the mixed convection heat transfer in an open isothermal cubic cavity. They concluded that the convective heat loss for the central solar receiver. A numerical analysis of laminar mixed convection in a channel with an open cavity and a heated wall bounded by a horizontally insulated plate was presented by Manca et al., 2003, where the authors considered three heating modes: assisting flow, opposing flow and heating from below. Later, a similar problem for the case of assisting forced flow configuration was tested experimentally by Manca et al., 2006.

The double-diffusive mixed convection in a channel with an open enclosure has also found wide applications in engineering, such as cooling of electronic components, finned heat exchangers, cavity of solar central receivers, chemical processing, thermal and pollution control, evaporative cooling and fire control in buildings. There are several studies related to mixed convection for combined heat, and mass transfer. Deng et al., 2004, made a numerical study for a laminar double diffusive mixed convection in a two-dimensional ventilated enclosure with discrete heat and contaminant sources. They investigated the characteristics of the airflow and heat/contaminant transport structures in the indoor air environment by means of a convection transport visualization technique. At the same time, Costa, 2004, carried out a numerical study for double-diffusive natural convection in parallelogrammic enclosures filled with moist air. Chamkha and Naser, 2001, studied the problem of unsteady, laminar double-diffusive convective flow of a binary gas mixture in an inclined rectangular enclosure filled with a uniform porous medium. A numerical simulation of double diffusive natural convection in rectangular enclosure in the presence of magnetic field and heat source was performed by Teamah, 2008. Teamah and El-Maghlany, 2010, numerically simulated double-diffusive mixed convective flow in a rectangular enclosure with insulated moving lid. Brown and Lai 2005 numerically investigated a horizontal channel with an open cavity and obtained correlations for combined heat and mass transfer which covered the entire convection regime from natural, mixed to forced convection.

In view of the aforesaid statements, it is seemed that the double diffusive mixed convection in a channel with a circular cavity has not been addressed yet. In the present study, we undertake this task varying the Lewis number *Le* ( $0.1 \leq Le \leq 10$) and Rayleigh number *Ra* ($10^3 \leq Ra \leq 10^5$) .A comprehensive study of the flow field, temperature and concentration distribution with detailed analysis on heat and mass transfer evaluation will be done.

## 2. Physical model

The physical system under study with the system of coordinates is sketched in Fig. 1. The problem deals with a two-dimensional open channel of length $L$ with semi-circular heater of diameter $0.5L$. The flat walls are adiabatic and semi-circular wall is under the boundary conditions with a high temperature $T_h$ and high concentration $C_h$. It is assumed that the height of the channel $w = 0\ 25L$. The forced flow of fresh air, imposed at the inlet, has a temperature $T_i$, a concentration $c_i$ and a horizontal velocity, $u_i$. Gravity acts in the vertical direction.

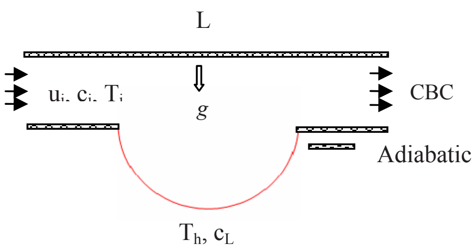


Fig.1. Physical configuration for the problem with boundary conditions

## 3. Mathematical formulation

The flow is considered steady, laminar, incompressible and two-dimensional. The field equations governing the heat transfer and fluid flow include the continuity equation, the Navier–Stokes equations and the energy equation, which can be expressed in non-dimensional form as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)+\frac{Ra}{\mathrm{Re}^2\,\mathrm{Pr}}\left(\theta+BrC\right) \quad (3)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re\,Pr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (4)$$

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{\mathrm{Re\,Pr}\,Le}\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right) \quad (5)$$

where the dimensionless variables are introduced as:

$$X=\frac{x}{L},Y=\frac{Y}{L},U=\frac{u}{u_i},V=\frac{v}{u_i},P=\frac{p}{\rho u_i^2},\theta=\frac{T-T_i}{T_h-T_i}\ \text{and}\ C=\frac{c-c_i}{c_h-c_i}$$

As can be seen from the Eqs. (1)-(5), five parameters that preside over this problem are the Reynolds number (*Re*), Prandtl number (*Pr*), Rayleigh number (*Ra*), Lewis number (*Le*) and buoyancy ratio (*Br*), which are defined respectively as

$$\mathrm{Re}=\frac{u_i L}{\nu},\ \mathrm{Pr}=\frac{\nu}{\alpha},\ Ra=\frac{g\beta\left(T_h-T_i\right)L^3}{\nu\alpha},\ Le=\frac{\alpha}{D},\text{and}\ Br=\frac{\beta_c\left(c_h-c_i\right)}{\beta_T\left(T_h-T_i\right)}$$

The dimensionless boundary conditions corresponding to the considered problem are as follows

at inlet: $U=1,\ V=0,\ \theta=0, C=0$

at outlet: $\frac{\partial U}{\partial X}=0, V=0, \frac{\partial \theta}{\partial X}=0, \frac{\partial C}{\partial X}=0$

at all solid boundaries other than semi-circle: $U,V=0,\ \frac{\partial \theta}{\partial N}=\frac{\partial C}{\partial N}=0$

on semi-circle: $U=V=0, \theta=C=1$

where *N* is the non-dimensional distances either *X* or *Y* direction acting normal to the surface.

The average heat and mass transfer rates on the surface of heat and contaminant sources can be evaluated by the average Nusselt and Sherwood numbers, which are defined respectively as

$$Nu=-\int_0^1 \frac{\partial \theta}{\partial Y}dX\ \text{and}\ Sh=-\int_0^1 \frac{\partial C}{\partial Y}dX$$

## 4. Solution scheme

The Galerkin weighted residual method of finite element formulation has been used as a numerical procedure. The finite element method begins by the partition of the continuum area of interest into a number of simply shaped regions called elements. These elements may be different shapes and sizes. Within each element, the dependent variables are approximated using interpolation functions. In the current study erratic grid size system is considered especially near the walls to capture the rapid changes in the dependent variables. The coupled governing equations (2)-(5) are transformed into sets of algebraic equations using finite element method to reduce the continuum domain into discrete triangular domains.

The system of algebraic equations is solved by iteration technique. The solution process is iterated until the subsequent convergence condition is satisfied:
$\left|\Gamma^{m+1} - \Gamma^{m}\right| \leq 10^{-6}$ where $m$ is number of iteration and $\Gamma$ is the general dependent variable.

## 5. Results and discussion

A numerical study has been performed to determine the effects of the double-diffusive mixed convection flow in a horizontal channel with an open cavity. For the intention of discussing the results, the numerical calculations are presented in the form of streamlines, isotherms, and concentration. With this aim, different parameters such as, Lewis number (*Le*) and Rayleigh number (*Ra)* are considered. In addition, the Lewis number *Le*, characterizes the mass transfer exchange between the two different concentration zones Moreover, Reynolds number (*Re*), Prandtl number (*Pr*), and buoyancy ratio (*Br*) are held fixed at 100, 0.7, and 1, respectively.

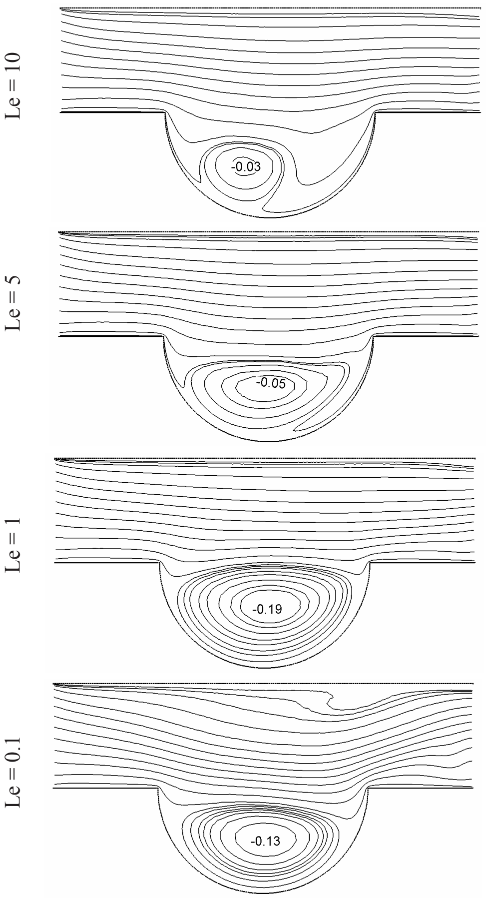


Fig. 2. Streamlines for selected values of Lewis number, *Le* at Ra = 1.E+5.

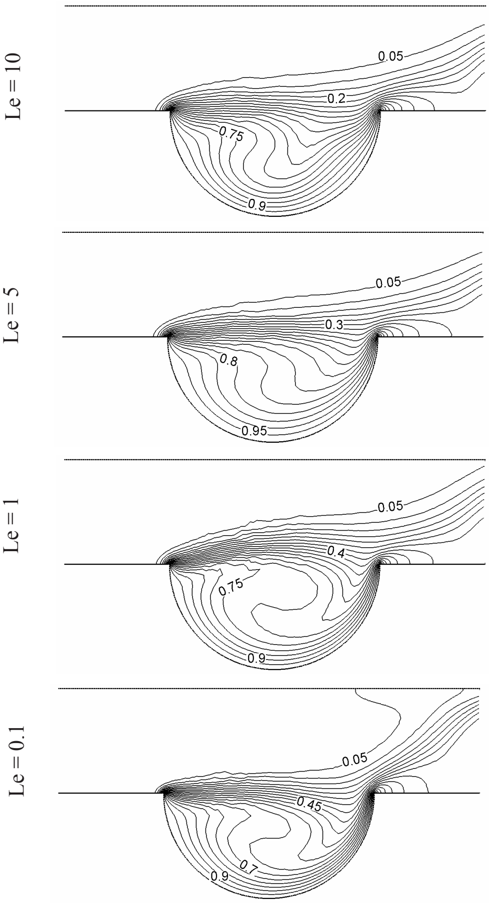


Fig. 3. Isotherms for selected values of Lewis number, *Le* at Ra = 1.E+5.

The influence of Lewis number, *Le* on streamlines and isotherms are demonstrated in Figs. 2-3. The Lewis number *Le* varied from 0.1 to 10 while keeping *Br* = 1 fixed. In Fig. 2, the streamlines in the through flow sprint almost analogous, apart from for those near the opening to the cavity, which twist near the cavity inside. Besides, the flow inside the lower part of the cavity is seen as a large rotating cell in a clockwise direction. Shape change was detected in the rotating cell due to the variation of *Le* inside the cavity. On the other hand, Lewis number has a visible significant effect on the strength of the rotating cell inside the cavity.

The corresponding isotherms are clustered deeply near the bottom surface of the cavity which indicates steep temperature gradients at the vertical direction in this cavity as shown in Fig. 3. In the residual region of the cavity, the temperature gradients are feeble and this implies that the temperature differences are very minute in the interior region of the cavity due to the strong effects of the mechanically-driven circulations.

Fig. 4 illustrates a concentration for different Lewis number at *Br* = 1. From this figure, it is seen that for *Le* = 0.1, the concentration contours in the cavity align almost parallel to the horizontal wall, which is evocative of the supremacy of high mass transfer. Moreover, a significant change in the concentration contours is noticeable for the higher values of Lewis number.

The effect of Lewis number on the average value of Nusselt and Sherwood numbers at $Ra = 10^3, 10^4, 10^5$ is illustrated in Fig. 5. For lower values of Ra (= $10^3$, $10^4$), the average Nusselt number decreases very slowly as the Lewis number is increased. But the average Sherwood number increases swiftly with the increasing of Lewis number for lower Ra. On the other hand, for Ra = $10^5$, the average Nusselt number decreases constantly with the increasing *Le* up to 5, later it increases. It is also observed that the average Sherwood number with the increasing *Sh* up to 1 then it decreases for Ra = $10^5$ .

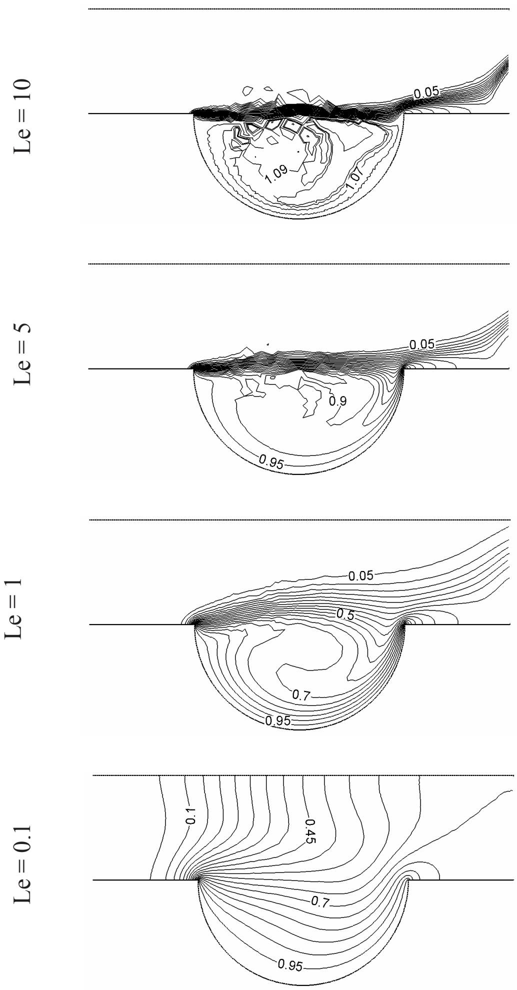


Fig. 4. Isoconcentration for selected values of Lewis number, Le at Ra = 1.E+5.

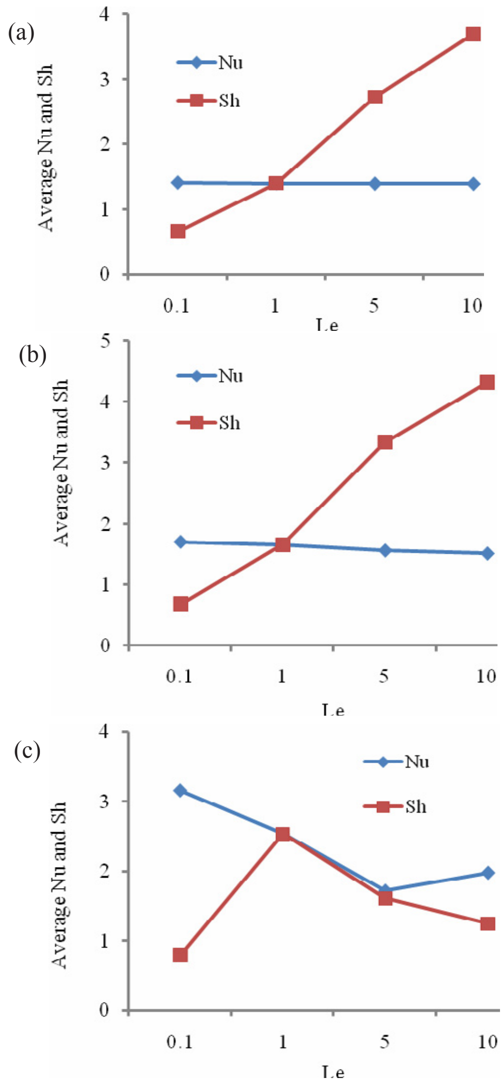


Fig. 5. Average Nusselt and Sherwood numbers for (a) Ra = 1.E+3, (b) Ra = 1.E+4 and (c) Ra = 1.E+5 versus Lewis number

## 6. Concluding remarks

This work deals with the effect of double-diffusive mixed convection in a horizontal channel along with an open semi circular cavity. The finite element method is engaged for numerical simulation. Graphical results of the flow structure, temperature, and concentration for Lewis number *Le* is presented and discussed. The following conclusions may be drawn from the above mentioned study:

- The control of the Lewis number on the isotherms is noteworthy. Moreover, strength of buoyancy-induced vortex decreases with the increase of the Lewis number in the streamlines.
- Overall mass transfer rate in terms of average Sherwood number increases with the augmented of Lewis number *Le*.

The results of this study suggest several guidelines for the thermal design of cooling of electronic components, finned heat exchangers, cavity of solar central receivers, etc.

## Acknowledgements

The authors like to express their gratitude to the Department of Mathematics, Bangladesh University of Engineering and Technology (BUET), Dhaka, Bangladesh.

## References

[1] Pavlovic MD, Penot F. Experiments in the mixed convection regime in an isothermal open cubic cavity. Exp Therm Fluid Sci 1991; 4 **:** 648–55.
[2] Manca O, Nardini S., Khanafer K, Vafai K. Effect of heated wall position on mixed convection in a channel with an open cavity. J Numer Heat Transfer 2003; 43 (3); 259-82.
[3] Manca O, Nardini S, Vafai K. Experimental investigation mixed convection in a channel with an open cavity. Exp Heat Transfer 2006; 19 (1): 53–68.
[4] Deng QH, Zhou J, Mei C, Shen YM. Fluid, heat and contaminant transport structures of laminar double-diffusive mixed convection in a two-dimensional ventilated enclosure. Int J Heat Mass Transfer 2004; 47**:** 5257–69.
[5] Costa VAF. Double-diffusive natural convection in parallelogrammic enclosures. Int J Heat Mass Transfer 2004; 47: 2913–26.
[6] Chamkha AJ, Naser HA. Double-diffusive convection in an inclined porous enclosure with opposing temperature and concentration gradients. Int J Therm Sci 2001; 40: 227-44.
[7] Teamah MA. Numerical simulation of double diffusive natural convection in rectangular enclosure in the presences of magnetic field and heat source. Int J Therm Sci 2008; 47: 237-48.
[8] Teamah MA, El-Maghlany WM. Numerical simulation of double-diffusive mixed convective flow in a rectangular enclosure with insulated moving lid. Int J Therm Sci 2010; 49: 1625–38.
[9] Brown NM, Lai FC. Correlations for combined heat and mass transfer from an open cavity in a horizontal channel. Int Commun Heat Mass Transfer 2005; 32(8): 1000-8.