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CPT/Lorentz invariance violation and neutrino oscillation

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Abstract

We analyze the consequences of violation of Lorentz and CPT invariance in the massless neutrino sector by deforming the canonical anti-commutation relations for the fields. We show that, for particular choices of the deformation, oscillation between massless neutrino species takes place when only Lorentz invariance is violated. On the other hand, if both Lorentz and CPT invariances are violated, we show that there is no oscillation between massless neutrino species. Comparing with the existing experimental data on neutrino oscillations, we obtain bounds on the parameter for Lorentz invariance violation.

1. Introduction

There has been an increased interest in the possibility that Lorentz and CPT symmetries may be violated at very high energies. For example, recent developments in quantum gravity suggest that Lorentz invariance may not be an exact symmetry at high energies [1] and CPT invariance has also been questioned within such contexts [2]. Spontaneous violation of CPT and Lorentz symmetries can arise in string theories [3] and the violation of Lorentz invariance in non-commutative field theories is well known [4]. On the experimental side, the UHE (ultra high energy) cosmic ray events seen at AGASA [5] and presently under study by AUGER [6] further support the possibility that Lorentz and CPT invariances may not hold at such energies. Of course, there already exist very stringent bounds on Lorentz and CPT violation from laboratory experiments in the kaon and the lepton sectors and any violation of these symmetries has to be compatible with these limits. Nonetheless, it is possible that even a tiny violation of CPT and Lorentz invariance can lead to interesting mechanisms for physical phenomena. In a recent Letter, for example, we have shown [7] how such a violation can lead to baryogenesis in thermal equilibrium (evading one of the criteria of Sakharov). In this Letter, we analyze the consequences of Lorentz and CPT violation in the neutrino sector. We would like to emphasize that several papers have already dealt with the effects of Lorentz [8,9] and CPT violation in the neutrino sector, particularly in connection with a qualitative discussion of neutrino oscillation in this scenario [10] (in another related context see [11]). In this Letter, we carry out a quantitative study of such phenomena within the context of a simple model and derive bounds on such symmetry violating parameters from the existing experimental results on neutrino oscillation.

Neutrino oscillation is an interesting phenomenon proposed about fifty years ago by Pontecorvo (in a different context) which is used to explain the deficit of solar and atmospheric neutrinos in fluxes measured on earth [12–14] (for other recent analysis see [15]). This mechanism which is responsible for the resolution of these puzzles is closely related to the K^0–$\bar{K}^0$ oscillation [16]. In its simplest form, the probability for oscillation between two species of particles i, j with a mixing angle θ_{ij} and

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energy levels E_i, E_j in a time interval t is given by

$$P_{i\to j}(t) = \sin^2(2\theta_{ij})\ \sin^2\left(\frac{\Delta E_{ij}t}{2}\right), \tag{1}$$

where

$$\Delta E_{ij} = E_i - E_j. \tag{2}$$

If the oscillation is between two neutrino species ν_i, ν_j with small masses m_i, m_j respectively, then in the conventional scenario one expands (this assumes Lorentz invariance and $c = 1$)

$$E_i = \sqrt{p^2 + m_i^2} \approx p + \frac{m_i^2}{2p} = p + \frac{m_i^2}{2E_i}, \tag{3}$$

so that we have

$$\Delta E_{ij} = E_i - E_j \approx \frac{m_i^2 - m_j^2}{2E} = \frac{\Delta m_{ij}^2}{2E}, \tag{4}$$

where we have assumed that for neutrinos of small mass, $E_i \approx E_j = E$. In this case, the probability for oscillation between the two neutrino species in traversing a path length L can be written as (see (1) and (4))

$$P_{\nu_i\to\nu_j}(L) = \sin^2(2\theta_{ij})\sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right) \\ = \sin^2(2\theta_{ij})\sin^2\left(\frac{1.27\Delta m_{ij}^2 L}{E}\right), \tag{5}$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$ is taken in $(\mathrm{eV})^2$, the neutrino energy E in MeV and the length of path traversed in 'm' (meters). (In the last line of the above formula, we have restored all the nontrivial constants as well as traded the time interval for the path length assuming that the neutrino travels almost at the speed of light.)

It follows from Eq. (5) that neutrino oscillation does not take place in free space if neutrinos are massless or (when massive) are degenerate in mass. With three families of neutrinos, there can only be two independent combinations of squared mass differences, say Δm_{12}^2, Δm_{23}^2 which are sufficient to find a solution for the solar neutrino as well as the atmospheric neutrino puzzles. Within the Standard Model, this can be achieved with the bounds [17]

$$\Delta m_{12}^2 \leqslant 10^{-4}\ \mathrm{eV}^2, \qquad 10^{-3}\ \mathrm{eV}^2 \leqslant \Delta m_{23}^2 \leqslant 10^{-2}\ \mathrm{eV}^2. \tag{6}$$

Given these, the bound on $\Delta m_{13}^2 = \Delta m_{12}^2 + \Delta m_{23}^2$ is determined. There is no further freedom within a model with three families of neutrinos.

Several experiments by now have looked for neutrino oscillations. One such experiment, namely, the LSND (Liquid Scintillator Neutrino Detector at Los Alamos) [18] has used muon sources from the decay $\pi^+ \to \mu^+ + \nu_\mu$. The experiment looks for neutrino oscillation in the subsequent decay of the muon through $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$. After a path length of $L = 30$ m, the experiment finds the oscillation channel $\bar{\nu}_\mu \to \bar{\nu}_e$ (with $20\ \mathrm{MeV} \leqslant E_{\nu_\mu} \leqslant 58.2\ \mathrm{MeV}$) with a probability of 0.26%. The experiment also reports the existence of the oscillation $\nu_\mu \to \nu_e$ with the same probability. Furthermore, the analysis of the results of this experiment, following (5), leads to a bound on the difference of the relevant squared mass difference to be

$$\Delta m^2 < 1\ \mathrm{eV}^2. \tag{7}$$

However, the MiniBoone experiement, which was expected to verify the results of LSND, has recently reported their first result [19] and excludes the mass region in (7). As a result, the simple explanation of the LSND results, based on the two flavor neutrino oscillation is ruled out.

All of the above discussion has been within the context of the Standard Model with a massive neutrino where both Lorentz invariance and *CPT* are assumed to hold. On the other hand, if Lorentz invariance or *CPT* or both are violated in the neutrino sector, it has been suggested that neutrino oscillation can take place in free space even for massless neutrinos (in contrast to Eq. (5) where Lorentz invariance is assumed). This was pointed out by Coleman and Glashow [20] and developed more extensively by Kostelecky and collaborators [21,22]. This is particularly clear from Eq. (1) where we see that the probability of oscillation really depends on the difference in the energy of the two neutrino species and if $\Delta E_{ij} = E_i - E_j \neq 0$ even when the masses vanish, the probability of oscillation will be nontrivial. This can happen, for example, if the two neutrino species have different (energy) dispersion relations. This possibility has been discussed extensively in the last few years by various groups [21,23]. In particular, Ref. [21] analyzes the structure of the most general Lagrangian with violations of Lorentz invariance and *CPT* in an attempt to understand the discrepancy between solar and atmospheric neutrinos and the LSND anomaly [18].

The goal of this Letter is to analyze the consequences of violations of Lorentz and *CPT* invariances in the massless neutrino sector from the point of view of a non-commutative field theory where such violations are more natural. In such a model, the violation of Lorentz and *CPT* invariances is implemented through a deformation of the canonical anti-commutation relations for the neutrino fields. Such a model can be thought of as a subclass of the general model proposed in [21], but since, depending on the deformation, we have fewer arbitrary parameters, we naturally have more predictive power. The result of our analysis can be summarized as follows. If there is violation of only Lorentz invariance, then oscillations between massless neutrino species can take place and comparing with the existing experimental data, we can determine bounds on the parameter characterizing Lorentz invariance violation. On the other hand, if both Lorentz and *CPT* invariances are violated, there is no oscillation between massless neutrino species.

2. The model and the phenomenology of neutrino oscillation

The model that we will describe below is inspired by the quantum theory of non-commutative fields developed in [24]. The quantum theory of fermionic non-commutative fields (neutrinos) is obtained from the standard fermionic quantum field theory by deforming the anti-commutation relations while retaining the usual Hamiltonian. In order to explain in some detail the construction, let us consider the conventional Lagrangian density for two flavors of massless fermions (neutrinos) given by

$$\mathcal{L} = i\bar{\psi}^i\gamma^\mu\partial_\mu\psi^i, \tag{8}$$

where the superscript $i = \{1, 2\}$ runs over the flavor quantum number (sum over repeated indices is understood).

The Hamiltonian density has the form

$$\mathcal{H} = -i\left(\psi^{i\dagger}\vec{\alpha}\cdot\vec{\nabla}\psi^i\right), \tag{9}$$

where $\vec{\alpha} = \gamma^0\vec{\gamma}$. With the conventional canonical anti-commutation relations for the fermion fields, one would obtain the standard relativistic equations for the massless neutrinos using the Hamiltonian following from (9). However, the noncommutative theory is obtained by deforming the canonical anti-commutation relations while maintaining the form of the Hamiltonian density (9).

We postulate the deformed equal-time anti-commutation relations to have the form (with all others vanishing)

$$\left\{\psi^i_\alpha(\mathbf{x}), \psi^{j\dagger}_\beta(\mathbf{y})\right\} = \mathcal{A}^{ij}_{\alpha\beta}\delta^{(3)}(\mathbf{x}-\mathbf{y}), \tag{10}$$

where $\alpha, \beta, \ldots = 1, 2, 3, 4$ are spinor indices and $\mathcal{A}^{ij}_{\alpha\beta}$ is a constant matrix. In this Letter we consider the following two special choices for the structure of the deformation matrix $\mathcal{A}$:

(1) $\mathcal{A}$ has a nontrivial structure only in the flavor space.
(2) $\mathcal{A}$ depends on both the flavor and the spinor indices nontrivially through a constant background vector.

As we will see, the first choice leads to a violation of Lorentz invariance whereas both Lorentz and *CPT* invariances are violated with the second choice.

2.1. Deformation depending only on flavor indices

In this case, $\mathcal{A}$ is a 2×2 constant matrix with complex elements in general, which, for simplicity, can be chosen to have the form

$$\mathcal{A}^{ij} = \begin{pmatrix} 1 & \alpha \\ \alpha^* & 1 \end{pmatrix}, \tag{11}$$

so that the complex parameters α can be thought of as the parameters of deformation. Clearly, the deformed anti-commutation relations reduce to the conventional ones when the parameters of deformation vanish.

Given the deformed anti-commutation relations (10) and the Hamiltonian density (9), the dynamical equation takes the form

$$\dot{\psi}^i = -\mathcal{A}^{ij}\left(\vec{\alpha}\cdot\vec{\nabla}\psi^j\right), \tag{12}$$

which in momentum space takes the form

$$E\psi^i = \mathcal{A}^{ij}\left(\vec{\alpha}\cdot\vec{p}\,\psi^j\right). \tag{13}$$

In order to determine the energy eigenvalues for this system, let us consider the unitary matrix D

$$D = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{|\alpha|}{\alpha} & 1 \\ -\frac{|\alpha|}{\alpha} & 1 \end{pmatrix},$$

$$D^\dagger = D^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\alpha}{|\alpha|} & -\frac{\alpha}{|\alpha|} \\ 1 & 1 \end{pmatrix}. \tag{14}$$

It is straightforward to check that D diagonalizes $\mathcal{A}$ and as a result, the energy spectrum for the fermions follows to be ($c = 1$)

$$E^1_\pm = \pm\left(1+|\alpha|\right)|\vec{p}|,$$
$$E^2_\pm = \pm\left(1-|\alpha|\right)|\vec{p}|. \tag{15}$$

Here $E^{1,2}$ are the energies of the two species of the neutrinos considered. We, therefore, conclude that with this choice of the deformation, this system exhibits violation of Lorentz invariance, as can be seen from the dispersion relations (15). However, *CPT* symmetry remains intact in this case which can be easily seen as follows.

As we have emphasized, our deformation can be thought of as a subclass of the extended Standard Model (ESM) [2]. Indeed, if we restrict to the part of the Lagrangian in [2] given by

$$\mathcal{L} = i\bar{\psi}^i\gamma^\mu\partial_\mu\psi^i + i\bar{\psi}^i c^{\mu\nu,ij}\gamma_\mu\partial_\nu\psi^j, \tag{16}$$

with the constant background field $c^{\mu\nu,ij}$ diagonal in the space–time indices, then we obtain $\pi^i = i(\delta^{ij}+c^{00,ij})\psi^{j\dagger}$. This leads to the canonical anti-commutation relations

$$\begin{aligned}\left\{\psi^i(\mathbf{x}), \psi^{j\dagger}(\mathbf{y})\right\} &= \left(\delta^{ij}+c^{00,ij}\right)^{-1}\delta(\mathbf{x}-\mathbf{y}) \\ &= \mathcal{A}^{ij}\delta(\mathbf{x}-\mathbf{y}),\end{aligned} \tag{17}$$

which is in agreement with our deformation for the noncommutative fields. In this case, since the background field is a constant second rank tensor, the extra term

$$\bar{\psi}^i c^{\mu\nu}_{ij}\gamma_\mu\partial_\nu\psi^j,$$

violates Lorentz invariance but not *CPT* symmetry as was also pointed out in Ref. [2].

The energy eigenstates corresponding to the eigenvalues (15) can now be determined directly through the application of the diagonalizing matrix

$$D\begin{pmatrix}\psi^1\\ \psi^2\end{pmatrix} = \begin{pmatrix}\tilde{\psi}^1\\ \tilde{\psi}^2\end{pmatrix}, \tag{18}$$

where $\tilde{\psi}^1$ and $\tilde{\psi}^2$ are eigenstates with energy values E^1 and E^2 respectively.

The time evolution for the energy eigenstates, is determined to be

$$\tilde{\psi}^1(t) = e^{-iE^1_+t+i\vec{p}\cdot\vec{x}}\tilde{\psi}^1(0), \tag{19}$$
$$\tilde{\psi}^2(t) = e^{-iE^2_+t+i\vec{p}\cdot\vec{x}}\tilde{\psi}^2(0), \tag{20}$$

where $E^{1,2}_+$ are the energy eigenvalues determined in (15). The diagonal wavefunctions can be seen from (14) and (18) to have the explicit forms

$$\tilde{\psi}^1 = \frac{1}{\sqrt{2}}\left(\frac{|\alpha|}{\alpha}\psi^1+\psi^2\right),$$
$$\tilde{\psi}^2 = \frac{1}{\sqrt{2}}\left(-\frac{|\alpha|}{\alpha}\psi^1+\psi^2\right). \tag{21}$$

Thus, we see that if $\alpha \in \Re$, then the diagonal wavefunctions can be thought of as resulting from a rotation by $\pi/4$ in the flavor space. This is, in fact, consistent with the hypothesis of large mixing angle (LMA) [25] and, therefore, for simplicity let us choose α to be real. In this case, we can parametrize (21) as

$$\tilde{\psi}^1 = \cos\theta_{12}\psi^1 + \sin\theta_{12}\psi^2, \tag{22}$$

$$\tilde{\psi}^2 = -\sin\theta_{12}\psi^1 + \cos\theta_{12}\psi^2, \tag{23}$$

with the mixing angle $\theta_{12} = 45°$.

Relations (22) and (23) can now be inverted to give

$$\psi^1 = \cos\theta_{12}\tilde{\psi}^1 - \sin\theta_{12}\tilde{\psi}^2,$$
$$\psi^2 = \sin\theta_{12}\tilde{\psi}^1 + \cos\theta_{12}\tilde{\psi}^2. \tag{24}$$

Thus, a neutrino initially in the state ψ^1 would evolve in time as

$$\begin{aligned}\psi^1(t) &= \cos\theta_{12}\tilde{\psi}^1(t) - \sin\theta_{12}\tilde{\psi}^2(t)\\ &= \Big[\big(\cos^2\theta_{12}e^{-iE^1_+ t} + \sin^2\theta_{12}e^{-iE^2_+ t}\big)\psi^1(0)\\ &\quad + \frac{1}{2}\sin 2\theta_{12}\big(e^{-iE^1_+ t} - e^{-iE^2_+ t}\big)\psi^2(0)\Big]e^{i\vec{p}\cdot\vec{x}}.\end{aligned}$$

Therefore, at a later time t, the probability of finding the state ψ^2 in the beam is given by

$$\begin{aligned}P_{\nu_1\to\nu_2} &= \left|\frac{1}{2}\sin 2\theta_{12}\big(e^{-iE^1_+ t} - e^{-iE^2_+ t}\big)e^{i\vec{p}\cdot\vec{x}}\right|^2\\ &= \sin^2(2\theta_{12})\sin^2\big(|\alpha||\vec{p}|t\big),\end{aligned} \tag{25}$$

and since we are considering particles with velocities close to c, we can replace

$$t \to L,$$

where L denotes the path length traversed by the neutrino. Let us note here that in our theory, the velocity of the neutrino can in principle be different from c, but any further correction is suppressed by terms of the order $\mathcal{O}(\alpha^2)$ which is extremely small. Thus, the probability for oscillation (25) becomes

$$P_{\nu_1\to\nu_2} = \sin^2(2\theta_{12})\sin^2\big(|\alpha|EL\big), \tag{26}$$

where we have used the fact that for $|\alpha| \ll 1$, $E \approx |\vec{p}|$.

There are several things to note from the expression (26). First, the deformation parameter α leads to rotations in the flavor space and thereby determines the mixing angles. However, differences from the conventional description of massive neutrinos arise because these deformation parameters also determine the nontrivial dispersion relations for the energy eigenvalues and lead to a nontrivial energy difference even in the absence of masses. Consequently, oscillation takes place even for massless neutrinos. The difference from the conventional description of neutrino oscillation shows up in (26) in the fact that the energy dependence is linear as opposed to the inverse dependence in (5).

2.2. Vector-dependent deformations

In order to define the minimal matrix $\mathcal{A}$ which breaks both *CPT* and Lorentz symmetries and which connects different flavors—that is $\mathcal{A}^{12} \neq 0$ for two flavor indices—we need to include a constant background (real) vector e^{ij}_μ. With this, the minimal $\mathcal{A}$ has the form

$$[\mathcal{A}]^{ij}_{\alpha\beta} = \delta_{\alpha\beta}\delta^{ij} + \big(\gamma^\mu\big)_{\alpha\beta}e^{ij}_\mu. \tag{27}$$

If we assume rotational invariance, we can set the space components of e^{ij}_μ to zero. The simplest case mixing flavors would then correspond to choosing $e^{12}_0 = e = e^{21}_0$ and the equations of motion in this case would have the forms

$$\dot{\psi}^1 = -\vec{\alpha}\cdot\vec{\nabla}\psi^1 - e\vec{\gamma}\cdot\vec{\nabla}\psi^2,$$
$$\dot{\psi}^2 = -\vec{\alpha}\cdot\vec{\nabla}\psi^2 - e\vec{\gamma}\cdot\vec{\nabla}\psi^1. \tag{28}$$

These can be diagonalized and lead to the dispersion relations

$$E^1_\pm = \pm\sqrt{1+e^2}\,|\vec{p}|,$$
$$E^2_\pm = \pm\sqrt{1+e^2}\,|\vec{p}|. \tag{29}$$

This is identical to the conventional dispersion relation for massless neutrinos, expect for a scale factor, and shows in particular that massless neutrinos and anti-neutrinos of different species are degenerate in energy. As a consequence, oscillations between massless neutrino species cannot take place (see Eq. (2)) if Lorentz and *CPT* symmetries are simultaneously violated in this model.

Eqs. (28) can be also obtained from the Hamiltonian density

$$H = -i\psi^{i\dagger}\vec{\alpha}\cdot\vec{\nabla}\psi^i - ie^{ij}_0\psi^{i\dagger}\vec{\gamma}\cdot\vec{\nabla}\psi^j, \tag{30}$$

with canonical anti-conmutation relations instead of (10). Our deformation can also be understood as a subclass of the extended Standard Model of [2] as follows. Let us consider the part of the extended model [2] of the form

$$\mathcal{L} = i\bar{\psi}^i\gamma^\mu\partial_\mu\psi^i + i\bar{\psi}^i e^\mu_{ij}\partial_\mu\psi^j, \tag{31}$$

with our minimal choice for the constant background vector e^{ij}_μ. In this case, the derivation of the canonical anti-commutation relation leads to the deformation discussed above up to a field rescaling. Since the constant background field is a vector, the extra term in the Lagrangian violates *CPT* invariance and the Lorentz violation of this term is manifest as well. Thus, the second choice of the deformation violates both Lorentz and *CPT* invariances. The *CPT* violation can also be seen from the analysis of the energy eigenstates. The diagonalized fermions involve a combination of left- and right-handed fields and, as a consequence, do not have well defined *CPT* transformation properties.

2.3. Bounds for α (Lorentz invariance violation)

As we have shown, oscillation between massless neutrinos can take place in our model if only Lorentz invariance is violated. Therefore, let us use the existing experimental data on neutrino oscillations to derive bounds on the parameter α of Lorentz invariance violation (LIV). We note that our earlier analysis for two neutrino flavors can be extended to incorporate more flavors easily. For example, to accommodate three neutrino flavors, we need to generalize the deformation parameter (as well as the mixing angles) as

$$\alpha \to \alpha_{ij}, \qquad \theta_{12} \to \theta_{ij}, \tag{32}$$

where $i, j = 1, 2, 3$. In this case, the probability for oscillation between neutrino flavors can be written as

$$P_{\nu_i \to \nu_j}(L) = \sin^2(2\theta_{ij}) \sin^2\big(|\alpha_{ij}|EL\big). \tag{33}$$

The same formula also holds for anti-neutrino oscillations since *CPT* is not violated in this case.

Therefore, comparing with the conventional analysis of oscillation for massive neutrinos given in (5), we can identify ($c = 1$)

$$|\alpha_{ij}| = \frac{\Delta m_{ij}^2}{4E^2}. \tag{34}$$

We note here that there are three deformation parameters α_{ij}, without any further constraint unlike the constraint on the difference of the squared masses in the conventional scenario.

Let us next note that from the solar neutrino experiments, we know that this involves oscillations of the flavors $1 \to 2$ with

$$\Delta m_{12}^2 < 8 \times 10^{-5}\ \text{eV}^2, \qquad E \sim 1\ \text{MeV}. \tag{35}$$

From (34), this translates into a deformation parameter

$$|\alpha_{12}| < 10^{-17}. \tag{36}$$

The atmospheric neutrino results, on the other hand, involve an oscillation of the type $2 \to 3$ with

$$\Delta m_{23}^2 < 2.6 \times 10^{-3}\ \text{eV}^2, \qquad E \sim 1\ \text{GeV}. \tag{37}$$

From (34), we see that this would translate into a deformation parameter

$$|\alpha_{23}| < 10^{-22}. \tag{38}$$

Finally, we note that although the simple interpretation of the LSND results has been disproved by the MiniBooNe experiment, it is nonetheless interesting to recognize that here the oscillations involve flavors of the type $1 \to 2$ (both in the neutrino as well as the anti-neutrino channels) with

$$\Delta m_{12}^2 < 1\ \text{eV}^2, \qquad E \sim 50\ \text{MeV}. \tag{39}$$

In this case, the analog of (34) for the anti-neutrinos leads to

$$|\alpha_{12}| < 10^{-16}. \tag{40}$$

It is clear now that within this scenario, all the experimental results can be naturally explained without any particular puzzle. We would like to note here that our analysis for solar neutrinos are not in contradiction with the data from the KamLAND experiment [26].

In this discussion, we have assumed the neutrinos to be completely massless in which case, the conventional oscillation does not take place. It is possible that the neutrinos have a small mass and that both mechanisms do contribute to the phenomena of neutrino oscillation. In this case, a careful analysis of the atmospheric neutrino oscillation results can lead to even a more stringent bound on the parameter α_{23} [11].

3. Conclusion and outlook

In this Letter, we have carried out a quantitative analysis of the consequences of *CPT* and Lorentz invariance violation in the massless neutrino sector. While it has already been suggested that in such a case, neutrino oscillation can take place even for massless neutrinos, we have presented a simple model of a theory of non-commutative fermions to study this phenomenon quantitatively. The model contains a minimal number of symmetry violating parameters that are introduced as deformation parameters in the equal-time anti-commutation relations for the fermion fields. Real values of these deformation parameters naturally lead to the large mixing angle scenario. While the deformation parameters directly lead to mixing between different neutrino flavors, they also lead to non-standard (energy) dispersion relations (through Lorentz and *CPT* violation), which leads to oscillations between massless neutrino species if only Lorentz invariance is violated.

In the case that there is violation of only Lorentz invariance, we have determined bounds on the parameters of deformation (parameters characterizing Lorentz invariance violation) from the existing experimental data on solar and atmospheric neutrinos as well as from the LSND data. The bounds on the deformation parameters within this minimal model are obtained to have the values

$$|\alpha_{23}| < 10^{-22}, \qquad |\alpha_{12}| < 10^{-16}. \tag{41}$$

We note that bounds for α were obtained from the LMA scenario by using $\theta \sim \pi/4$. For the solar and atmospheric neutrinos, this is indeed consistent with the experimental determination.

On the other hand, if Lorentz and *CPT* invariances are violated simultaneously, then oscillation between massless neutrinos disappears in our model, although oscillations can take place for massive neutrinos in the standard scenario. This fact could indicate that at very high energy where masses can be neglected, neutrino oscillation would signal a violation of Lorentz invariance and not of *CPT* symmetry.

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