Computers and Mathematics with Applications 59 (2010) 622–628

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Extracting a general iterative method from an Adomian decomposition method and comparing it to the variational iteration method

J. Biazar [a], M. Gholami Porshokuhi [b,*], B. Ghanbari [a]

[a] Department of Mathematics, Faculty of Science, Guilan University, P.O Box 1914, P.C. 41938, Rasht, Iran

[b] Department of Mathematics, Faculty of Science, Islamic Azad University, Takestan Branch, Iran

ARTICLE INFO

Article history:
Received 24 June 2009
Received in revised form 2 November 2009
Accepted 2 November 2009

Keywords:
Linear and nonlinear operators
Adomian decomposition method
Adomian polynomials
The variational iteration method

ABSTRACT

In this work, a new form of Adomian decomposition method (ADM) is presented; by this form a general iterative method can be achieved in which there is no need of calculating Adomian polynomials. Also, this general iterative method is compared with the Adomian decomposition method and variational iteration method (VIM) and its advantages are expressed.

1. Introduction

Recently, various mathematical methods, such as Adomian decomposition [1,2], variational iteration [3,4], homotopy perturbation [5,6], Exp function [7,8], and others [9,10], have been proposed for obtaining exact or analytical approximation solutions for nonlinear problems.

In this paper, for solving functional equations, by using an Adomian decomposition method procedure, we derive a general iteration method and compare it with the variational iteration method for a class of especial equations.

The Adomian decomposition method developed by Adomian at the beginning of 1980 is used in [11] to solve the Burger's–Huxley and Burger's–Fisher equations. The authors of [12] applied the Adomian decomposition method to find solution of systems of integral–differential equations. In [13], the Adomian decomposition method is employed to solve the wave equation which has special importance in engineering and sciences. The interested reader can see [14–16] for some other applications of the method. The convergence of the method is systematically discussed by Babolian and Biazar [17].

The variational iteration method developed by the Chinese mathematician Ji-Huan He was successfully applied to various sciences and engineering problems, for example, the variational iteration method is used in [18] to solve delay differential equations. This method is employed in [19] to solve a system of two nonlinear integro-differential equations, which arise in biology, describing Biological species living together. The authors of [20] used the variational iteration method to solve fourth-order parabolic equations. The interested reader can see [21–23] for some other applications of the method. The convergence of the method is systematically discussed by Tatari and Dehghan [24].

* Corresponding author.
E-mail addresses: biazar@guilan.ac.ir, jafar.biazar@gmail.com (J. Biazar), m_gholami_p@yahoo.com (M. Gholami Porshokuhi), b.ghanbary@yahoo.com (B. Ghanbari).

0898-1221/$ – see front matter Crown Copyright © 2009 Published by Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2009.11.001

2. Basic idea of He's variational iteration method

To clarify the basic ideas of VIM, we consider the following differential equation:

$$Lu(t) + Nu(t) = g(t) \tag{1}$$

where, L is a linear operator, N is a nonlinear operator and $g(t)$ is an inhomogeneous term.

According to VIM, we can write down a correction functional as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(\xi)(Lu_n(\xi) + N\tilde{u}_n(\xi) - g(\xi))\mathrm{d}\xi, \tag{2}$$

where λ is a general Lagrange's multiplier, which can be identified optimally via the variational theory, and $\tilde{u}_n$ is a restricted variation which means $\delta\tilde{u}_n = 0$.

It is obvious now that the main steps of He's variational iteration method require first the determination of the Lagrangian multiplier λ that will be identified optimally. Having determined the Lagrange multiplier, the successive approximations u_{n+1}, $n \geq 0$, of the solution u will be readily obtained upon using any selective function u_0. Consequently, the solution

$$u(x) = \lim_{n\to\infty} u_n(x).$$

In other words, the correction functional (2) will give several approximations, and therefore the exact solution is obtained at the limit of the resulting successive approximations.

3. Basic idea of Adomian's decomposition method and new modified Adomian's decomposition method

Consider the functional equation

$$F(u(x)) = g(x), \tag{3}$$

where F is a functional operator and $g(x)$ is an inhomogeneous term. Suppose that the functional operator F can be decomposed into three operators, i.e.

$$F = I + R + N, \tag{4}$$

where I is an invertible operator, R is a linear and N is an analytic nonlinear operator.

Now we can write the Eq. (3) as the following:

$$I(u(x)) + R(u(x)) + N(u(x)) = g(x). \tag{5}$$

Applying the inverse operator I^{-1} to both sides of Eq. (5) and using given conditions, we obtain

$$u(x) = f(x) - I^{-1}(R(u(x))) - I^{-1}(N(u(x))). \tag{6}$$

ADM defines the unknown function $u(x)$ by an infinite series, say

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \tag{7}$$

where the components $u_n(x)$ are usually determined recurrently. Substituting this infinite series into Eq. (6) leads to

$$\sum_{n=0}^{\infty} u_n(x) = f(x) - I^{-1}\left(R\left(\sum_{n=0}^{\infty} u_n(x)\right)\right) - I^{-1}\left(N\left(\sum_{n=0}^{\infty} u_n(x)\right)\right). \tag{8}$$

Adomian also considers $N(u)$ as the summation of an infinite series of polynomials, say

$$N(u) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n). \tag{9}$$

Polynomials A_n, which are called Adomian polynomials, are generated for all kinds of nonlinearity so that A_0 depends only on u_0, A_1 depends on u_0 and u_1 and in general A_n depends on $u_0, u_1, \ldots, u_n$. Adomian introduces these polynomials as

$$A_n(u_0, u_1, \ldots, u_n) = \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\left[N\left(\sum_{i=0}^{n} \lambda^i u_i\right)\right]_{\lambda=0}, \qquad n = 0, 1, 2, \ldots. \tag{10}$$

The Adomian procedure can be presented as the following:

$$\begin{aligned} &u_0(x) = f(x), \\ &u_{n+1} = -I^{-1}(R(u_n)) - I^{-1}(A_n(u_0, u_1, \ldots, u_n)), \qquad n = 0, 1, 2, \ldots. \end{aligned} \tag{11}$$

Substituting Eq. (9) into Eq. (8) leads to

$$\sum_{n=0}^{\infty} u_n(x,t) = f(x) - I^{-1}\left(R\left(\sum_{n=0}^{\infty} u_n(x)\right)\right) - I^{-1}\left(\sum_{n=0}^{\infty} A_n\right). \tag{12}$$

It can be written as

$$u_0 + u_1 + \cdots + u_n + \cdots = f(x) - I^{-1}(R(u_0 + u_1 + \cdots + u_n + \cdots)) - I^{-1}(A_0 + A_1 + \cdots + A_n + \cdots). \tag{13}$$

By considering $v_n = \sum_{i=0}^{n} u_i$ and Eq. (13), the following procedure can be constructed:

$$\begin{aligned} &v_0 = f(x), \\ &v_{n+1} = v_0 - I^{-1}(R(v_n)) - I^{-1}\left(\sum_{i=0}^{n} A_i\right), \qquad n = 0, 1, 2, \ldots. \end{aligned} \tag{14}$$

Consequently, the exact solution may be obtained by

$$u = \lim_{n\to\infty} v_n = \lim_{n\to\infty} \sum_{i=0}^{n} u_i. \tag{15}$$

4. The general iterative method

In this section, we are going to construct a general iteration method to solve partial differential equations.

For the analytic nonlinear operator N, we can write

$$\lim_{n\to\infty} N\left(\sum_{i=0}^{n} u_i\right) = \lim_{n\to\infty} \sum_{i=0}^{n} A_i. \tag{16}$$

By considering, Eqs. (14) and (16) can be reconstructed as

$$\begin{aligned} &v'_0 = f(x), \\ &v'_{n+1} = v'_0 - I^{-1}(R(v'_n)) - I^{-1}(N(v'_n)). \end{aligned} \tag{17}$$

Eq. (17) is a general iteration method for solving the functional equations. One of the advantages of the general iteration method with respect to the Adomian decomposition method is that in this method computing the Adomian polynomials is not needed.

In this section, we apply the general iteration method to solve the following problems.

Example 1. Consider the following equation with initial condition

$$\begin{aligned} &\frac{\partial u}{\partial t} - 3\frac{\partial(u^2)}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0, \\ &u(x, 0) = 6x. \end{aligned} \tag{18}$$

By using $\left[\frac{\partial}{\partial t}\right]^{-1}$ yields

$$u(x,t) = 6x + \int_0^t \left(3\frac{\partial(u^2(x,\tau))}{\partial x} - \frac{\partial^3 u(x,\tau)}{\partial x^3}\right) \mathrm{d}\tau. \tag{19}$$

From (19), we get

$$\begin{aligned} &u_0 = 6x, \\ &u_{n+1} = u_0 + \int_0^t \left(3\frac{\partial(u_n^2)}{\partial x} - \frac{\partial^3 u_n}{\partial x^3}\right) \mathrm{d}\tau, \quad n = 0, 1, \ldots. \end{aligned} \tag{20}$$

For the first few n, we have

$$\begin{aligned} &u_1 = 6x(1 + 36t), \\ &u_2 = 6x(1 + 36t + 1296t^2 + 15\,552t^3), \\ &u_3 = 6x(1 + 36t + 1296t^2 + 15\,552t^3 + 1\,119\,744t^4 + 20\,155\,392t^5) + \text{small terms} \\ &\vdots \\ &u_n = 6x(1 + 36t + 1296t^2 + 46\,656t^3 + 1\,679\,616t^4 + 60\,466\,176t^5 + 21\,767\,866t^6 \\ &\qquad + 78\,364\,164\,096t^2 + \cdots). \end{aligned} \tag{21}$$

Recall that

$$u = \lim_{n\to\infty} u_n. \tag{22}$$

That gives

$$u(x,t) = \frac{6x}{1-36t}, \qquad |36t| < 1. \tag{23}$$

This is an exact solution.

Example 2. Consider the partial differential equation with the initial conditions

$$\begin{aligned} &\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 (u^3)}{\partial x^2} - \frac{\partial^4 u}{\partial x^4} = 0,\\ &u(x,0) = \frac{1}{x}, \qquad \frac{\partial u(x,0)}{\partial t} = -\frac{1}{x^2}. \end{aligned} \tag{24}$$

Similarly we obtain the following iteration formulation

$$\begin{aligned} &u_0(x,t) = \frac{1}{x} - \frac{t}{x^2},\\ &u_{n+1} = u_0(x,t) + \int_0^t\int_0^t \left(\frac{\partial^2 u_n(x,t)}{\partial x^2} - 2\frac{\partial^2 (u_n(x,t))^3}{\partial x^2} + \frac{\partial^4 u_n(x,t)}{\partial x^4}\right) \mathrm{d}t\mathrm{d}t, \qquad n = 0, 1, \ldots. \end{aligned} \tag{25}$$

We obtain the following successive approximations

$$\begin{aligned} &u_1(x,t) = \frac{1}{x} - \frac{t}{x^2} + \frac{t^2}{x^3} - \frac{t^3}{x^4} + \text{small terms},\\ &u_2(x,t) = \frac{1}{x} - \frac{t}{x^2} + \frac{t^2}{x^3} - \frac{t^3}{x^4} + \frac{t^4}{x^5} - \frac{t^6}{x^7} + \text{small terms},\\ &\vdots\\ &u_n(x,t) = \frac{1}{x} - \frac{t}{x^2} + \frac{t^2}{x^3} - \frac{t^3}{x^4} + \frac{t^4}{x^5} - \frac{t^6}{x^7} + \cdots, \end{aligned} \tag{26}$$

and in the closed form by

$$u(x,t) = \frac{1}{x+t}. \tag{27}$$

Example 3. Let us solve the following ordinary differential equation:

$$\begin{aligned} &\frac{\partial^3 u(x)}{\partial x^3} + \frac{1}{2}u(x)\frac{\partial^2 u(x)}{\partial x^2} = 0,\\ &u(0) = 0, \qquad u'(0) = 1, \qquad u''(0) = A, \qquad u'(\infty) = 0. \end{aligned} \tag{28}$$

For (17), we obtain

$$\begin{aligned} &u_0(x) = x + \frac{1}{2}Ax^2,\\ &u_{n+1}(x) = u_0(x) - \frac{1}{2}\int_0^x\int_0^x\int_0^x \left(u_n(x)\frac{\partial^2 u_n(x)}{\partial x^2}\right) \mathrm{d}x\mathrm{d}x\mathrm{d}x, \qquad n = 0, 1, 2 \ldots. \end{aligned} \tag{29}$$

Using (29), we obtain the following successive approximations

$$\begin{aligned} u_1(x) &= x + \frac{1}{2}x^2A - \frac{1}{240}A^2x^5 - \frac{1}{48}Ax^4,\\ u_2(x) &= x + \frac{1}{2}x^2A - \frac{1}{5\,702\,400}A^4x^{11} - \frac{1}{193\,536}A^2x^9 + \frac{11}{161\,280}A^3x^8\\ &\quad + \frac{11}{20\,160}A^2x^7 + \frac{1}{960}A^2x^6 - \frac{1}{240}A^2x^5 - \frac{1}{48}x^4A, \end{aligned}$$

$$
\begin{aligned}
u_3(x) &= x+\frac{x^2A}{2}-\frac{A^7x^{23}}{6\,282\,355\,064\,832\,000}-\frac{197A^6x^{21}}{6\,290\,632\,212\,480\,000}\\
&\quad-\frac{\left(-\frac{83}{752\,467\,968\,000}A^7+\frac{1}{211\,805\,798\,400}A^5\right)x^{20}}{40}\\
&\quad-\frac{\left(-\frac{83}{752\,467\,968\,000}A^6+\frac{1}{159\,188\,779\,008}A^4\right)x^{19}}{38}+\frac{14\,057A^5x^{18}}{656\,653\,713\,713\,408\,000}\\
&\quad-\frac{\left(\frac{5449}{3\,678\,732\,288\,000}A^6-\frac{1829}{15\,918\,877\,152\,000}A^4\right)x^{17}}{34}\\
&\quad-\frac{\left(\frac{5449}{229\,920\,768\,000}A^5-\frac{17}{6\,502\,809\,600}A^3\right)x^{16}}{32}-\frac{1147A^4x^{15}}{253\,609\,574\,400}\\
&\quad-\frac{\left(\frac{967}{3\,019\,161\,600}A^3-\frac{10\,033}{49\,816\,166\,400}A^5\right)x^{14}}{28}-\frac{\left(\frac{1}{4\,055\,040}A^2-\frac{10\,033}{3\,832\,012\,800}A^4\right)x^{13}}{26}\\
&\quad+\frac{1157A^3x^{12}}{2\,554\,675\,200}-\frac{\left(-\frac{23}{1\,612\,800}A^2+\frac{5}{193\,536}A^4\right)x^{11}}{22}-\frac{5A^3x^{10}}{387\,072}-\frac{43A^2x^9}{967\,680}\\
&\quad-\frac{\left(\frac{1}{1344}A-\frac{11}{10\,080}A^3\right)x^8}{16}+\frac{11A^2x^7}{20\,160}+\frac{Ax^6}{960}-\frac{A^2x^5}{240}-\frac{x^4A}{48},\\
&\quad\vdots
\end{aligned}
$$

The results derive so far, for these examples, are exactly the same as those obtained by the variational iteration method [21,25].

5. Equivalence of VIM and general iteration method for a class of especial equations

In this section, we show that the VIM and general iteration method for especial forms are equivalence equations.

5.1. *We first consider the following general nonlinear differential equation*

$$
\begin{aligned}
&Lu(x,t)+Ru(x,t)+N(x,t)=g(x,t),\\
&u(x,0)=u_0
\end{aligned}
\tag{30}
$$

where $L=\frac{\partial}{\partial t}$, R and N are linear and nonlinear operator respectively and $g(x,t)$ is an inhomogeneous term.

Using VIM to solve Eq. (30), the following variational iteration formula can be obtained:

$$
u_{n+1}(x,t)=u_n(x,t)+\int_0^t\lambda\{Lu_n(x,\xi)+R\tilde{u}_n(x,\xi)+N\tilde{u}_n(x,\xi)-g(x,\xi)\}\mathrm{d}\xi. \tag{31}
$$

Its stationary conditions can be obtained as follows:

$$
\begin{aligned}
&\lambda'(\xi)|_{\xi=t}=0,\\
&1+\lambda(\xi)|_{\xi=t}=0.
\end{aligned}
\tag{32}
$$

The Lagrange multipliers can be identified as follows:

$$
\lambda(\xi)=-1. \tag{33}
$$

Substituting the identified multiplier into Eq. (31) results in the following iteration formula:

$$
\begin{aligned}
u_{n+1}(x,t)&=u_n(x,t)-\int_0^t\{Lu_n(x,\xi)+Ru_n(x,\xi)+Nu_n(x,\xi)-g(x,\xi)\}\mathrm{d}\xi\\
&=u_n(x,t)-\int_0^t\{Lu_n(x,\xi)\}\mathrm{d}\xi-\int_0^t\{Ru_n(x,\xi)+Nu_n(x,\xi)-g(x,\xi)\}\mathrm{d}\xi\\
&=u_0(x,t)-\int_0^t\{Ru_n(x,\xi)+Nu_n(x,\xi)-g(x,\xi)\}\mathrm{d}\xi.
\end{aligned}
\tag{34}
$$

We have used

$$
u_0(x,t)=u_n(x,0)=u(x,0). \tag{35}
$$

Now if we consider the general iterative method for solving Eq. (30), we can write

$$
\begin{aligned}
&v_{n+1}(x,t)=v_0(x,t)-\int_0^t\{Rv_n(x,t)+Nv_n(x,t)-g(x,t)\}\mathrm{d}t,\\
&v_0(x,t)=u(x,0).
\end{aligned}
\tag{36}
$$

It is clear that Eq. (34) is an analogy form of Eq. (36).

5.2. We now consider the following general nonlinear partial differential equation

$$\begin{aligned} &Lu(x,t)+Ru(x,t)+N(x,t)=g(x,t),\\ &u(x,0)=f_0(x), \qquad \frac{\partial u(x,0)}{\partial t}=f_1(x), \end{aligned} \tag{37}$$

where $L=\frac{\partial^2}{\partial t^2}$, R and N are linear and nonlinear operator respectively and $g(x,t)$ is an inhomogeneous term.

Using VIM to solve Eq. (37), the following variational iteration formula can be obtained:

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t \lambda\{Lu_n(x,\xi)+R\tilde{u}_n(x,\xi)+N\tilde{u}_n(x,\xi)-g(x,\xi)\}\mathrm{d}\xi. \tag{38}$$

Its stationary conditions can be obtained as follows:

$$\begin{aligned} &\lambda''(\xi)=0,\\ &\lambda(\xi)|_{\xi=t}=0,\\ &1-\lambda'(\xi)|_{\xi=t}=0. \end{aligned} \tag{39}$$

The Lagrange multipliers can be identified as follows:

$$\lambda(\xi)=\xi-t. \tag{40}$$

Substituting the identified multiplier into Eq. (38) results in the following iteration formula:

$$\begin{aligned} u_{n+1}(x,t) &= u_n(x,t)+\int_0^t(\xi-t)\{Lu_n(x,\xi)+Ru_n(x,\xi)+Nu_n(x,\xi)-g(x,\xi)\}\mathrm{d}\xi\\ &= u_n(x,t)+\int_0^t \xi\{Lu_n(x,\xi)\}\mathrm{d}\xi-\int_0^t t\{Lu_n(x,\xi)\}\mathrm{d}\xi+\int_0^t(\xi-t)\{Ru_n(x,\xi)+Nu_n(x,\xi)-g(x,\xi)\}\mathrm{d}\xi\\ &= u_0(x,t)-\int_0^t(t-\xi)\{Ru_n(x,\xi)+Nu_n(x,\xi)-g(x,\xi)\}\mathrm{d}\xi. \end{aligned} \tag{41}$$

But

$$u_0(x,t)=u_n(x,0)+t\frac{\partial u_n(x,0)}{\partial t}=f_0(x)+tf_1(x). \tag{42}$$

Now if we consider the general iteration method for solving Eq. (37), we can write

$$\begin{aligned} &v_{n+1}(x,t)=v_0(x,t)-\int_0^t\int_0^t\{Rv_n(x,t)+Nv_n(x,t)-g(x,t)\}\mathrm{d}t\mathrm{d}t,\\ &v_0(x,t)=f_0(x)+tf_1(x). \end{aligned} \tag{43}$$

For prove equivalence between Eqs. (41) and (43), it is sufficient show that

$$\int_0^t\int_0^t w(x,t)\mathrm{d}t\mathrm{d}t=\int_0^t(t-\xi)w(x,\xi)\mathrm{d}\xi, \tag{44}$$

where

$$w(x,t)=Ru_n(x,t)+N(x,t)-g(x,t). \tag{45}$$

For Eq. (44), if we take the partial derivative with respect to t, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_0^t\int_0^t w(x,t)\mathrm{d}t\mathrm{d}t\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_0^t(t-\xi)w(x,\xi)\mathrm{d}\xi\right). \tag{46}$$

Consequently

$$\int_0^t w(x,t)\mathrm{d}t=\int_0^t w(x,\xi)\mathrm{d}\xi. \tag{47}$$

So the main terms differ in a constant, say $\int_0^t\int_0^t w(x,t)\mathrm{d}t\mathrm{d}t=\int_0^t(t-\xi)w(x,\xi)\mathrm{d}\xi+c$.

By taking $w(x,t)=1$, we derive $c=0$.

6. Conclusion

In this paper, we have been looking for a general iteration formula, having the Adomian decomposition method in mined.

By using the Adomian decomposition method, we have achieved this goal and presented a general iteration formula, which seems to be effective.

Three examples, which illustrate the method and its simplicity efficiency, lead to the results which are exactly the same as those obtained by the variational iteration method. This method can be used for solving other functional equations as well.

The convergence of the method is under study in our research group.

Also in this article, equivalence of this method with the variational iteration method for some especial functional equations is indicated.

Acknowledgement

We thank the anonymous referee for his/her comments, which greatly helped to improve the quality of paper.

References

[1] G. Adomian, A review of the decomposition method in applied mathematics, J. Math. Anal. Appl. 135 (1988) 501–544.
[2] G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Academic Publishers, Boston, 1994.
[3] J.H. He, A new approach to nonlinear partial differential equations, Commun. Nonlinear Sci. Numer. Simul. 2 (4) (1997) 203–205.
[4] J.H. He, Variational iteration method—A kind of nonlinear analytical technique: Some examples, Internat. J. Non-Linear Mech. 34 (1999) 708–799.
[5] J.H. He, A coupling method of a homotopy technique and a perturbation technique for non-linear problems, Internat. J. Non-Linear Mech. 35 (1) (2000) 37–43.
[6] J.H. He, New interpretation of homotopy-perturbation method, Internat. J. Modern Phys. B 20 (18) (2006) 2561–2568.
[7] J.H. He, Xu-Hong Wu, Exp-function method for nonlinear wave equations, Chaos Solitons Fractals 30 (3) (2006) 700–708.
[8] J.H. He, M.A. Abdou, New periodic solutions for nonlinear evolution equations using Exp-function method, Chaos Solitons Fractals 34 (2007) 1421–1429.
[9] J.H. He, Some asymptotic methods for strongly nonlinear equations, Internat. J. Modern Phys. B 20 (10) (2006) 1141–1199.
[10] J.H. He, Non-perturbative methods for strongly nonlinear problems. Berlin: Dissertation. de-Verlag in Internet GmbH., 2006.
[11] N.A. Ismail, K. Raslan, A. Abd Rabboh, Adomian decomposition method for Burger's–Huxley and Burger's–Fisher equations, Appl. Math. Comput. 159 (2004) 291–301.
[12] J. Biazar, Solution of systems of integral–differential equations by Adomian decomposition method, Appl. Math. Comput. 168 (2005) 1232–1238.
[13] J. Biazar, R. Islam, Solution of wave equaton by Adomian decomposition method and the restrictions of the method, Appl. Math. Comput. 149 (2004) 807–814.
[14] A.M. Wazwaz, Adomian decomposition method for a reliable treatment of the Bratu-type equations, Appl. Math. Comput. 166 (2005) 652–663.
[15] J. Biazar, H. Ebrahimi, An approximation to the solution of hyperbolic equations by Adomian decomposition method and comparison with charactistics method, Appl. Math. Comput. 163 (2005) 633–638.
[16] A.M. Wazwaz, A comparison between the variational iteration method and Adomian decomposition method, J. Comput. Appl. Math. 207 (2007) 129–136.
[17] E. Babolian, J. Biazar, On the order of convergence of Adomian method, Appl. Math. Comput. 130 (2002) 383–387.
[18] J.H. He, Variational iteration method for delay differential equations, Commun. Nonlinear Sci. Numer. Simul. (1997) 235–236.
[19] F. Shakeri, M. Dehghan, Solution of a model describing biological species living together using the variational iteration method, Math. Comput. Modelling 48 (2008) 685–699.
[20] J. Biazar, H. Ghazvini, He's variational iteration method for fourth-order parabolic equations, Comput. Math. Appl. (2006).
[21] A.M. Wazwaz, The variational iteration method for rational solutions for KdV, K (2,2), Burgers, and cubic Boussinesq equations, J. Comput. Appl. Math. 207 (2007) 18–23.
[22] M.A. Abdou, A. Soliman, Variational iteration method for solving Burger's and coupled Burger's equations, J. Comput. Appl. Math. 181 (2005) 245–251.
[23] S. Momani, S. Abuasad, Application of He's variational iteration method to Helmholtz equation, Chaos Solitons Fractals 27 (2006) 1119–1123.
[24] M. Tatari, M. Dehghan, On the convergence of He's variational iteration method, J. Comput. Appl. Math. 207 (2007) 121–128.
[25] A.M. Wazwaz, The variational iteration method for solving two forms of Blasius equation on a half-infinite domain, Appl. Math. Comput. 188 (2007) 485–491.