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Brane worlds in 5D and warped compactifications in IIB

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Abstract

In computing potentials for moduli in, for instance, type IIB string theory in the presence of fluxes and branes a factorisable ansatz for the ten-dimensional metric is usually made. We investigate the validity of this ansatz by examining the cosmology of a brane world in a five-dimensional bulk and find that it contradicts the results obtained by using a factorisable ansatz. We explicitly identify the problem with the latter in the IIB case. These arguments support our previous work on this question.

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1. Introduction

Following the work of Giddings et al. [1] (GKP)[1] there has been much activity in computing potentials for the moduli in type IIB theories in the presence of fluxes and D-branes/orientifold planes [2–15]. In all these calculations the following ansatz is made for reducing the theory to an effective four-dimensional one:

$$ds^2 = g_{MN}\,dx^M\,dx^N = e^{2\omega(y)-6u(x)}\bar{g}_{\mu\nu}(x) + e^{-2\omega(y)+2u(x)}\big(\tilde{g}_{mn}(y) + z_i(x)\phi^i_{mn}(y) + \cdots\big)\,dy^m\,dy^n. \tag{1}$$

In the above $\tilde{g}_{mn}(y)$ is the metric on the internal Calabi–Yau manifold, $u(x)$ is the volume modulus and the $z_i(x)$ are the other Kähler and complex structure moduli. The question we wish to address is the following: does the potential obtained by using this ansatz in the ten-dimensional action, yield the correct four-dimensional equations on a ((3 + 1)-dimensional) brane obtained by projecting the ten-dimensional equations?

In a previous paper [6] we made the following observations. The no-go theorem, which in effect states that the strong energy condition is satisfied in the effective four-dimensional theory, if it is satisfied in the ten-dimensional theory (as is the case in string theory) was shown to be inapplicable here, since the volume modulus is not stabilized.

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[1] This work was based on earlier works [32–38].

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It followed from this that to have a consistent derivation of the potential one needed to keep the time dependence of the volume modulus. In [6] it was pointed out that doing so, in the presence of non-trivial warping seemed to require one to keep all the Kaluza–Klein excitations as well. In effect, while the static solutions in the presence of fluxes and branes was certainly valid, the actual form of the 4D potential was not really established.

In this Letter we will highlight the problems associated with the factorized form of the metric ansatz used in the literature, by considering the far simpler case of the original [16] (RS1) construction of a $(3+1)$-dimensional brane world embedded in five dimensions. We will see explicitly that the (non-static) five-dimensional equations (assuming homogeneity in three-space) projected on to the brane [17–23], (for recent reviews see [24–26]) are manifestly different from the effective four-dimensional equations obtained from the above factorized metric ansatz. The latter fails to capture the correct four-dimensional physics.[2] Finally we will revisit the full ten-dimensional theory and discuss precisely where the factorization ansatz fails, in the light of our five-dimensional investigation.

2. 4D effective action from metric ansatz

The action for the theory is

$$S=\frac{1}{2\kappa_5^2}\int d^5x\,\sqrt{g}R^{(5)}-\int d^5x\,\sqrt{g}\left(\frac{1}{2}(\partial\phi)^2+V(\phi)\right)-\sum_{i=0,\pi}\int d^4x\,\sqrt{g_i^{(4)}}L_i. \tag{2}$$

We have included the action for two branes in the above with the energy density of each brane being split up into a tension part (which in general will depend on the bulk scalar) and a matter density. The fifth dimension is taken to be an S^1/Z_2 orbifold and we have chosen a gauge such that the branes are located at the fixed points $y=0,\pi l_5$ ($l_5^3=\kappa_5^2$) of the Z_2 action. The metric is taken to be block diagonal,

$$ds^2=g_{\mu\nu}(x,y)\,dx^\mu\,dx^\nu+b^2(x,y)\,dy^2.$$

If one were to proceed in analogy with what is done in the analysis of the corresponding type IIB case one would use the metric ansatz

$$ds^2=e^{2A(y)-u(x)}\tilde{g}_{\mu\nu}(x)\,dx^\mu\,dx^\nu+e^{2u(x)}\,dy^2. \tag{3}$$

The salient feature of the above is the assumption of factorisability of x and y dependences (the e^{-u} factor in the first term is inserted in order to decouple the modulus field u from the four-dimensional metric to get the Einstein frame). One could have of course chosen the y-dependent factor in the second term to be as in (1) but this would amount to a trivial redefinition of the coordinate. Inserting this ansatz into the five-dimensional action (2) we get

$$S=\frac{1}{2\kappa_5^2}\int dy\,e^{2A(y)}\int d^4x\,\sqrt{\tilde{g}}\left[\tilde{R}^{(4)}-\frac{3}{2}\big(\widetilde{\partial_x u}\big)^2\right]-\int d^4x\,\sqrt{\tilde{g}}\int dy\,\frac{1}{2}e^{2A(y)}\big(\widetilde{\partial_x\phi}\big)^2-\int d^4x\,\sqrt{\tilde{g}}\,U(u,\phi). \tag{4}$$

The tilde in the above denotes contraction with the tilde metric in (3) and the potential U is given by

$$U(u,\phi)=-ae^{-3u(x)}+\int dy\left[\frac{1}{2}e^{4A(y)-3u(x)}(\partial_y\phi)^2+e^{4A(y)-u(x)}V(\phi)\right]+e^{-2u(x)}\big\{e^{4A(0)}L_0+e^{4A(\pi)}L_\pi\big\}. \tag{5}$$

In the above the constant $a=\frac{1}{2\kappa_5^2}\int dy\,12e^{4A(y)}(\partial_y A)^2>0$ and $L_{0,\pi}$ is the Lagrangian on each brane. We should point out that this calculation was done in the "upstairs" version of the orbifold so that the y-integral is over a circle—thus there is no boundary term(s) and in evaluating this constant a in the potential a term involving the

[2] For related work showing that the effective four-dimensional approach can be misleading, see [39].

second y-derivative of A was turned by integration by parts into one involving only $\partial_y A$ to get the stated expression. If we had worked in the downstairs version there would be two boundaries at the locations of the branes. In this case one needs to add the Gibbons–Hawking (extrinsic curvature) term to the action which, of course, just serves to cancel the boundary contribution in the above integration by parts—resulting in the same expression for a.

Several remarks are in order here. Firstly we have an effective four-dimensional theory very much like the one in the type IIB case of GKP. There is a potential for the modulus $u(x)$ that depends on the bulk scalar field ϕ. The scalar field is the analog of the four-form field in the GKP case and to make the correspondence one would also require a static solution to the bulk scalar field equation to be substituted in to the above. Note also that there is a critical point for the potential for $u(x)$ even in the absence of a bulk scalar field. Finally it is clear that the four-dimensional gravitational equations will be of the usual form, and in particular the Friedman equation describing four-dimensional cosmology will be the usual one.

This analysis is in sharp conflict with what emerges from the projection to one or other brane of the five-dimensional equations, as we shall see in the next section.

3. 4D projection of 5D equations

There is a large literature on brane world cosmology and the relevant original works were quoted in the introduction. As in those works we look for spatially homogeneous solutions so the metric is parametrized as

$$ds^2 = -n^2(t,y)\,dt^2 + a^2(t,y)(dx_i)^2 + b^2(t,y)\,dy^2.$$

Note that we can always use a gauge where $n(t,y) = b(t,y)$ but we will not do so here. The gravitational field equations then become

$$\frac{\dot a^2}{a^2} + \frac{\dot a \dot b}{ab} - \frac{n^2}{b^2}\left(\frac{a''}{a} + \frac{a'^2}{a^2} - \frac{a'b'}{ab}\right) = \frac{\kappa_5^2}{3}\left(\rho_0 n^2 \frac{\delta(y)}{b} + \rho_\pi n^2 \frac{\delta(y-\pi)}{b} + T_{00}^\phi\right), \tag{6}$$

$$\frac{a^2}{b^2}\left[\frac{a'}{a}\left(\frac{a'}{a} + 2\frac{n'}{n}\right) - \frac{b'}{b}\left(\frac{n'}{n} + 2\frac{a'}{a}\right) + 2\frac{a''}{a} + \frac{n''}{n}\right] + \frac{a^2}{n^2}\left[\frac{\dot a}{a}\left(-\frac{\dot a}{a} + 2\frac{\dot n}{n}\right) - 2\frac{\ddot a}{a} + \frac{\dot b}{b}\left(-2\frac{\dot a}{a} + \frac{\dot n}{n}\right)\right]$$
$$= \kappa_5^2\left(p_0 a_0^2 \frac{\delta(y)}{b} + p_\pi a_\pi^2 \frac{\delta(y-\pi)}{b} + T_{ii}^\phi\right), \tag{7}$$

$$\frac{n'}{n}\frac{\dot a}{a} + \frac{a'}{a}\frac{\dot b}{b} - \frac{\dot a'}{a} = \frac{\kappa_5}{3} T_{05}^\phi, \tag{8}$$

$$\frac{a'}{a}\left(\frac{a'}{a} + \frac{n'}{n}\right) - \frac{b^2}{n^2}\left[\frac{\dot a}{a}\left(\frac{\dot a}{a} - \frac{\dot n}{n}\right) + \frac{\ddot a}{a}\right] = \frac{\kappa_5^2}{3} T_{55}^\phi. \tag{9}$$

In the above a dot denotes differentiation with respect to time and a prime with respect to y.

We also have the scalar field equation

$$-\frac{1}{na^3 b}\partial_0\left(n^{-1}a^3 b\partial_0\phi\right) + \frac{1}{na^3 b}\partial_y\left(na^3 b^{-1}\partial_y\phi\right) = \frac{dV}{d\phi} + \frac{dT_0}{d\phi}\frac{\delta(y)}{b} + \frac{dT_0}{d\phi}\frac{\delta(y-\pi)}{b}.$$

In the above we have split the Lagrangian on the brane into a ϕ-dependent tension $T_{0,\pi}(\phi)$ and a ϕ-independent matter term $L_i = T_i(\phi) + L_{\mathrm{im}}$.

Now we may write

$$\ln a(t,y) = \ln a_0(t) + \frac{1}{2}A_1(t,y)|y| + A_2(t,y), \tag{10}$$

$$\ln n(t,y) = \ln n_0(t) + \frac{1}{2}N_1(t,y)|y| + N_2(t,y), \tag{11}$$

$$\ln b(t,y) = \ln b_0(t) + \frac{1}{2}B_1(t,y)|y| + B_2(t,y), \tag{12}$$

where $A_i(t,y) = \sum_{n=0}^{\infty} a_{in}(t)\cos ny = A_i(t) + O(y^2)$ with $A_2(t) = 0$, and $\frac{1}{2}A_1(t,\pi)\pi + A_2(t,\pi) = \ln\frac{a_\pi(t)}{a_0(t)}$, and N_i, B_i satisfy similar relations. These expansions are uniquely determined since $a_0(t) \equiv a(t,0)$, etc., and the second and third terms in each expansion are the sum of all odd powers and even powers of y, respectively, such that they are consistent with the orbifold symmetry. We note in passing that this parametrization imposes no topological constraint on the function $A_1(t,y)$, since the integral $\oint (a'/a)'\,dy$ is identically zero when the RHS of (10) is used to calculate the integrand. The same remarks are valid for the other functions N_1, B_1.

From Eq. (6) we get, by integrating over vanishing intervals around $y = 0$ and $y = \pi$, the boundary conditions

$$A_1(t,0) = \frac{\kappa_5^2}{3}\rho_0(t)b_0(t), \qquad A_1(t,\pi) = -\frac{\kappa_5^2}{3}\rho_\pi(t)b_\pi(t). \tag{13}$$

Similarly, from Eq. (7) we get

$$N_1(t,0) = \frac{\kappa_5^2}{3}b_0(t)\big(2\rho_0(t) + 3p_0(t)\big), \qquad N_1(t,\pi) = -\frac{\kappa_5^2}{3}b_\pi(t)\big(2\rho_\pi(t) + 3p_\pi(t)\big). \tag{14}$$

Also from the scalar field equation we have the matching condition

$$\phi_{0,\pi}'^2 = \frac{b_{0,\pi}^2}{4}\left(\frac{dT_{0,\pi}}{d\phi}\right)^2. \tag{15}$$

Note that if we had assumed that $A_1(t,y)$, $N_1(t,y)$ are y-independent (as is done in much of the literature and corresponds to the RS solution which is valid in the absence of a bulk field) then we would have been forced to the constraint

$$\rho_0(t)b_0(t) = -\rho_\pi(t)b_\pi(t), \qquad p_0(t)b_0(t) = -p_\pi(t)b_\pi(t). \tag{16}$$

Of course, in the absence of a bulk scalar field one would have the static RS solution which essentially results in this constraint. Thus avoidance of this constraint seems to require a bulk field. Note that our result here is somewhat different from the conclusion of [22] where it is argued that the issue depends on the existence of a stabilization mechanism for the modulus $b(t)$. The argument above shows that the real problem is the assumption of linearity in y as in the RS1 solution (where of course it is a consequence of the equations of motion). The point is that if one did assume this linearity even in the presence of the scalar field, one would be still forced to the result (16) thus it is essential to consider an ansatz going beyond linearity.

The boundary conditions (13), (14) highlight the problem with the metric ansatz equation (3). They show that in a dynamic situation the factorization of the metric components into a y-dependent and a t- (or x-) dependent factor is simply not valid, since according to them $A_1(t,y)$ and $N_1(t,y)$ are necessarily time-dependent, whilst the factorization ansatz would imply that they are purely y-dependent. It follows that the equations of motion coming from the effective four-dimensional action (4) will be incorrect in the sense that they would not be compatible with the 5-dimensional equations of motion except in the static case.

Let us, for instance, discuss the analog of the Friedman equation for this case. This has been discussed at length in the literature beginning with the work of Binetruy et al. [17]. Nevertheless, for completeness we will rederive it (especially since most derivations are done in the absence of a bulk scalar).

Using Eqs. (10), (11), (13), (14) in Eqs. (9) and (8) evaluated at $y = 0$ we get

$$\frac{\ddot{a}_0}{a_0} + \frac{\dot{a}_0^2}{a_0^2} = -\frac{\kappa_5^2}{3b_0^2}T_{55}^{\phi}\Big|_0 - \frac{\kappa_5^2}{36}\rho_0(\rho_0 + p_0), \qquad \dot{\rho}_0 + 3(\rho_0 + p_0)\frac{\dot{a}_0}{a_0} = 2T_{05}^{\phi}\big|_{0+} = \frac{2}{b_0}\phi_{0+}'\dot{\phi}_0.$$

Eliminating p and integrating (after multiplying by an integrating factor a^4) we get

$$H_0^2 = \frac{\kappa_5^4}{36}\rho_0^2 - \frac{\kappa_5^4}{9}a_0^{-4}\int \frac{\rho_0}{b_0}\dot{\phi}_0\phi_0' a_0^3\,da_0 - \frac{2\kappa_5^2}{3}a_0^{-4}\int^{a_0} da\,a^3\left(\frac{\phi'^2}{2b_0^2} + \frac{\dot{\phi}_0^2}{2} - V_0\right) + \frac{\mu}{a_0^4}, \tag{17}$$

where the last term (with μ an integration constant) is often referred to as 'dark radiation' in the literature.

This equation looks very different from the usual four-dimensional one. To see the connection let us first ignore the scalar field and put $V = \Lambda$ a bulk cosmological constant as in RS1 and set the arbitrary constant $\mu = 0$. Then we split the energy density on the brane into a tension piece and a matter piece, i.e.,

$$\rho_0 = T_0 + \rho_{m0},$$

to get

$$H_0^2 = \frac{\kappa_5^4}{18} T_0 \rho_{m0} \left(1 + \frac{\rho_{m0}}{2T_0}\right) + \frac{\kappa_5^2}{6}\left(\frac{\kappa_5^2}{6} T_0^2 + \Lambda\right). \tag{18}$$

The second term on the RHS is an effective four-dimensional cosmological constant and to obtain a static solution in the absence of brane matter one would need to use the RS1 fine-tuning condition $\frac{\kappa_5^2}{6} T_0^2 + \Lambda = 0$. In any case we see that for $\rho_{m0} \ll T_0$, we get the usual equation provided that we identify the four-dimensional gravitational coupling as $\kappa_4^2 = \frac{\kappa_5^4}{6} T_0$. All this is well known and can be found in several of the papers quoted in the introduction. However, in the absence of a bulk scalar (or some other equivalent bulk physics) the modulus $b(t)$ cannot be stabilized [27,28] and will appear as a zero mass particle coupling with gravitational strength in four dimensions. Thus if one wants to get a phenomenologically viable brane world one needs to stabilize the modulus by, for instance, including the bulk scalar.[3]

It is instructive also to consider the static limit of the modified Friedman equation (17). Putting $H = \dot{\phi} = 0$, $\rho_m = 0$ (so that $\rho_0 = T_0$) we get after using (15),

$$V_0 = \frac{1}{8}\left(\left.\frac{dT_0}{d\phi}\right|_0\right)^2 - \frac{\kappa_5^2}{6} T_0^2.$$

This is the generalization [28] of the RS fine tuning condition in the presence of a scalar field. As pointed out by DeWolfe et al. this equation by itself is not a fine tuning condition, it just serves to determine ϕ_0, but taken in conjunction with the boundary condition at $y = \pi$ the modulus b_0 gets fixed but one fine tuning is required.

Eq. (17) thus reduces to equations investigated in earlier work in the absence of scalar fields as well as to the static equation in the presence of scalar fields. However, there is a peculiar feature of this equation which appears to defy interpretation in terms of a four-dimensional effective action. This is the fact that the kinetic term for the scalar field (as well as the y-derivative term) appears with a negative sign, although the potential appears with the right (i.e., positive) sign. This is not a situation like that of the dilaton (or the volume modulus) which mixes with the graviton and would appear to come with the wrong sign kinetic term in the original (string or Jordan) frame. For instance, here if we choose the tension T independent of ϕ, the effective four-dimensional Newton constant ($\kappa_4^2 = \frac{\kappa_5^2}{6} T$) is constant and the system is already in the Einstein frame.

It should be noted that in this system the four-dimensional Friedman equation comes from the G_{55} equation rather than from the G_{00} equation which is the five-dimensional Friedman equation. It is instructive to compare the results of projecting the latter equation to the brane with the equation obtained from projecting the G_{55} equation, i.e., (17). We shall do this in the case that the bulk scalar as well as the radion are stabilized, i.e., $\dot{\phi}$, $\dot{b} = 0$ (with $b_0 = 1$). In this case also the integral in (17) is trivial and we get (using $\rho_0 = T_0 + \rho_m$ and (15))

$$H_0^2 = \frac{\kappa_5^4}{18} T_0 \rho_m \left(1 + \frac{\rho_m}{2T_0}\right) + \frac{\mu}{a^4} + \frac{\kappa_5^2}{6}\left(\frac{1}{6}\kappa_5^2 T_0^2 - \frac{1}{8}\left(\frac{dT_0}{d\phi}\right)^2 + V\right).$$

[3] An exception to this, pointed out by the referee, is when we live on the positive tension brane of an RSI system. Now if the brane separation is large enough one would get a scalar–tensor theory with the Brans–Dicke parameter being large enough to satisfy experimental bounds. Of course, in this case one would not solve the hierarchy problem. If one wishes to solve that as in RSI, the observed world needs to be on the negative tension brane and in this case one would need to stabilize the modulus.

On the other hand, from the G_{00} equation (6) at $y=0$ after using the expansions for the metric functions (10), (11), (12) as well as (13) and (15) we get (again with $b_0=1$)

$$H_0^2=\frac{\kappa_5^4}{9}T_0\rho_m\left(1+\frac{\rho_m}{2T_0}\right)+\frac{\kappa_5^2}{3}\left(\frac{1}{6}\kappa_5^2T_0^2+\frac{1}{8}\left(\frac{dT_0}{d\phi}\right)^2+V\right)+\frac{\kappa_5^2}{3}\frac{B_1(t,0)\rho}{4}+A_2''(t,0).$$

Comparing these two equations shows that although $B_1(t,0)$ may be set to zero $A_2''(t,0)$ cannot be zero and in fact

$$A_2''(t,0)=-\frac{\kappa_5^4}{18}T_0\rho_m\left(1+\frac{\rho_m}{2T_0}\right)+\frac{\mu}{a^4}-\frac{\kappa_5^2}{6}\left(\frac{1}{6}\kappa_5^2T_0^2+\frac{3}{8}\left(\frac{dT_0}{d\phi}\right)^2+V\right).$$

In other words, the linear approximation $A_2=0$ (and $A_1(y,t)$ independent of y) is invalid except in the static RS case.

To recapitulate, the cosmology of the brane world is radically different from that which would arise from dimensional reduction using the metric ansatz (3). One would expect similarly that the cosmology on a brane in type IIB string theory would not be correctly described by a naive 4D reduction using the ansatz (1).

Additional differences arise between the projection of the five-dimensional equations and the effective action obtained by using the metric ansatz (3) in the expression for the potential for the modulus b. This is important for it is precisely the analog of this ansatz that is used in discussions of the derivation of the moduli potential in the type IIB case. To see this we use Eqs. (10), (11), (13), (14) in Eq. (7) minus twice Eq. (9) (this linear combination is taken to eliminate $\ddot{a}$ terms) evaluated at $y=0$, to get

$$\frac{\ddot{b}_0}{b_0}+(\dot{a},\ \dot{b},\ \dot{n}\text{ terms})=U'(b). \tag{19}$$

$U(b)$ is an effective potential for the modulus b and (after putting $n_0=1$)

$$U'(b)=\kappa_5^2\left(-\frac{1}{a_0^2}T_{ii}^{\phi}\bigg|_0+\frac{2}{3b_0^2}T_{55}^{\phi}\bigg|_0\right)+\frac{\kappa_5^2}{18}\rho_0(\rho_0+3p_0)+\frac{\kappa_5^2}{6}\left(\kappa_5^2T_0^2+\frac{BT_0}{b_0^2}\right)+\frac{1}{b_0^2}(2A_2+N_2). \tag{20}$$

Now specialize to the RS case where there is no scalar field and $T_{55}^{\phi}|_0=-b_0^2\Lambda$, $T_{ii}^{\phi}|_0=-\Lambda a_0^2$, $\rho_0=T_0a_0^2$, $p=-T_0$. The cosmic acceleration is given by

$$\frac{\ddot{a}_0}{a_0}+\frac{\dot{a}_0^2}{a_0^2}=\frac{\kappa_5^2}{3}\Lambda-\frac{\kappa_5^4}{36}T_0(T_0-3T_0).$$

So we have a static solution if the RS condition $\Lambda=-\frac{\kappa_5^2}{6}T_0^2$ is imposed. Now let us look at the force on the modulus b under the same conditions. Noting that the RS solution implies $A_2=N_2=B=0$ we get

$$U'(b)=\frac{\kappa_5^2}{3}\left(\Lambda+\frac{1}{6}\kappa_5^2T_0^2\right)=0,$$

where the last equality follows from the RS fine-tuning condition. Thus (as expected) the modulus b is undetermined in the absence of a scalar field and implies that the RS theory has a zero mass particle coupling with gravitational strength.

However, this contradicts what we would find if the metric ansatz (3) is used to get the effective potential. For in that case, after setting $\phi=0$ and using the RS solution for the warp factor

$$A(y)=-k|y|,\qquad k=\frac{\kappa_5^2}{6}T_0,\qquad \kappa_5^2T_0^2=-6\Lambda,\qquad T_\pi=-T_0<0,$$

we get from (5)

$$U(u) = \frac{T_0}{2}\left(e^{-4k\pi l_5} - 1\right)\left(e^{-3u(x)} + e^{-u(x)} - 2e^{-2u(x)}\right)$$

so that

$$U'(u) = \frac{T_0}{2}\left(e^{-4k\pi l_5} - 1\right)\left(-3e^{-3u(x)} - e^{-u(x)} + 4e^{-2u(x)}\right),$$

giving an unstable critical point at $u = 0$!

Clearly the metric ansatz (3) gives the wrong physics. The root of the problem is the assumption of factorisability of the metric coefficients into x-dependent and y-dependent factors. As can be seen from Eqs. (13), (14), inserted into Eqs. (10), (11), the metric does not factorize. Or to put it another way the assumption of factorisable is inconsistent with the boundary conditions at the branes. This error in turn gives an incorrect expression for the potential for the radion modulus which, in particular, leads to the incorrect result that there is a critical point even in the absence of a scalar field.

4. Conclusions: lessons for IIB

What lessons can one draw from this exercise for the system that interests us—type IIB compactified on a Calabi–Yau orientifold with a stack of D3 branes. The ten-dimensional low energy effective action for this theory (in the Einstein frame with $2\kappa_{10}^2 = 1$) is

$$S = \int d^{10}X\,\sqrt{-g}\left\{R - \frac{1}{2\tau_I^2}\partial_M\tau\partial^M\bar{\tau} - \frac{1}{2\cdot 3!\tau_I}G_{MNP}\bar{G}^{MNP} - \frac{1}{4\cdot 5!}\tilde{F}_{MNPQR}\tilde{F}^{MNPQR}\right\}$$
$$+ \frac{1}{4i}\int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\tau_I}.$$

In the above $\tau = C_0 + ie^{-\phi}$, $G_3 = F_3 - \tau H_3$, with $F_3 = dC_2$ and $H_3 = dB_2$. Also $\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$ with the self-duality condition $\tilde{F}_5 = *\tilde{F}_5$ being imposed by hand at the level of the equations of motion.

In addition there is the action for the D3 branes and orientifold 3-planes in Einstein frame

$$S_{\text{loc}} = \sum_i \left(-\int_i d^4x\, T_3\sqrt{|g^{(4)}|} + \mu_3 \int_i C_4\right).$$

Here the integrals are taken over the 4D non-compact space at a point i in the internal manifold and $T_3 = \mu_3 > 0$ (< 0) for a D-brane (orientifold plane). The self-duality of the five form is satisfied by the following ansatz,

$$\tilde{F}_5 = \frac{1}{4!}(1+*)\sqrt{\bar{g}_4(x)}\,d\alpha(x,y) \wedge dx^0 \wedge \cdots \wedge dx^3, \tag{21}$$

where $\alpha(x,y)$ is a scalar function. The four-dimensional effective action may now be derived by introducing the metric ansatz

$$ds^2 = e^{2\omega(y)-6u(x)}\tilde{g}_{\mu\nu}(x)\,dx^\mu\,dx^\nu + e^{-2\omega(y)+2u(x)}\tilde{g}_{mn}(x,y)\,dy^m\,dy^n \tag{22}$$

with $\partial_\mu \det \tilde{g}_{mn} = 0$.

The effective potential was derived in [1] by reducing the ten D action using the static version of this ansatz (i.e., with $u = 0$ and $\partial_\mu g_{mn} = 0$) and the expression (the tilde denotes the use of the metric $\tilde{g}$ in the inner product)

$$V = \int d^6y\,\sqrt{\tilde{g}^{(6)}}\,\frac{e^{4\omega-12u}}{24\tau_I}|\widetilde{iG_3 - *_6 G_3}|^2 \tag{23}$$

was obtained. However, except at the minimum of the potential the static ansatz cannot really be used and immediately leads to the no-go theorem forbidding positive potentials [29–31] and the resolution, as pointed out in [6], is to include time dependence of the volume modulus $u(x)$. An attempt was made to include the moduli and a non-trivial warp factor in [28] but it was shown in [6] that a consistent derivation was not possible without including all the Kaluza–Klein (KK) modes. In fact it was argued that the full ten-dimensional equations with time-dependent moduli (and except at the minimum of the potential the moduli are necessarily time-dependent) and non-trivial warp factor, imply that the metric ansatz (22) is invalid. The argument in the previous section for the five-dimensional theory highlights this inconsistency. To see this directly in the current context consider the Bianchi identity for $\tilde{F}_5$. After using the above metric ansatz it becomes,

$$\tilde{\nabla}^2\alpha = \frac{i}{12\tau_I} e^{8\omega-4u} \widetilde{G_{mnp} *_6 \bar{G}^{mnp}} + 8\widetilde{\partial_m\alpha\partial^m\omega} + e^{8\omega-4u}\sum_i \mu_3 \frac{\delta^{(6)}(y-y^i)}{\sqrt{\widetilde{g^{(6)}}}}. \tag{24}$$

Integrating this over a small ball of radius ε around the point $y = y^i$ and letting $\varepsilon \to 0$ we get

$$\lim_{\varepsilon\to 0} \oint_{y_i} \nabla_m \alpha d\sigma^m = e^{8\omega(y^i)-4u(x)}\mu_3. \tag{25}$$

In particular, this equation implies that the function α cannot be independent of space–time since (except at the minimum of the potential) $u(x)$ is space–time-dependent. Also from the Einstein equation with the metric ansatz (22), after using (24) to eliminate the local source term, we get (for more details see [6])

$$\begin{aligned}\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{R}^{(4)}\tilde{g}_{\mu\nu} = -\frac{1}{4}\tilde{g}_{\mu\nu}\Bigg[&\frac{e^{2\omega}}{12\tau_I}|iG_3 - *_6 G_3|^2 + e^{-4\omega-8u}\Big(\partial_m\widetilde{\big(\alpha - e^{4\omega}\big)}\Big)^2 \\ &+ e^{-8u}\Big(\widetilde{\nabla^2}\big(\alpha - e^{4\omega}\big) + e^{-4\omega}\partial_m e^{4\omega}\partial^m\big(\alpha - e^{4\omega}\big)\Big)\Bigg] + \cdots,\end{aligned} \tag{26}$$

the ellipses denoting first order derivative terms. Again integrating this equation over a ball of radius ε centered at $y = y_i$ and taking the radius to zero we have,

$$\lim_{\varepsilon\to 0} \oint_{y_i} \nabla_m \alpha\, d\sigma^m = \lim_{\varepsilon\to 0} \oint_{y_i} \nabla_m e^{4\omega(y)}\, d\sigma^m.$$

Comparing with (25) we see as expected that the warp factor cannot be trivial in the presence of a brane and also that consistency requires $\partial_\mu u(x) = 0$. This in turn is valid only at the minimum of the potential. Essentially the problem as in the five-dimensional case is that the factorization ansatz is not valid in the presence of branes.

In conclusion, we have shown that the factorized ansatz for getting an effective action in four dimensions is likely to give an incorrect result for the moduli potential. At the (global) minimum of the potential the condition on the fluxes will of course remain unchanged (this is essentially determined by supersymmetry) so that arguments that depend only on static solutions to the classical equations (such as those in [3] (KKLT) where the complex structure moduli and dilaton are integrated out classically) will remain unchanged. However, arguments that depend on the potential (away from the global minimum as in some of the computations in [11,12,14]) may not be valid. For instance, as observed by the authors of the first two papers the calculation of the soft scalar masses for none ISD fluxes from the potential (23) disagree with that obtained directly from the D-brane action. At such points the volume modulus u is time-dependent and the arguments of this Letter (and [6]) will apply.

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