Application of Particle Swarm Optimization Algorithm for Computing Critical Depth of Horseshoe Cross Section Tunnel

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Abstract

Critical depth is an important parameter in the design, operation and maintenance of open channels and analysis of gradually varied flow. For horseshoe cross section channels, the governing equations are highly nonlinear in the critical flow depth and thus solution of the implicit equations involves time consuming numerical methods. In current research, through conversion of critical depth equation to an objective function and then its minimization by using Particle Swarm Optimization algorithm, we calculate critical depth in horseshoe channels. The accuracy of the proposed model was also evaluated by comparing with existing equations. Furthermore this method can be used to deal with other optimization problems in hydraulic engineering.

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1. Introduction

Critical depth is an important parameter for hydraulic engineers in the analysis of varied flow and play major role in the design, operation and maintenance of open channels. Critical depth is the flow depth
corresponding to the minimum specific energy for a given discharge in an open channel and is described by the following relationship [1], [2]:

\[
\frac{\alpha Q^2 T}{gA \cos \phi} = 1
\]  

(1)

where \(A\), \(T\), \(Q\), \(\alpha\) and \(g\) are cross sectional area of flow, top width, flow discharge, kinetic energy correction factor and gravitational acceleration, respectively (see Fig. 1). Because \(A\) and \(T\) depend on the flow depth \((y)\), so the equation (1) is explainable according to \(y\) and critical depth value is obtained by solving this relation.

Fig. 1. Longitudinal profile and cross-section geometry of an open channel

The standard horseshoe cross sections because of hydraulic characteristics are widely used in many tunnels for free surface and pressurized conduit water conveyance. Horseshoe cross sections have been designed and used by USBR [3] and frequently is used in several countries [4]. Because the complex geometry of horseshoe cross sections, equation (1) is implicit in critical depth and no analytical solution exist. There are few studies for determining normal and critical depth with satisfactory accuracy. In recent years some researchers developed iterative formulas and methods for computing the normal depth of the horseshoe cross section tunnel [5], [6]. In 2011 for the first time, Vatankhah and Easa presented an explicit regression-based equations for critical depth of horseshoe channels and has a maximum error of less than 1% [7].

In this research study, we use standard horseshoe cross section to demonstrate the capacity of Particle Swarm Optimization (PSO) algorithm for calculating critical depth of this compound man-made sections of channels. Comparisons of the accuracy of the proposed and existing solutions are also presented. The proposed PSO model has high accuracy and wide application range over cross sections.

2. PSO Algorithm

Particle Swarm Optimization (PSO) algorithm for the first time was proposed by Kennedy and Eberhart in 1995 [8], [9]. The method has been founded based on simulating group behaviour of a specific species of animals such as birds and fishes [10]-[12]. In PSO algorithm, there are many creatures that are said particle and have been distributed in the search space of a function that we intend to optimize its value [13]. Each particle calculates the objective function value in a position of space, which has located in it. Each particle sets its course in proportion to its best previous position and also in proportion to the best position achieved by its neighbors [14]. One step of the algorithm finishes after doing the movement. These steps are repeated several times until the desired answer is obtained. PSO algorithm is becoming popular because of its simplicity and ability to converge to good result [15], [16].

The PSO algorithm analyzes the problem by the exchange of information between the populations (called swarms) that in fact are the same particles (called candidate solutions) of the group. Each particle sets its path towards its best previous position and also towards the best position achieved by its neighbors [14].

In this method, particles are made by random positions and velocities. During algorithm implementation, the position and velocity of each particle in the \(t+1\) step of the algorithm are made from information on the
previous step. The equations that alter the velocity and position of the particles are:

\[ v_i(t+1) = \omega v_i(t) + c_1 r_1 [\hat{x}_i(t) - x_i(t)] + c_2 r_2 [g(t) - x_i(t)] \]  
\[ x_i(t+1) = x_i(t) + v_i(t+1) \]  

(2)  

(3)

Thus, \( v_i(t) \) is the velocity of particle \( i \) at time \( t \) and \( x_i(t) \) is the position of particle \( i \) at time \( t \). Where \( \omega \) is the inertial coefficient on the interval \([0.5, 1.2]\), which can either dampen the particle inertia or accelerate the particle in its original direction. Generally, lower values of the inertial coefficient speed up the convergence of the swarm to optima, and higher values of the inertial coefficient encourage exploration of the entire search space [15].

\( r_1 \) and \( r_2 \) are random numbers in the interval \([0, 1]\), and \( c_1 \) and \( c_2 \) are cognitive coefficients. \( r_1 \) and \( r_2 \) lead to the kind of diversity in the answers and thus a thorough search is performed in the search space. \( c_1 \) is the cognitive coefficient of personal experiences of each particle and in contrast \( c_2 \) is the cognitive coefficient of the whole set’s experience [15]. The value \( \hat{x}(t) \) is the individual best candidate solution for particle \( i \) at time \( t \), and \( g(t) \) is the swarm’s global best candidate solution at time \( t \). From the above equations, it can be concluded that each particle while moving considers its previous moving direction, the best position that it has ever been and the best position that has ever been experienced by the entire set. Clerc and Kennedy demonstrate that by using \( \omega = 0.7298 \) and \( c_1 = c_2 = 1.4962 \), results with higher accuracy could be achieved, but in PSO algorithm usually \( c_1 = c_2 = 2 \) is suggested [16]. In this research all simulations were performed with 40 particles and 200 maximum iterations.

3. Critical depth calculation using PSO algorithm

If the flow is uniform and the slope of the channel is small (less than 10%) \( \cos \theta = 0 \) and \( \alpha = 1 \) are acceptable assumption, therefore relation (1) changes as follows:

\[ \frac{Q^2T}{gA^3} = 1 \]  

(4)

to calculate critical depth, firstly we should convert the above relation to an objective function and then minimize it. The considered objective function is minimized when depth is equal to critical depth, therefore:

\[ \frac{Q^2T}{gA^3} - 1 = 0 \Rightarrow F = \left| \frac{Q^2T}{gA^3} - 1 \right| \]  

(5)

as it is observed, the only constraint of this problem is depth value \((0 < y < y_{\text{max}})\), which enters optimization algorithm by a penalty function. To solve the problem, at first we rewrite \( T \) and \( A \) parameters according to the flow depth \((y)\). As a result, we obtain \( F \) function according to the flow depth and geometric parameters of channel and optimize it by using the PSO algorithm.

4. Geometric Properties

It is necessary to illustrate and describe the geometric properties of the cross section before presenting comparisons of the accuracy of the proposed and existing explicit solution. The standard horseshoe cross section as shown in Fig. 2 is defined by the intersection of four circles so it consists of four arc segments: a top arc with radius \( r \), two lateral arcs with radius \( R \), and a bottom arc with the same radius \( R \) but with different circular centers [3], [4]. General horseshoe cross sections can be classified using the characteristic parameter \( t \)
= \frac{R}{r}$. When $t = 3$, it is called as standard Type I horseshoe cross section, and when $t = 2$, it is called as standard Type II horseshoe cross section. When $t = 1$, the horseshoe cross section become circular cross section with radius $r$ [5]-[7].

Fig. 2 shows a general horseshoe cross section and the corresponding geometric elements and angels for three ranges of water depths ($y$): (a) $0 \leq y \leq e$, (b) $e \leq y \leq r$, (c) $r \leq y \leq 2r$. $e$ is the height of the bottom arc, which is given by $e = 0.12917r$ ($\theta = 0.294515$) for Type I, and $e = 0.17712r$ ($\theta = 0.424031$) for Type II cross sections, respectively [6], [7].

Formulas for computing the geometric elements of standard horseshoe cross sections such as flow area ($A$) and width of the channel at surface ($T$), which are required for developing the critical depth equation were presented in [6], [7]. These formulas are presented in Table 1.

Table 1. Formulas for computing geometric elements for three zones of flow depth of a horseshoe cross section

<table>
<thead>
<tr>
<th>Zones of flow depth</th>
<th>$0 \leq y \leq e$ or $0 \leq \beta \leq \theta$</th>
<th>$e \leq y \leq r$ or $0 \leq \alpha \leq \theta$</th>
<th>$r \leq y \leq H$ or $0 \leq \varphi \leq \pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = \arccos(1 - y / (tr))$</td>
<td>$\alpha = \arcsin(1 / t - y / (tr))$</td>
<td>$\varphi = 2 \arccos(y / r - 1)$</td>
<td>$\varphi = 2 \arccos(y / r - 1)$</td>
</tr>
<tr>
<td>$A = t^2 r^2 \left[ \beta - 0.5 \sin(2\beta) \right]$</td>
<td>$A = t^2 r^2 \left[ C - \alpha - 0.5 \sin(2\alpha) + 2 \sin \alpha (t - 1) / t \right]$</td>
<td>$A = t^2 \left[ t^2 C + 0.5 (\pi - \varphi + \sin \varphi) \right]$</td>
<td>$A = t^2 \left[ t^2 C + 0.5 (\pi - \varphi + \sin \varphi) \right]$</td>
</tr>
<tr>
<td>$T = tH \sin(\beta)$</td>
<td>$T = H \left[ 1 - t + t \cos(\alpha) \right]$</td>
<td>$T = H \sin(\varphi / 2)$</td>
<td>$T = H \sin(\varphi / 2)$</td>
</tr>
</tbody>
</table>

Note: $C = 2 \theta + 1 - \sin(2\theta) - \cos(2\theta)$, $H = 2r$ = height of the

Table 2. Critical depth using PSO and explicit method for horseshoe cross sections

<table>
<thead>
<tr>
<th>Horseshoe cross section</th>
<th>Geometrical specification</th>
<th>Discharge (m$^3$/s)</th>
<th>Critical depth by explicit method (m) [7]</th>
<th>Critical depth by PSO method (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>$r = 1.6$ m</td>
<td>45</td>
<td>2.6859</td>
<td>2.6899</td>
</tr>
<tr>
<td></td>
<td>$r = 2.0$ m</td>
<td>35</td>
<td>2.1594</td>
<td>2.1635</td>
</tr>
<tr>
<td></td>
<td>$r = 2.5$ m</td>
<td>55</td>
<td>2.5345</td>
<td>2.5379</td>
</tr>
<tr>
<td>Type II</td>
<td>$r = 1.7$ m</td>
<td>40</td>
<td>2.5569</td>
<td>2.5545</td>
</tr>
<tr>
<td></td>
<td>$r = 2.0$ m</td>
<td>35</td>
<td>2.2310</td>
<td>2.2331</td>
</tr>
<tr>
<td></td>
<td>$r = 2.5$ m</td>
<td>55</td>
<td>2.6244</td>
<td>2.6264</td>
</tr>
</tbody>
</table>

5. Numerical results and discussion

Typically we do not know that the critical depth belongs to which case of three cases given in Fig. 2 when
we need to determine the critical depth for a given discharge. Therefore, after characteristic angles $\beta$, $\alpha$, or $\varphi$ are determined by the depth flow ($y$) and substituting formulas for the flow area ($A$) and width of the channel at the water surface ($T$) in Table 1 into objective function, then we can optimize it by using the PSO algorithm. In Table 2, comparisons between the value of critical depth for some numerical examples of Type I and Type II horseshoe cross sections from presenting PSO model and explicit relations in [7] are tabulated.

The most striking point to note is that comparisons between the two methods shows the superiority of the PSO algorithm technique. The differences between the explicit and PSO solution is less than 0.2% and this shows the high performance of this method.

6. Conclusion

In this study, we presented the calculation of critical depth, an essential parameter in hydraulic engineering, in horseshoe cross section open channels, based on PSO algorithm. The consistency of the model is checked through certain examples, numerical examples demonstrated the capacity, accuracy and simplicity of the present PSO model. Application of this algorithm may be used for solving other similar hydraulic engineering problems and equations like normal depth.

References