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# Supersymmetry counterterms revisited

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**Abstract**

Superspace power-counting rules give estimates for the loop order at which divergences can first appear in non-renormalisable supersymmetric field theories. In some cases these estimates can be improved if harmonic superspace, rather than ordinary superspace, is used. The new estimates are in agreement with recent results derived from unitarity calculations for maximally supersymmetric Yang–Mills theories in five and six dimensions. For $N = 8$ supergravity in four dimensions, we speculate that the onset of divergences may correspondingly occur at the six loop order.


## 1. Introduction

In this Letter we review the question of the onset of ultraviolet divergences in super-Yang–Mills theories (in more than four dimensions) and in supergravity. One of the most distinctive features of supersymmetric quantum field theories is that they generally have improved ultraviolet behaviour compared to their non-supersymmetric counterparts [1]. This became evident shortly after the discovery of four-dimensional supersymmetry and all-orders results were subsequently established for renormalisable theories. These results are most clearly seen in the framework of superspace where non-renormalisation theorems can be established using superspace power-counting [2]. This approach can also be applied to non-renormalisable theories, in particular to supergravity and higher-dimensional super-Yang–Mill theories (SYM). Here, although early hopes that the maximally supersymmetric $N = 8$ supergravity might be finite do not seem to be justifiable,[1] superspace techniques can nevertheless be used to predict the lowest loop-order at which the onset of ultraviolet divergences can be expected to occur. Typically, the onset of divergences occurs at higher loop order than in non-supersymmetric theories, but no non-renormalisable field theory is rendered finite by supersymmetrisation.

In [5,6] predictions were made for the onset of divergences in supergravity and higher-dimensional SYM theories, using techniques from [7,8]. The status of these original predictions based upon standard superspace was reviewed in [9]. These predictions were based on two assumptions: firstly, that the coun-

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[1] 3-loop counterterms were constructed for $N = 1, 2$ supergravities in [3] and a 7-loop counterterm was found for $N = 8$ supergravity in [4].

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terterm corresponding to the lowest loop-order divergence should be invariant under the full "on-shell" $N$-extended supersymmetry[2] of the theory and secondly, that it should be expressible as a full superspace integral with respect to the portion $M$ of the full supersymmetry that is linearly realisable "off-shell" and so can be preserved manifestly in the quantisation procedure.

The non-renormalisation theorem of Refs. [5,7] is conveniently expressed using the background field method,[3] and draws its power from the fact that, in a background-quantum split, only certain combinations of the background fields appear in the Feynman rules used in calculating corrections to the quantum effective action $\Gamma$ (the generating functional of 1PI diagrams). Although it is convenient to solve superspace constraints to express the *quantum* fields appearing on the internal lines of 1PI superspace Feynman diagrams as differential constructions using unconstrained "prepotentials", this is never necessary for the *background* fields. The non-renormalisation theorem that follows from this is:

**Theorem 1.** *For gauge and supergravity multiplets at loops $\ell \geqslant 2$, and for matter multiplets at all loop orders, counterterms must be written as full superspace integrals for the maximal off-shell linearly realisable supersymmetry, with background gauge invariant integrands written without using prepotentials for the background fields.*

Additional restrictions on counterterms may also follow by combining the non-renormalisation theorem with the full on-shell supersymmetry. The on-shell supersymmetry is not as powerful as the off-shell, linearly-realisable supersymmetry because it is nonlinear and also has an algebra that closes only modulo field-equation transformations, thus allowing poorly controllable mixing between counterterms and the original action. However, imposing the original classical field equations removes variations of the original action from consideration, and restriction to the lowest order divergences leaves them nothing of lower order to mix with, so a simple statement of invariance with respect to the full on-shell supersymmetry, modulo the classical field equations, is obtained. This can have the effect of linking allowable counterterms under the linearly realisable $M$ supersymmetry to counterterms that are thereby disallowed, requiring thus an overall vanishing coefficient [6]. Consequently, the simple requirement of invariance for the leading divergences under the full on-shell supersymmetry modulo classical field equations is significantly strengthened upon combination with the non-renormalisation theorem.

In practice, for the maximally supersymmetric Yang–Mills and supergravity theories, with 16 and 32 supersymmetries respectively, it was assumed[4] in [5,6] that one-half of the supersymmetries can be preserved manifestly in the quantum theory; in other words, that one can quantise these theories, at least in principle, using superfields with 8 or 16 supersymmetries.

On the basis of these assumptions it was argued in [5,9] that the lowest counterterm would be of the form $R^4$ in supergravity (3 loops in $D = 4$), while for $D = 5$ SYM the first divergence should occur at 4 loops corresponding to a counterterm of the form $F^4$. However, recent computations [14] using unitarity cutting-rule techniques have indicated that the onset of divergences actually occurs at higher loop order in the $D = 5$ maximal SYM theory and also in $D = 4$ maximal supergravity. Specifically, the authors of [14] find that divergences start at 6 loops in $D = 5$ SYM and that their onset is postponed to at least

[2] In this Letter, we refer to minimal supersymmetry in a given dimension as $N = 1$ supersymmetry, etc. For clarity, we also indicate, when appropriate, the corresponding number of supercharges.

[3] It can also be derived straightforwardly using standard superspace Feynman rules.

[4] For $D = 4$, $N = 4$ SYM, a full quantum formalism was given in Refs. [5,8] in terms of $D = 4$, $M = 2$ superfields (corresponding to 8 supercharges). Similarly, the $D = 5$, $N = 2$ and $D = 6$, $N = 2$ SYM theories can be written in terms of $M = 1$ superfields in these dimensions [10,11]. Higher-dimensional analogues of this 8-supercharge formalism would linearly realise only a partial Lorentz covariance; for example, one could work with a formalism that linearly realises $D = 6$ Lorentz covariance and 8 supercharges in the maximal $D \geqslant 7$ SYM theory. For $D = 4$, $N = 8$ supergravity, linearised analysis reveals the possibility of a $D = 4$, $M = 4$ ordinary superspace formalism. Correspondingly, an off-shell version of linearised $D = 10$, $M = 1$ supergravity was constructed in [12]. Multiplet counting arguments at the linearised level [13] also suggest the existence of an $M = 1$ off-shell version of the $D = 10$ spin $3/2$ multiplet that is needed to complete the maximal $D = 10$, $N = 2$ theory, but the details of this have not been worked out.

5 loops in the $D = 4$, $N = 8$ supergravity theory. Previously it had been shown that the maximal SYM theories are finite at two loops in $D = 5, 6$ [15], but the new unitarity calculations strengthen this result substantially for the $D = 5$ case.

In [16] an attempt was made to explain these results using higher-dimensional gauge-invariance. If a theory in a given spacetime dimension somehow knows about the gauge-invariances of the theory in higher dimensions from which it can be derived by dimensional reduction, then one could expect to have improved ultraviolet divergence behaviour because, for example, terms involving undifferentiated scalar fields would be ruled out by the higher-dimensional gauge symmetry. In practice, this argument seems hard to justify, and indeed does not seem to be true in simpler non-supersymmetric examples. However, there is a simpler reason why one would expect to find better UV behaviour than predicted in [5,6,9], and that is that it may be possible to preserve a larger fraction of the supersymmetry manifestly in the quantum theory that had been assumed. The results of [5,6, 9] depend on power-counting in ordinary superspace, but one can obtain improved power-counting results if one makes us of harmonic superspace [17]. Indeed, this formalism has been available since the early 1980s but has been overlooked in the context of non-renormalisable theories.

It was shown in [18] that the maximal SYM theory in $D = 4$ admits an off-shell superfield formulation (and therefore a quantisation procedure) in $M = 3$ harmonic superspace, so that one can therefore preserve $3/4$ of the supersymmetries rather than just $1/2$. If one uses this formalism for higher-dimensional theories, even though the manifest higher-dimensional Lorentz symmetry is lost, one finds that the expected onset of divergences in $D = 5$ now agrees with the results of [14]. The situation in $D = 6$, however, is unchanged, and indeed the old results for this theory are in agreement with the new calculations. For the maximal supergravity theory it is not known how much supersymmetry can be preserved off-shell using harmonic superspace methods, but the similarity of the relationships between $D = 4$, $N = 3$ and $N = 4$ SYM and $D = 4$, $N = 7$ and $N = 8$ supergravity lead us to conjecture that there may be an off-shell version of $N = 8$ supergravity with $M = 7$ supersymmetry. If this is true, then one would expect the onset of divergences to occur at 6 loops, while if only $M = 6$ supersymmetry can be preserved, then the divergences would start at 5 loops.

## 2. Yang–Mills theory

The $D = 4$, $N = 4$ SYM theory is described in ordinary Minkowski superspace by a scalar superfield $W_{ij} = -W_{ji}$, $i, j = 1, \ldots, 4$ which transforms under the six-dimensional representation of the $SU(4)$ internal symmetry group, and which is also real in the sense that $\overline{W}^{ij} = \frac{1}{2}\epsilon^{ijkl} W_{kl}$. It obeys the constraints

$$\nabla_{\alpha i} W_{jk} = \nabla_{\alpha [i} W_{jk]}, \tag{1}$$

$$\overline{\nabla}^i_{\dot\alpha} W_{jk} = -\frac{2}{3}\delta_{[j}{}^i \overline{\nabla}^l_{\dot\alpha} W_{k]l}, \tag{2}$$

where $(\nabla_{\alpha i}, \overline{\nabla}^i_{\dot\alpha})$ are gauge-covariant spinorial derivatives. In fact, the reality constraint on $W_{ij}$ means that the above two constraints are equivalent. These equations follow from the imposition of the standard constraints on the superspace field-strength two-form; by use of the Bianchi identities one can show that they imply that the only component fields are those of the on-shell SYM multiplet, i.e., 6 scalar fields, 4 spin $1/2$ fields and 1 vector field; and furthermore, that these fields obey the usual classical equations of motion.

The same multiplet can also be described in $M = 3$ superspace by an irreducible $M = 3$ superspace field strength superfield obeying similar constraints to the $N = 4$ ones but with (2) replaced by

$$\overline{\nabla}^i_{\dot\alpha} W_{jk} = -\delta_{[j}{}^i \overline{\nabla}^l_{\dot\alpha} W_{k]l}. \tag{3}$$

Note that the $M = 3$ $W_{ij}$ transforms under the complex 3-dimensional representation of the $SU(3)$ internal symmetry group and is no longer subject to a self-duality condition. This multiplet has exactly the same field content as the $N = 4$ multiplet and is again on-shell. However, this version can be extended off-shell using harmonic superspace techniques whereas it is not known how to do this while maintaining manifest $N = 4$ supersymmetry.

We give an outline of the off-shell $M = 3$ harmonic superspace formalism for the $N = 4$ theory in Appendix A. The details of this off-shell theory are not essential for the superspace power-counting argument which follows, however. This is because it is sufficient to examine the leading counterterm from the old-style analysis, so we can simply look at the possible counterterms that can be constructed using the $M=3$ superfield $W_{ij}$. On the other hand, the existence of the off-shell harmonic superspace theory (for which a full quantum formalism can indeed be elaborated[5]), assures us that counterterms need also to be expressible as full superspace integrals in $M=3$ superspace.

The old power-counting rules, assuming that only one-half of the supersymmetry is preserved, predict that the simplest Lagrangian counterterm would be of the form $\int d^8\theta\, W^4 \sim F^4$ (where $F$ is the spacetime Yang–Mills field strength tensor). However, if we now make use of the harmonic superspace formalism, we expect the lowest order counterterms to be of the form $\int d^{12}\theta\, W^4 \sim \partial^2 F^4$ where $\partial$ denotes a spacetime derivative. This is under the assumption that we are quantising using superfields covariant with respect to $D=4$ Lorentz symmetry, even though the actual spacetime dimension is higher.

Let us now consider the maximal SYM theory in $D=6$. In the old analysis, this was to be quantised using ordinary $D=6$, $M=1$ superfields ($\leftrightarrow D=4$, $M=2$, i.e., 8 supercharges). The $D=6$, $M=1$ SYM field strength is a spinor $W^{\alpha i}$, $i=1,2$, and the off-shell counterterms must, as we have reviewed above, be expressible as gauge-invariant $M=1$ superspace integrals. Thus, the old $D=6$ predictions for maximal SYM theory is $\int d^8\theta\,(W^{\alpha i})^4 \sim \partial^2 F^4$. Hence, in the $D=6$ case there is no change between the old prediction based on preserving 8 supersymmetries with $D=6$ Lorentz covariance and the new prediction based on preserving 12 supersymmetries with $D=4$ Lorentz covariance; either way, one obtains a prediction of 3 loops for the first $D=6$ divergence. What has happened in $D=6$ is that the requirements of $D=6$ Lorentz plus gauge invariance and of 8-supercharge manifest supersymmetry coincide with those of gauge and $D=4$ Lorentz invariance and 12-supercharge supersymmetry.

Now consider the case of $D=5$ maximal SYM. In the old analysis, this was to be quantised using $D=5$, $M=1$ superfields (again $\leftrightarrow D=4$, $M=2$, i.e., 8 supercharges). The $D=5$, $M=1$ SYM field strength superfield is a scalar $W$, however. Thus, one would expect to find a divergence of the form $\int d^8\theta\, W^4 \sim F^4$. The new 12-supercharge harmonic superspace prediction improves this to $\partial^2 F^4$, exactly in agreement with the cutting-rule results of [14]. Note that the harmonic superspace prediction also reproduces exactly the bound one obtains by assuming that $D=6$ gauge invariance is still somehow active in $D=5$ as considered in [16]. Although there is no particular reason to believe that restrictions from higher-dimensional gauge invariance should continue to be applicable after dimensional reduction, the new 12-supercharge harmonic superspace analysis is robust, and should apply to the analysis of all quantum corrections in the theory.

To summarise, let us compare in various spacetime dimensions the predictions of the old ordinary superspace Feynman rules to the new ones from harmonic superspace, the latter agreeing fully with the cutting-rule results of [14]. Table 1 gives the results from the old analysis.

Table 1
Maximal SYM divergence expectations from 8-supercharge ordinary superspace Feynman rules

| Dimension | 10 | 8 | 7 | 6 | 5 | 4 |
|---|---|---|---|---|---|---|
| Loop $L$ | 1 | 1 | 2 | 3 | 4 | $\infty$ |
| Gen. form | $\partial^2 F^4$ | $F^4$ | $\partial^2 F^4$ | $\partial^2 F^4$ | $F^4$ | finite |

These predictions can be compared with those shown in Table 2 derived from the new harmonic superspace analysis, agreeing fully with the unitarity cutting-rule results of [14].

Table 2
Maximal SYM divergence expectations from 12-supercharge harmonic superspace Feynman rules or from cutting rules

| Dimension | 10 | 8 | 7 | 6 | 5 | 4 |
|---|---|---|---|---|---|---|
| Loop $L$ | 1 | 1 | 2 | 3 | 6 | $\infty$ |
| Gen. form | $\partial^2 F^4$ | $F^4$ | $\partial^2 F^4$ | $\partial^2 F^4$ | $\partial^2 F^4$ | finite |

## 3. Supergravity

In order to study supergravity counterterms for quantisation about flat space, it is sufficient to work at the linearised level. Linearised $N$-extended super-

[5] An example of a full quantum formalism using harmonic superspace, including the derivation of non-renormalisation theorems, was given in the context of $D=2$ $(4,0)$ non-linear supersymmetric sigma models in [19]. Quantisation using the harmonic $M=3$ formalism of $D=4$, $N=4$ SYM was carried out in [20].

gravity with $N \geqslant 5$ is described by a field strength superfield, $W_{ijkl}$, which is totally anti-symmetric in the $SU(N)$ indices $i, j = 1, \dots, N$, and obeys the constraints

$$D_{\alpha i} W_{jklm} = D_{\alpha[i} W_{jklm]}, \tag{4}$$

$$\overline{D}^i_{\dot\alpha} W_{jklm} = -\frac{4}{(N-3)} \delta^i_{[j} \overline{D}^n_{\dot\alpha} W_{klm]n}. \tag{5}$$

In addition, in the $N = 8$ theory, $W$ satisfies a self-duality condition

$$\overline{W}^{ijkl} = \frac{1}{4!} \epsilon^{ijklmnpq} W_{mnpq}. \tag{6}$$

The component fields contained in these superfields are precisely those of the supergravity multiplets and the above constraints imply that they all satisfy the corresponding linearised supergravity field equations. As in the SYM case, the maximal theory can be described by either an irreducible $N = 8$ or $N = 7$ multiplet.

Under the old rules it was assumed that the maximal theory could be quantised preserving $D = 4$, $M = 4$ supersymmetry; in this case, the lowest order counterterm that one can construct is at the 3 loop order and has the form $\int d^{16}\theta\, W^4 \sim R^4$, where $R$ is the spacetime curvature. This was shown to be fully $N = 8$ supersymmetric in [21] and was shown to have full $SU(8)$ internal symmetry in [22].[6] Now let us suppose, in analogy to the Yang–Mills case, that the theory can be quantised in a harmonic superspace formalism preserving $M = 7$ supersymmetry. In this case the lowest counterterm that would be allowed would be $\int d^{28}\theta\, W^4 \sim \partial^6 R^4$. This corresponds to a 6 loop counterterm. In fact, if $M \geqslant 4$ four-dimensional supersymmetries can be preserved, the expected lowest order counterterm would be $\int d^{4M}\theta\, W^4 \sim \partial^{2M-8} R^4$.

The results of [14] indicate that the onset of divergences in $N = 8$ supergravity occurs at the earliest at 5 loops, and this suggests that there must be an off-shell formulation of the theory with at least $M = 6$ supersymmetry (the 5 loop counterterm is of the form $\partial^4 R^4$). However, as we have mentioned above, the analogy with the maximal Yang–Mills theory suggests that the $N = 8$ theory may indeed admit an off-shell formulation with $M = 7$ supersymmetry. If this is the case, it will be interesting to see if the methods of [14] can be extended to confirming this explicitly. Alternatively, it would be interesting to see if harmonic superspace methods can be developed to find the off-shell theory. Results of the unitarity cutting-rule analysis might serve as a mathematical "experiment" revealing the possibility of an unknown off-shell superspace formalism for maximal supergravity. This could also have an important impact on the study of quantum M-theory.

[6] This action counterterm can be written in a very concise form in a certain harmonic superspace [23]; in this version all the symmetries are manifest.

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## Appendix A. $N = 3$ SYM in harmonic superspace

The constraints of $N = 3$ SYM in ordinary superspace are

$$F_{\alpha i \beta j} = \epsilon_{\alpha\beta} W_{ij}, \tag{A.1}$$

$$F_{\alpha i \dot\beta}{}^{j} = 0, \tag{A.2}$$

where $F$ is the superspace Yang–Mills field strength tensor and the components which are primarily constrained are those with bi-spinorial indices. The Bianchi identities then imply constraints on other components, and in particular imply that the field $W_{ij}$ satisfies the constraints (3) and also that the field equations of the component fields must be satisfied.

These constraints can be interpreted as integrability conditions in a harmonic superspace context. To do this, one enlarges the superspace by adjoining to it an internal manifold of the form[7] $U(1) \times U(1) \times U(1) \backslash U(3) := H \backslash U(3)$. This space is a compact complex manifold with complex dimension 3. We shall follow the standard practice of working with fields defined on the group $U(3)$; this is equivalent to

[7] This is the same space as $U(1) \times U(1) \backslash SU(3)$ which was used in [18].

working on the coset if all the fields are taken to be equivariant with respect to the isotropy group $H$. We denote an element of $U(3)$ by $u_I{}^i$ and its inverse by $(u^{-1})_i{}^I$ where the capital indices are acted on by the isotropy group and the small indices by $U(3)$. Using $u$ and its inverse we can convert $U(3)$ indices to $H$ indices and vice versa. Thus we have

$$\begin{aligned} D_{\alpha I} &:= u_I{}^i D_{\alpha i}, \\ \overline{D}^I_{\dot\alpha} &:= \overline{D}^i_{\dot\alpha}(u^{-1})_i{}^I. \end{aligned} \tag{A.3}$$

In order to differentiate with respect to the internal manifold we introduce the right-invariant vector fields $D_I{}^J$. They satisfy the constraints $D_I{}^I = 0$ and $\overline{D}^I{}_J = -D_J{}^I$. They also satisfy the commutation relations of $\mathfrak{u}(3)$. The diagonal derivatives correspond to the isotropy algebra while the remainder fall into two complex conjugate sets: $(D_1{}^2, D_1{}^3, D_2{}^3)$ and $(D_2{}^1, D_3{}^1, D_3{}^2)$. The former three derivatives can be thought of as the components of the $\bar\partial$ operator on the coset and the latter as the components of the conjugate operator $\partial$.

A superfield that is annihilated by $(D_1{}^2, D_1{}^3, D_2{}^3)$ is called H-analytic (H for harmonic), and will have a short harmonic expansion on the coset since this space is a compact complex manifold. A superfield which is annihilated by $(D_{\alpha 1}, \overline{D}^3_{\dot\alpha})$ is called G-analytic (G for Grassmann), and a superfield which annihilated by both sets of operators is called CR-analytic, or just analytic for short.

The SYM constraints can now be interpreted as integrability conditions in harmonic superspace. They correspond to the vanishing of the SYM curvature in the spinorial directions $(\alpha 1, \overset{3}{\dot\alpha})$. These constraints are

$$F_{\alpha 1 \beta 1} = F^{33}_{\dot\alpha\dot\beta} = F_{\alpha 1 \overset{3}{\dot\beta}} = 0. \tag{A.4}$$

At this stage we have not yet extended the gauge theory into the new directions so that the $F_{\alpha i \beta j}$, etc., do not depend on the harmonic coordinates. It therefore follows that the constraints (A.4) imply (A.1), (A.2) above, and these in turn imply the equations of motion. We can take the theory off-shell by introducing a connection for the harmonic directions and by allowing the components of the curvature in these directions to be non-vanishing. In this case, the constraints (A.4) no longer imply the original constraints (A.1), (A.2). To get the equations of motion we therefore only need to find an action which will imply that the curvature should vanish in the harmonic directions. Since this space has three complex dimensions one can use a Chern–Simons action for this. Remarkably, all of the dimensions and internal charges work out just right for this to work.

The resulting action is

$$I = \int d\mu^{11}_{33}\, Q^{(3)}{}^{33}_{11}, \tag{A.5}$$

where the superscripts indicate the $(U(1))^3$ charges and the measure $d\mu$ is defined by

$$d\mu^{11}_{33} := d^4x\, du\, \big(D_2\, D_3\, \overline{D}^1\, \overline{D}^2\big)^2. \tag{A.6}$$

Here $du$ is the usual measure for the coset space while $(D_2)^2 := D_{\alpha 2} D^{\alpha 2}$ etc. $Q^{(3)}$ is the Chern–Simons 3-form defined in the usual way (on the whole of harmonic superspace) by $dQ^{(3)} = \mathrm{tr}(F \wedge F)$. The component of $Q^{(3)}$ which appears in the action is the component in the anti-holomorphic harmonic direction, $Q^{(3)}{}_1{}^2{}_{,1}{}^3{}_{,2}{}^3 := Q^{(3)}{}^{33}_{11}$. This expression can be written explicitly in terms of the connection in the harmonic directions which has components $(A_1{}^2, A_1{}^3, A_2{}^3)$. The above constraints, together with the fact that the mixed harmonic/superspace curvatures are also zero imply that these components of the connection are G-analytic so that the action is manifestly supersymmetric.

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