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Forecasting and analyzing the competitive diffusion of mobile cellular broadband and fixed broadband in Taiwan with limited historical data ☆

Chiun-Sin Lin *

Department of Business and Entrepreneurial Management, Kainan University, No. 1 Kainan Road, Luzhu, Taoyuan 33857, Taiwan, ROC

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ABSTRACT

Taiwan experienced the rapid growth of mobile cellular broadband from 2005 by introducing 3G operations and had higher penetration than the average of the developing countries, the world, and even the developed countries. There are many forecasting models which were developed and successfully predicted the diffusion of long lifecycle product, but there are very few forecasting models which were developed for predicting new products with short lifecycle. Assumption of these models is always the growth of products follows an S-shaped curve. As for the products which were just introduced to the market, it is very difficult to identify if they follow an S-shaped curve with their limited historical data. This research aims to apply Grey system theory to predict the diffusion of mobile cellular broadband and fixed broadband in Taiwan since Grey system theory has a characteristic which requires very limited primitive data (the least 4 data) to build a differential forecasting model. We use penetration as an indicator to describe the diffusion of new products. The numerical data show that the Grey forecasting models GM(1,1) built in this paper have higher prediction accuracy than logistic models and grey Verhulst models. Moreover, we apply Lotka–Volterra model to analyze the competitive relationship between mobile cellular broadband and fixed broadband. The empirical data show that the relationship is commensalism rather than predator–prey. These results can be extended to contribute to other researches.

1. Introduction

Penetration of telecommunication service is used to depict the number of subscriptions per 100 inhabitants. In 2002, penetration of mobile cellular subscriptions exceeded 100 in Taiwan, reached 109.55 and was the world's number one of the year (ITU, 2012). The penetration which exceeds 100 implies that some users have 2 or more subscriptions. Popularization of mobile cellular networks indirectly pushes the introduction of mobile broadband services.

Upon the demand of Internet access over cellular network, new technologies, GPRS and EDGE, were developed in 2000 and 2003 respectively. GPRS and EDGE are so called 2.5G and 2.75G networks. Because of the long distance from the base station, the interference from radio signal and the poor indoor coverage of cellular network, the data access over cellular network is not widely accepted by the market until 2005. The growth of mobile broadband is very slow in the beginning (Chunghwa Telecom, 2011).

The introduction of 3G network is the key of rapid growth of mobile cellular. We can see the rapid growth from year 2005 which is the year that Taiwan starts 3G operations. The mobile cellular is equivalent to mobile cellular broadband from that year.

In 2011, the penetration of mobile cellular in Taiwan is up to 124.1% which is much higher than the world's average of 85.7% and even higher than the average of developed countries of 122.3%. The growth of mobile cellular in Taiwan is very fast. Table 1 shows the penetration rates of mobile cellular of developed, developing, world countries average and Taiwan.

There are many forecasting models which were developed and successfully predicted the diffusion of long product lifecycle, but there are very few forecasting models which were developed for predicting new products with short lifecycle. Assumption of these models is always the growth of products follows the S-shaped curve. As for the products which were just introduced to the market, it is very difficult to identify if they follow the S-shaped curve with their limited historical data. This research aims to apply Grey system theory to predict the diffusion of mobile cellular broadband and fixed broadband in Taiwan since Grey system theory has a characteristic which requires very limited primitive data (the least 4 data) to build a differential forecasting model. The proposed GM(1,1) model will be compared with the logistic model and the grey Verhulst model respectively. In order to find out the relationship between

* Tel.: +886 3 3412500x6224; fax: +886 2 23120078.
E-mail address: netec7@yahoo.com.tw.

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Table 1
Penetration rates of mobile cellular of developed, developing, world countries average and Taiwan (ITU, 2012) (Unit: subscriptions per 100 inhabitants).

	2005	2006	2007	2008	2009	2010	2011
Developed	82.1	92.9	102.0	108.3	112.6	114.5	122.3
Developing	22.9	30.2	39.1	49.1	58.4	68.9	77.8
World	33.9	41.7	50.6	59.9	68.2	77.1	85.7
Taiwan	97.6	101.7	105.7	110.2	116.4	119.9	124.1

mobile cellular broadband and fixed broadband, we apply the Lotka–Volterra model to analyze the competitive mechanism.

The remainder of this paper is organized as follows. Section 2 provides a review of relevant literature. Section 3 describes methodology. Section 4 presents empirical analyses and results. Section 5 includes policy implications. The final section concludes the whole article.

2. Literature review

Applying an S-shaped diffusion model is the first step (Chu et al., 2009) in analyzing product diffusion. Chuang and Hsu (2010) applied Bass model and KK model to forecast multinational diffusion in LCD TV industry with data from Asia and North America. Wu and Chu (2010) applied Logistic, Gompertz and Bass models to analyze the penetration of mobile cellular subscriptions during 1988–2007 in Taiwan and compare the performance of these models. They found that the Gompertz model outperformed the other models for the period prior to diffusion take-off. However, the logistic model surpassed the other models after diffusion inflection. There is no diffusion model which can completely fit the whole lifecycle of a product.

Michalakelis et al. (2008) applied the Bass model, Fisher–Pry model and Gompertz model to examine the diffusion rate of mobile telephone subscriptions in Greece in 1994–2005(Q3). They presented that these S-shaped models could accurately fit and forecast the diffusion of mobile cellular subscriptions. Gruber and Verboven (2001) studied the diffusion of mobile telecommunication services in the European Union during 1984–1997 using a logistic model. The transition from analogue to digital networks was the major impact to the diffusion of mobile telecommunications.

However, applying these models needs some amount of data points to identify the fitness of models. As for products with new technologies, the lifecycles are usually only a few years. It has not enough data points to identify the suitable diffusion models. This becomes a big problem.

Trappey and Wu (2008) developed a modified logistic model, Time-Varying Extended Logistic model, to forecast 6 new services with penetration rate and 16 new products with cumulative sales volume. They argued that the Time-Varying Extended Logistic model outperformed the simple logistic and Gompertz models in most of product datasets. Although this modified model has no need of upper limit of the curve to be estimated prior to forecasting, it is also only suitable for data that grows as an S-shaped curve. In addition, it may not be suitable for linear data or for curves with many anomalous data points.

Deng (1982, 1989) firstly proposes Grey system theory to predict a system whose data is ambiguous, insufficient or uncertain. It does not require that the dataset fits any curve. It is a kind of time series prediction which is sampled equally in time interval. Grey system theory predicts the future values based on the inputs obtained in the previous and current data. That is why it has the ability of self-adaptation. Grey system theory has a characteristic to build a differential model, so called grey model (GM), by using the least 4 data to replace difference modeling in vast quantities of data.

Lin et al. (2011) applied Grey model to predict the future CO2 emission in Taiwan from 2008 to 2012 using primitive data from 2000 to 2007. It provided a reference with which the Taiwan government could establish measures to reduce CO2 emissions. Rezaei and Khalaj (2005) applied Grey prediction theory to the handoffs algorithm of cellular networks to reduce both the number of handoffs and handoff delay. Zhang et al. (2010) applied Grey system theory to predict the population growth of China. Lim et al. (2010) used GM(1,1) model to predict the online auction's closing price to maximize the bidder's profit. Kayacan et al. (2010) also argued that grey system theory based approaches can achieve good performance characteristics when applied to real-time systems, since grey predictors adapt their parameters to new conditions as new outputs become available. Because of this reason, grey predictors are more robust with respect to noise and lack of modeling information when compared to conventional methods. Wen and Huang (2005) used Grey Verhulst method to construct population prediction model. They pointed out that the difference of Grey Verhulst model was the added item in the response side of GM(1,1) model and it was to limit the whole development for real system. In empirical analyses, we will compare these two models and show that Grey Verhulst model is not suitable for predicting the diffusion of new products.

Lotka–Volterra model comprises a competitive mechanism in the diffusion process. This model includes the dynamics of competition between species in ecology system. Lee et al. (2005) used it to analyze the competitive relationship between KSE and KOSDAQ, two competing stock markets in Korea. Prior to their paper, Modis (1999) analyzed the behavior of common stocks by Lotka–Volterra model as if they were species competing for investors' money. Chiang and Wong (2011) applied the Lotka–Volterra model to explore the competitive diffusion of desktop computers and notebook computers. Afterwards, Chiang (2012) continued to use this model to study the relationship between 200 mm silicon wafers and 300 mm silicon wafers and show that they have a prey–predator relationship under the assumption of natural competition in the global semiconductor market. Lotka–Volterra model is appropriate to analyze the competitive relationship between mobile cellular broadband and fixed broadband in this research.

3. Methodology

3.1. Grey system theory and grey prediction

Grey system theory is firstly proposed by Dr. J. L. Deng (1982). It is used to predict a system whose data is ambiguous, insufficient or uncertain. A system is called "WHITE" if all information in system is completely known. On the contrary, a system is called "BLACK" if all information in system is unknown. A Grey system is the one that some information in system is known and some is not. It is widely used in predication for many different areas such as information engineering, computer science, electronics, electricity, mechanical engineering, automation, industrial engineering, medical science, etc.

Grey prediction is based on Grey Model (GM). The basic Grey Model is GM(m, n) where m indicates the order of differential equation and n indicates the number of variables. The simplest form is GM(1,1). It means that the model uses the first order differential equation and has only one variable. Grey predication has some characteristics that it doesn't need too many historical data and to fit a typical distribution (Wen and Huang, 2005).

3.2. Proposed grey prediction model

Grey model is a non-function time series forecasting model. It has the ability of self-adaptation. It uses the output of previous state as input of current state then generates the output of current state together with other inputs of current state. Grey model GM(1, 1) is most widely used in researches but it only can be used in positive data sequence (Deng, 1989). According to the definition of Grey system theory, the shadow equation of GM(1,1) is a first-order ordinary linear differential equation which can be defined as (Deng, 1989):

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \tag{1}$$

where t is the independent variable, a is the developing coefficient and b is the Grey influence coefficient.

The differential equation in Eq. (1) is solved to obtain the values of parameters a and b. It requires an algorithm to solve the differential equation.

First, identify the primitive series $X^{(0)}$.

$$X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \ldots\ldots, x^{(0)}(n)\right\}, \quad n \geq 4 \tag{2}$$

where $X^{(0)}$ is the non-negative series and n is the sample size.

Second, construct Accumulated Generating Operation (AGO).

The AGO is defined as:

$$\begin{aligned} X^{(1)} &= \left\{x^{(1)}(1), x^{(1)}, \ldots\ldots, x^{(1)}(n)\right\} \\ &= \left\{\sum_{t=1}^{1} x^{(0)}(t), \sum_{t=1}^{2} x^{(0)}(t), \ldots\ldots, \sum_{t=1}^{n} x^{(0)}(t)\right\}, \quad n \geq 4. \end{aligned} \tag{3}$$

Third, solve the developing coefficient a and influence coefficient b of Grey differential equation.

According to the least square method, we have

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = \left(B^T B\right)^{-1} B^T Y \tag{4}$$

where

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{5}$$

$$B = \begin{bmatrix} -\frac{1}{2}\left[x^{(1)}(1) + x^{(1)}(2)\right] & 1 \\ -\frac{1}{2}\left[x^{(1)}(2) + x^{(1)}(3)\right] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}\left[x^{(1)}(n-1) + x^{(1)}(n)\right] & 1 \end{bmatrix}. \tag{6}$$

Then, we can obtain the first order predicted value $\hat{x}^{(1)}$ and the predicted value $\hat{x}^{(0)}$.

After the estimation of the coefficients a and b, the grey prediction equation can be obtained and solved in the differential Eq. (1) as follows:

$$\begin{aligned} &\hat{x}^{(1)}(t+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-at} + \frac{b}{a}, t = 1, 2, \ldots, n-1 \\ &\hat{x}^{(1)}(1) = x^{(0)}(1). \end{aligned} \tag{7}$$

To obtain the predicted value of the primitive data at time $(t + 1)$, we use Inverse Accumulated Generating Operation (IAGO) to establish the Grey prediction model.

$$\hat{x}^{(0)}(t+1) = \hat{x}^{(1)}(t+1) - \hat{x}^{(1)}(t), t = 1, 2, \ldots, n-1. \tag{8}$$

Finally, forecast the value at time $(n + k)$

$$\hat{x}^{(0)}(n+k) = \hat{x}^{(1)}(n+k) - \hat{x}^{(1)}(n+k-1), \text{ where } k \geq 1. \tag{9}$$

3.3. Grey Verhulst model

Grey Verhulst model is defined as (Deng, 1989; Wen and Huang, 2005)

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b\left(x^{(1)}\right)^2.$$

We note that the main difference of grey Verhulst model is the added item in the response side of GM(1,1) model and the analysis steps are the same as the approach of GM(1,1) model.

Use the least square method to estimate the parameters, we have

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = \left(B^T B\right)^{-1} B^T Y$$

where

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

$$B = \begin{bmatrix} -\frac{1}{2}\left[x^{(1)}(1) + x^{(1)}(2)\right] & \left[\frac{1}{2}\left[x^{(1)}(1) + x^{(1)}(2)\right]\right]^2 \\ -\frac{1}{2}\left[x^{(1)}(2) + x^{(1)}(3)\right] & \left[\frac{1}{2}\left[x^{(1)}(2) + x^{(1)}(3)\right]\right]^2 \\ \vdots & \vdots \\ -\frac{1}{2}\left[x^{(1)}(n-1) + x^{(1)}(n)\right] & \left[\frac{1}{2}\left[x^{(1)}(n-1) + x^{(1)}(n)\right]\right]^2 \end{bmatrix}.$$

The time response sequence can be obtained as follows:

$$\begin{aligned} &\hat{x}^{(1)}(t+1) = \frac{ax^{(1)}(0)}{bx^{(1)}(0) + \left[a - bx^{(1)}(0)\right]e^{ak}}, t = 1, 2, \ldots, n-1 \\ &\hat{x}^{(1)}(1) = x^{(0)}(1). \end{aligned}$$

Inverse Accumulated Generating Operation (IAGO) is

$$\hat{x}^{(0)}(t+1) = \hat{x}^{(1)}(t+1) - \hat{x}^{(1)}(t), t = 1, 2, \ldots, n = 1.$$

3.4. Lotka–Volterra model

The Lotka–Volterra model has been developed to study the interaction between two competitors (Lee et al., 2005). It can be expressed in two differential equations as follows:

$$\frac{dx}{dt} = a_1 x(t) - b_1 x(t)^2 - c_1 x(t) y(t) \tag{10}$$

$$\frac{dy}{dt} = a_2 y(t) - b_2 y(t)^2 - c_2 y(t) x(t) \tag{11}$$

where x and y are population of two competing species at time t, a_i is the logistic parameter for the species i when it is living alone, b_i is the limitation parameter of the niche capacity related to the niche size for the species i, and c_i is the interaction parameter with the other species.

Table 2
Prediction capability levels of MAPE.
Source: Lewis (1982).

MAPE (%)	Prediction capability
<10	Highly accurate
10–20	Good
20–50	Reasonable
>50	Inaccurate

To use discrete time data, it is necessary to convert the Lotka–Volterra equation, which is a continuous time model, into discrete time version. Leslie (1957) proved that above differential equations are transformed to difference equations as follows:

$$x(t+1) = \frac{\alpha_1 x(t)}{1+\beta_1 x(t)+\gamma_1 y(t)}, t = 1, 2, \ldots, n-1 \tag{12}$$

$$y(t+1) = \frac{\alpha_2 y(t)}{1+\beta_2 y(t)+\gamma_2 x(t)}, t = 1, 2, \ldots\infty, n-1 \tag{13}$$

where α_i and β_i are the logistics parameters for the species i when it is living alone (Leslie, 1957). The positive constant γ_i is the magnitude of the effect which each species has on the rate of increase of the other. Leslie derived the relation between coefficients of differential equations and those of difference equation can be described as follows:

$$a_i = \ln \alpha_i \tag{14}$$

$$b_i = \frac{\beta_i a_i}{\alpha_i - 1} = \frac{\beta_i \ln \alpha_i}{\alpha_i - 1} \tag{15}$$

$$c_i = \gamma_i \frac{b_i}{\beta_i} = \frac{\gamma_i}{\beta_i} \frac{\beta_i \ln \alpha_i}{\alpha_i - 1} = \frac{\gamma_i \ln \alpha_i}{\alpha_i - 1}. \tag{16}$$

3.5. PE and MAPE

In order to evaluate the fitting and forecasting performance of the proposed model, this study used the percentage error (PE) and the mean absolute percentage error (MAPE) to compare the actual value with predicted value. PE and MAPE can be calculated as follows:

$$\mathrm{PE} = \frac{A_t - P_t}{A_t} \times 100\%$$
$$\mathrm{MAPE} = \frac{1}{n} \times \sum_{t=1}^{n} \left| \frac{At - Pt}{At} \right| \times 100\%$$

where A_t is actual value and P_t is predicted value.

Prediction capability levels of MAPE are shown in Table 2.

4. Empirical analyses and results

4.1. Forecasting mobile cellular broadband penetration in Taiwan

International Telecommunication Union (ITU) collects the numbers of subscriptions of different telecommunication services and economic information from up to 222 different countries and areas for many years. The services include the subscriptions of fixed line, fixed Internet, fixed broadband, and mobile cellular. The penetrations of these services are calculated per 100 inhabitants. Meanwhile the data are also categorized into developed, developing countries based on the classifications of UN M49. Therefore, the data collected by ITU provides us a good source to study and forecast the trend.

The introduction of 3G network is a critical point of rapid growth of mobile cellular. We can observe the rapid growth from year 2005. Mobile cellular is equivalent to mobile cellular broadband. The data about the penetration of mobile cellular broadband in Taiwan is presented in Table 3 (ITU, 2012).

In-sample data from 2005 to 2010 was used for model fitting and out-of-sample data 2011 was reserved for validation.

We estimate the data by using the proposed model. Initially,

$$X^{(0)} = \left\{ x^{(0)}(1), x^{(0)}(2), \ldots\ldots, x^{(0)}(6) \right\}$$
$$= \{97.55, 101.72, 105.73, 110.16, 116.44, 119.91\}.$$

The predicted values can be obtained by following the steps of the proposed GM(1,1) model as follows:

$$\widehat{X}^{(0)} = \{97.55, 101.56, 105.97, 110.57, 115.38, 120.38\}.$$

Table 3 shows the actual value, predicted value, PE, and MAPE of mobile cellular broadband penetration in Taiwan. Fitting MAPE of the proposed GM(1,1) predicted data is 0.3431%. The maximum error is 0.9103%. According to Table 2, the model fitting performance is highly accurate.

We use the penetration rates of year 2011 as out-of-sample data to evaluate forecasting performance. The forecasted value is 125.61 and forecasting MAPE is 1.2412%. Forecasting performance is also highly accurate. Moreover, we forecast the penetration rate for year 2012 and have 131.07.

4.1.1. Comparing GM(1,1) with logistic model

In order to examine the comparative advantage of the proposed model, we compare the proposed GM(1,1) model with the traditional logistic model and show the numerical results in Table 3.

When we estimate the logistic model, we have to set an upper bound for the model. Different upper bound will get different result. The forecasting result is not stable. This is a problem of logistic model. In this case, we use 2005–2010 dataset as sample data with upper bounds of 120 and 130 respectively to fit the logistic curve and then predict the penetration and calculate MAPE.

Table 3
Actual value, predicted value, PE, and MAPE of mobile cellular broadband penetration in Taiwan (Unit: subscriptions per 100 inhabitants).

Year	Actual value	GM(1,1)	PE(%)	Logistic-ub120	PE(%)	Logistic-ub130	PE(%)
2005	97.55	97.55	0.0000	77.31	20.7483	95.14	2.4705
2006	101.72	101.56	0.1573	99.43	2.2513	101.76	−0.0393
2007	105.73	105.97	−0.2270	111.36	−5.3249	107.42	−1.5984
2008	110.16	110.57	−0.3722	116.61	−5.8551	112.15	−1.8065
2009	116.44	115.38	0.9103	118.71	−1.9495	116.01	0.3693
2010	119.91	120.38	−0.3920	119.51	0.3336	119.12	0.6588
MAPE			0.3431		6.0771		1.1571
2011	124.07	125.61	−1.2412	119.82	3.4255	121.59	1.9989
MAPE			1.2412		3.4255		1.9989
2012		131.07		119.93		123.53	

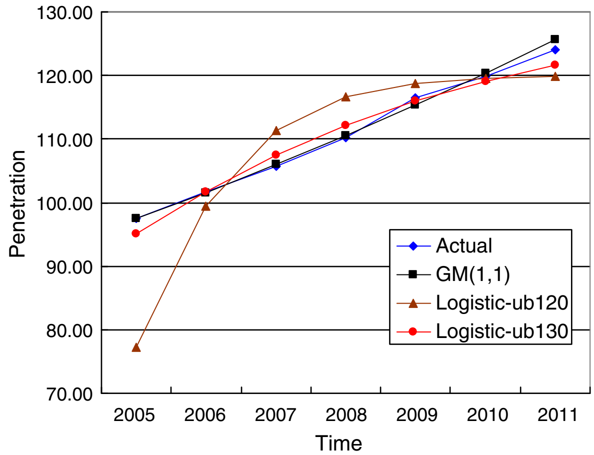

Fig. 1. Penetration curves for actual, GM(1,1) predicted, Logistic-ub120 predicted, and Logistic-ub130 predicted values of mobile cellular broadband penetration.

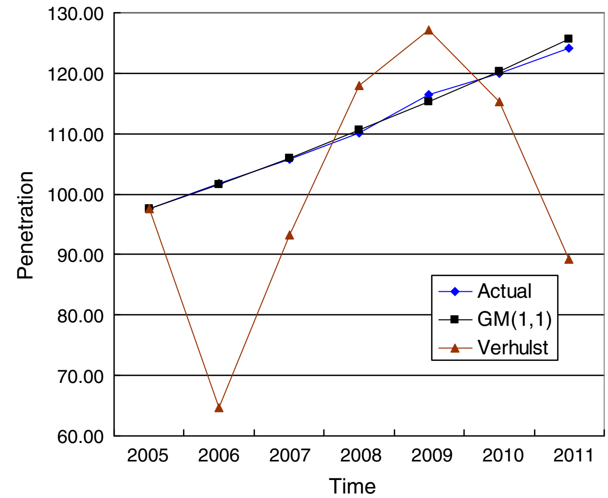

Fig. 2. Comparison among the actual, GM(1,1) predicted, and grey Verhulst predicted values of mobile cellular broadband penetration.

The results show that the fitting MAPE of logistic model with upper bound 120 is 6.0771 and the fitting MAPE of logistic model with upper bound 130 is 1.1571. Both numbers are all larger than the fitting MAPE of GM(1,1) model. Forecasting MAPEs of logistic models are all larger. We can conclude that the proposed GM(1,1) is better than logistic models in this case.

Fig. 1 shows comparison among the actual, GM(1,1) predicted, Logistic-ub120 predicted, and Logistic-ub130 predicted values of mobile cellular penetration. GM(1,1) curve can fit and forecast very well.

4.1.2. Comparing GM(1,1) with grey Verhulst model

We use 2005–2010 dataset as sample data. Actual values and predicted values of GM(1,1) and grey Verhulst model are presented in Table 4. Both fitting MAPE and forecasting MAPE of proposed GM(1,1) model are far better than those of grey Verhulst model. We observe that grey Verhulst model is not suitable for the short product lifecycle.

The curve of the grey Verhulst model is illustrated in Fig. 2. Compared with actual and GM(1,1) curves, it fluctuates a lot.

4.2. Forecasting fixed broadband penetration in Taiwan

In the same time period, we want to observe the behavior of the fixed broadband penetration in Taiwan. We use 2005–2010 dataset of the fixed broadband penetration as sample data. Following the similar calculation steps, we have the numerical results as shown in Tables 5 and 6.

In this case, the fitting MAPE of GM(1,1) is 0.8266% and forecasting MAPE of GM(1,1) is 0.7170%. Both numbers are less 1%. The forecasting performance of GM(1,1) is highly accurate and far better than that of logistic model. We also observe the data in Table 6 and discover that grey Verhulst model has large fitting and forecasting MAPEs and it is not suitable for forecasting in this kind of case.

Figs. 3 and 4 illustrate that the GM(1,1) curve is compared with logistic model and grey Verhulst model for fixed broadband respectively and show that GM(1,1) can fit and forecast very accurately with very limited historical data.

Table 4
Comparing GM(1,1) with grey Verhulst model for forecasting mobile cellular broadband (Unit: subscriptions per 100 inhabitants).

Year	Actual value	GM(1,1)	PE(%)	Verhulst model	PE(%)
2005	97.55	97.55	0.0000	97.55	0.0000
2006	101.72	101.56	0.1573	64.75	36.3449
2007	105.73	105.97	−0.2270	93.26	11.7942
2008	110.16	110.57	−0.3722	118.02	−7.1351
2009	116.44	115.38	0.9103	127.13	−9.1807
2010	119.91	120.38	−0.3920	115.24	3.8946
MAPE			0.3431		11.3916
2011	124.07	125.61	−1.2412	89.22	28.0890
MAPE			1.2412		28.0890
2012		131.07		61.02	

4.3. Analyzing competitive relationship between mobile cellular broadband and fixed broadband

4.3.1. Estimating Lotka–Volterra equations

First, we have to estimate the parameters of Eqs. (12) and (13). The yearly penetration of mobile cellular broadband was designated as $x(t)$, and that of fixed broadband as $y(t)$.

We use the nonlinear least square method to estimate the parameters. The nonlinear least square method utilizes an iterative algorithm to achieve the convergence. The estimation results are presented in Table 7.

4.3.2. Analyzing competitive relationship

According to the signs of interaction parameters, c_1 and c_2, in Eqs. (10) and (11), types of competitive roles can be classified as presented in Table 8 (Modis, 1999).

To determine the sign of c_i, we have to use Eq. (16). The sign of c_i must be the same as the sign of γ_i since $\frac{\ln \alpha_i}{\alpha_i - 1}$ is always positive if $\alpha_i > 0$ and $\alpha_i \neq 1$ in Eq. (16). Thus, we can use the sign of γ_i to determine the type of competitive relationship between mobile cellular broadband and fixed broadband.

In Table 7, the parameter for the interaction in mobile cellular broadband, γ_1, is negative, and that in fixed broadband, γ_2, is also negative. However, the t-statistic of γ_2 indicates that the effect of mobile cellular broadband diffusion on the changes in the penetration of fixed broadband is not statistically significant, with a significance level of 10%. Therefore, it is reasonable to assume that γ_2 is 0. In this case, we may conclude that the competitive relationship between mobile cellular broadband and fixed broadband is commensalism. This implies that the penetration of fixed broadband is not influenced by the fluctuating penetration of mobile cellular broadband, while the penetration of mobile cellular broadband increases as the penetration of fixed broadband increases.

Table 5
Actual value, predicted value, PE, and MAPE of fixed broadband penetration in Taiwan (Unit: subscriptions per 100 inhabitants).

Year	Actual value	GM(1,1)	PE(%)	Logistic-ub23	PE(%)	Logistic-ub24	PE(%)
2005	19.10	19.10	0.0000	17.5	8.3770	18.67	2.2513
2006	19.71	19.84	−0.6596	19.67	0.2029	19.82	−0.5581
2007	20.64	20.53	0.5329	21.08	−2.1318	20.77	−0.6298
2008	21.54	21.24	1.3928	21.92	−1.7642	21.53	0.0464
2009	21.59	21.98	−1.8064	22.41	−3.7981	22.13	−2.5012
2010	22.88	22.75	0.5682	22.68	0.8741	22.59	1.2675
MAPE			0.8266		2.8580		1.2091
2011	23.71	23.54	0.7170	22.82	3.7537	22.94	3.2476
MAPE			0.7170		3.7537		3.2476
2012		24.36		22.91		23.21	

5. Managerial implications

According to ITU key statistical highlights released in June 2012, the percentage of individuals using the Internet continues to grow worldwide and 2.3 billion people were online by the end of 2011. In developing countries, the number of Internet users doubled between 2007 and 2011 and the total international Internet bandwidth increased seven-fold over the last five years reaching 76,000 Gbit/s by the end of 2011.

Mobile broadband has become the single most vibrant ICT service reaching a 40% annual subscription growth in 2011. By the end of 2011, there were more than 1 billion mobile broadband subscriptions worldwide. Although developing countries are catching up in terms of 3G coverage, huge disparities remain between mobile broadband penetration in the developing and the developed world (ITU, 2012). Taiwan started the rapid growth of mobile cellular broadband from year 2005 by introducing 3G operations and achieved higher penetration than the average of the developing countries, the world, and even the developed countries.

Taiwanese government and enterprises were urged to know the trend of both mobile cellular broadband and fixed broadband. The accurate forecast will help government make correct policies and regulations. As to enterprises, the correct forecast will help them estimate the size of market and the timing of development for the related broadband products or services and put in resources in advance.

Conventionally, the logistic models with S-shaped curves were developed and successfully predicted the diffusion of long product lifecycle, but there are very few forecasting models which were developed for forecasting new products with short lifecycle. As to the products which were just introduced to the market, it is very difficult to identify if they follow an S-shaped curve with their limited historical data. Broadband with only a few years' diffusion is just the case. In order to precisely and simply forecast the diffusion for such products, we propose GM(1,1) to solve the problem and discover that it can forecast very accurately with a few data points. The mechanism is GM(1,1) predicts the future values based on the inputs obtained in the previous and current data and does not require that the dataset fits any conventional curve. The numerical data show that the forecasting performance of GM(1,1) is highly accurate and far better than that of logistic model.

As to fixed broadband, there were 590 million fixed broadband subscriptions worldwide by the end of 2011. Fixed broadband growth in developed countries is slowing (5% increase in 2011) (ITU, 2012). The average annual fixed broadband growth rate is about 4% from 2005 to 2011 in Taiwan. We are curious about whether mobile cellular broadband would gradually replace fixed broadband in Taiwan

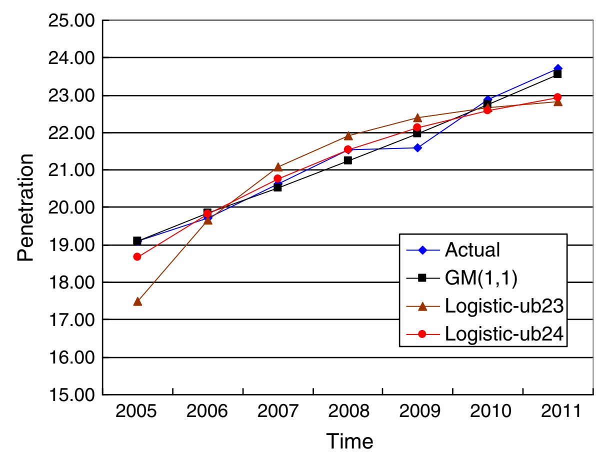

Fig. 3. Comparison among the actual, GM(1,1) predicted, and logistic predicted values of fixed broadband penetration.

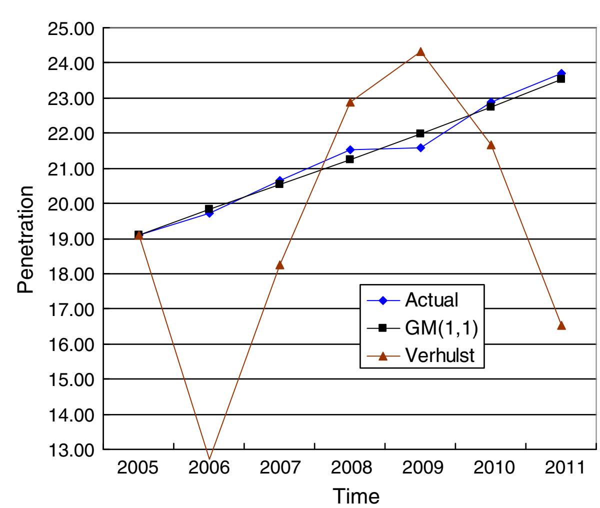

Fig. 4. Fixed broadband penetration curves for comparing GM(1,1) with grey Verhulst model.

Table 6
Comparing GM(1,1) with grey Verhulst model for forecasting fixed broadband (Unit: subscriptions per 100 inhabitants).

Year	Actual value	GM(1,1)	PE(%)	Verhulst model	PE(%)
2005	19.10	19.10	0.0000	19.1	0.0000
2006	19.71	19.84	−0.6596	12.72	35.4642
2007	20.64	20.53	0.5329	18.24	11.6279
2008	21.54	21.24	1.3928	22.87	−6.1746
2009	21.59	21.98	−1.8064	24.31	−12.5984
2010	22.88	22.75	0.5682	21.68	5.2448
MAPE			0.8266		11.8516
2011	23.71	23.54	0.7170	16.52	30.3248
MAPE			0.7170		30.3248
2012		24.36		11.14	

Table 7
Estimation results of the Lotka–Volterra model.

Mobile cellular broadband			Fixed broadband		
Parameter	Estimate	t-Statistic	Parameter	Estimate	t-Statistic
α_1	0.939439	20.55648***	α_2	1.375556	4.224186*
β_1	0.00352	4.049*	β_2	0.069567	1.545055
γ_1	−0.023045	−4.132559*	γ_2	−0.01035	−1.671601
R^2	0.997306		R^2	0.957398	
Adjusted R^2	0.994612		Adjusted R^2	0.914797	

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 8
Types of competitive relationship according to the signs of c_1 and c_2.

c_1	c_2	Type	Explanation
+	+	Pure competition	Occurs when both species suffer from each other's existence
+	−	Predator–prey	Occurs when one of them serves as direct food to the other
−	−	Mutualism	Occurs in case of symbiosis or a win–win situation
−	0	Commensalism	Occurs in a parasitic type of relationship in which one benefits from the existence of the other, who nevertheless remains unaffected
+	0	Amensalism	One suffers from the existence of the other, who is impervious to what is happening
0	0	Neutralism	Occurs if there is no interaction whatsoever

market. We apply Lotka–Volterra model to analyze the competitive relationship between mobile cellular broadband and fixed broadband. The empirical data show that the relationship is commensalism rather than predator–prey. Mobile cellular broadband benefits from the existence of fixed broadband, while fixed broadband is not affected by mobile cellular broadband. It can be interpreted that the 3G mobile cellular broadband with limited bandwidth could not fully replace fixed broadband with wider bandwidth because 4G (in terms of WiMAX) is not developed successfully in Taiwan. As to fixed broadband helping mobile cellular broadband, we can explain that fixed broadband with wider bandwidth promotes cloud computing applications and which will stimulate the diffusion of mobile cellular broadband.

Taiwan's telecom regulator will accept bids for 4G (in terms of LTE) spectrum late in 2013, with up to eight licences to be granted by December of that year. 4G, which operates five times faster than the current 3G network, allows users to download large email attachments quickly, watch live TV without buffering, make high-quality video calls, and play live games on the go (Dawn.com, 2012). 4G will bring Taiwan's mobile broadband to a new stage.

6. Conclusion

This research uses GM(1,1) model to predict penetration rates of mobile cellular broadband and fixed broadband in Taiwan with a few data points. Empirical results show that the forecasting performance of GM(1,1) is highly accurate with very small MAPEs. Compared with logistic model and grey Verhulst model, this proposed model is far better. We can conclude that the proposed GM(1,1) is very suitable for forecasting new product diffusion with limited historical data.

In addition, we apply Lotka–Volterra model to analyze the competitive relationship between mobile cellular broadband and fixed broadband. The empirical data show that the relationship is commensalism rather than predator–prey. In managerial implications section, we try to explain that the 3G mobile cellular broadband could not gradually replace fixed broadband because it has limited bandwidth and 4G is not developed well in Taiwan. Whereas, the fixed broadband with wider bandwidth promotes cloud computing applications and which will help the diffusion of mobile cellular broadband.

Further research can be conducted in the new stage of 4G to examine the relationship between mobile broadband and fixed broadband. Considering the emerging new products, future study can extend to analyze the relationship between mobile broadband and smart phone or the relationship between mobile broadband and pad computer. The proposed GM(1,1) also can contribute to forecast the diffusion of other new products.

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