Second-order direct CP asymmetry in $B_{(s)} \to X\ell\nu$

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ABSTRACT

A direct CP asymmetry in inclusive semileptonic $B_{(s)}$ decays vanishes by CPT to lowest order in weak interactions. Calculating the asymmetry at second-order-weak interactions in the Cabibbo–Kobayashi–Maskawa framework we find $A_{sl} = (-3.2 \pm 0.9) \times 10^{-9}$. A maximal asymmetry which is two orders of magnitude larger is estimated in a left–right-symmetric model. This quite generic upper bound implies negligible effects on wrong-sign lepton asymmetries in B^0 and B_s decays.

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1. Introduction

The D0 Collaboration working at the Fermilab Tevatron has reported recently a charge asymmetry in like-sign dimuon events produced in $\bar{p}p$ collisions. The measured asymmetry [1,2],

$$A_{sl}^b \equiv \frac{N^{++} - N^{--}}{N^{++} + N^{--}}$$
$$= \left[-0.957 \pm 0.251(\text{stat}) \pm 0.146(\text{syst})\right]\%, \qquad (1)$$

was interpreted as due to CP violation in B^0–$\bar{B}^0$ or B_s–$\bar{B}_s$ mixing,

$$A_{sl}^b = (0.506 \pm 0.043)A_{sl}^d + (0.494 \pm 0.043)A_{sl}^s. \qquad (2)$$

(An asymmetry consistent with zero was measured a few years ago by the CDF Collaboration using fewer same-sign dimuon events produced in $\bar{p}p$ collisions with a lower integrated luminosity [3].) The experimental result (1) differs by 3.2 standard deviations from much smaller asymmetries predicted within the Cabibbo–Kobayashi–Maskawa (CKM) framework [4],

$$A_{sl}^d = \left(-4.8^{+1.0}_{-1.2}\right) \times 10^{-4}, \qquad A_{sl}^s = (2.06 \pm 0.57) \times 10^{-5}. \qquad (3)$$

Unambiguous evidence for physics beyond the Standard Model requires (a) showing that the measured asymmetry is, indeed, due to B^0 or B_s semileptonic decays and not due to background processes [5], and (b) confirming an anomalously large negative asymmetry with a somewhat higher statistical significance than the current one.

In view of the tiny CKM predictions (3) for A_{sl}^k $(k = d, s)$ from second-order $|\Delta B| = 2$ weak transitions, attention has been often drawn to potentially larger corrections to the two asymmetries from new $|\Delta B| = 2$ flavor physics at a TeV or higher energy scale [6]. In this Letter we will study $|\Delta B| = 1$ contributions to A_{sl}^k from CP violation in inclusive semileptonic decays, which have been systematically neglected in all earlier studies.

In Section 2 we review briefly the usual treatment of the asymmetry A_{sl}^k involving CP violation in B_k^0–$\bar{B}_k^0$ mixing, introducing a new contribution from direct CP violation. Section 3 discusses an argument based on CPT for the vanishing of the new contribution at lowest order in weak interactions. We perform a second-order calculation of the inclusive semileptonic asymmetry within the CKM framework. Maximal values of the asymmetry in a left–right extension of the Standard Model are studied in Section 4. In Section 5 we estimate for completeness second-order amplitudes for neutral B mesons decaying directly into wrong-sign leptons while Section 6 concludes.

2. Including a direct asymmetry in A_{sl}^k

One starts by defining neutral B mass eigenstates B_L^k and B_H^k, with mass and width differences Δm_k and $\Delta \Gamma_k$, in terms of flavor states B_k^0 and $\bar{B}_k^0$,

$$|B_L^k\rangle = p_k|B_k^0\rangle + q_k|\bar{B}_k^0\rangle,$$
$$|B_H^k\rangle = p_k|B_k^0\rangle - q_k|\bar{B}_k^0\rangle. \qquad (4)$$

Time evolution of flavor states [7,8],

$$|B_k^0(t)\rangle = g_+^k(t)|B_k^0\rangle - (q_k/p_k)g_-^k(t)|\bar{B}_k^0\rangle,$$

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$$\big|\bar{B}_k^0(t)\big\rangle = g_+^k(t)\big|\bar{B}_k^0\big\rangle - (p_k/q_k)g_-^k(t)\big|B_k^0\big\rangle, \qquad (5)$$

implies time-dependent decay rates for inclusive semileptonic decays for wrong-sign leptons $B_k^0(t) \to X\ell^-\bar{\nu}_\ell$ ($\ell = e, \mu$) and their charge conjugates,

$$d\Gamma\big[B_k^0(t) \to X\ell^-\bar{\nu}_\ell\big]/dt = \left|\frac{q_k}{p_k}\bar{A}_\ell^k\right|^2 \big|g_-^k(t)\big|^2,$$

$$d\Gamma\big[\bar{B}_k^0(t) \to X\ell^+\nu_\ell\big]/dt = \left|\frac{p_k}{q_k}A_{\bar\ell}^k\right|^2 \big|g_-^k(t)\big|^2, \qquad (6)$$

where

$$\bar{A}_\ell^k \equiv A\big(\bar{B}_k^0 \to X\ell^-\bar{\nu}_\ell\big), \qquad A_{\bar\ell}^k \equiv A\big(B_k^0 \to X\ell^+\nu_\ell\big), \qquad (7)$$

$$\big|g_-^k\big|^2 = \frac{1}{2}e^{-\Gamma_k t}\big[\cosh(\Delta\Gamma_k t/2) - \cos(\Delta m_k t)\big]. \qquad (8)$$

By definition A_{sl}^k is the time-dependent asymmetry of wrong-sign leptons due to mixing,

$$A_{sl}^k(t) \equiv \frac{d\Gamma[\bar{B}_k^0(t) \to X\ell^+\nu_\ell]/dt - d\Gamma[B_k^0(t) \to X\ell^-\bar{\nu}_\ell]/dt}{d\Gamma[\bar{B}_k^0(t) \to X\ell^+\nu_\ell]/dt + d\Gamma[B_k^0(t) \to X\ell^-\bar{\nu}_\ell]/dt}. \qquad (9)$$

Usually, one neglects a direct CP asymmetry in $B_k^0 \to X\ell^+\nu_\ell$. Assuming $|\bar{A}_\ell^k| = |A_{\bar\ell}^k|$ one finds [7,8],

$$A_{sl}^k(\text{mixing}) = \frac{1 - |q_k/p_k|^4}{1 + |q_k/p_k|^4} \approx \text{Im}\left(\frac{\Gamma_{12}^k}{M_{12}^k}\right). \qquad (10)$$

That is, the asymmetry caused by CP violation in B_k^0–$\bar{B}_k^0$ mixing is given by $\text{Im}(\Gamma_{12}^k/M_{12}^k)$ where M_{12}^k and Γ_{12}^k are off-diagonal elements of Hermitian matrices representing $B_k^0 \leftrightarrow \bar{B}_k^0$ transitions via off-shell (dispersive) and on-shell (absorptive) intermediate states, respectively. The CKM predictions (3) for $A_{sl}^k(\text{mixing})$ are based on calculations of this imaginary part for B^0 and B_s [4].

We now define a direct CP asymmetry in inclusive semileptonic decays,

$$A_{sl}^k(\text{direct}) \equiv \frac{|\bar{A}_\ell^k|^2 - |A_{\bar\ell}^k|^2}{|\bar{A}_\ell^k|^2 + |A_{\bar\ell}^k|^2}. \qquad (11)$$

Eqs. (6) and (9) imply an expression for $A_{sl}^k(t)$ which includes both the asymmetry in mixing and the direct asymmetry. Neglecting terms quadratic in the $A_{sl}^k(\text{mixing})$ and $A_{sl}^k(\text{direct})$, one has

$$A_{sl}^k(t) = \frac{A_{sl}^k(\text{mixing}) - A_{sl}^k(\text{direct})}{1 - A_{sl}^k(\text{mixing})A_{sl}^k(\text{direct})}$$
$$\approx A_{sl}^k(\text{mixing}) - A_{sl}^k(\text{direct}). \qquad (12)$$

We note that this asymmetry of time-dependent decay rates is actually time-independent as in the special case (10) of CP violation in mixing alone. In the following two sections we will study $A_{sl}^k(\text{direct})$.

3. Direct asymmetry in the CKM framework

The inclusive semileptonic direct asymmetries in B^0 and B_s decays are equal to each other to a good approximation as $\Gamma(B_s \to X\ell^+\nu_\ell) = \Gamma(B^0 \to \ell^+X\nu_\ell) + \mathcal{O}(m_s\Lambda_{\text{QCD}}/m_b^2)$. (See also calculation below.) Furthermore, in the isospin symmetry limit the asymmetry in B^0 decays is equal to that measured directly in self-tagged $B^+ \to X\ell^+\nu_\ell$ by comparing the rate for this inclusive process with
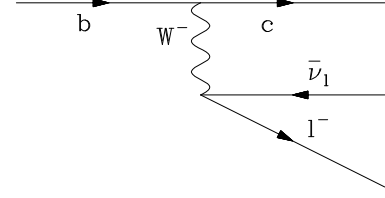

Fig. 1. Tree diagram for $b \to c\ell^-\bar{\nu}_\ell$ representing $\bar{B} \to X_{(C=1)}\ell^-\bar{\nu}_\ell$.

that of its charge-conjugate. For this reason we omit the superscript k in $A_{sl}^k(\text{direct})$ by defining a generalized direct semileptonic asymmetry for non-strange (charged or neutral) or strange B mesons,

$$A_{sl} \equiv \frac{\Gamma(\bar{B} \to X\ell^-\bar{\nu}_\ell) - \Gamma(B \to X\ell^+\nu_\ell)}{\Gamma(\bar{B} \to X\ell^-\bar{\nu}_\ell) + \Gamma(B \to X\ell^+\nu_\ell)}. \qquad (13)$$

It has often been stated that A_{sl} vanishes because of CPT invariance [9–11]. CPT implies equal total decay widths for a particle and its antiparticle. A generalization of this theorem applies to partial decay rates for a set of final states, connected among themselves by strong and electromagnetic final state interactions but not connected by such interactions to other states. The inclusive decays $B \to X_{(C=-1)}\ell^+\nu_\ell$ and $B \to X_{(C=0)}\ell^+\nu_\ell$ are two special cases to which this generalization applies.

A violation of the theorem of equal partial rates for $B \to X\ell^+\nu_\ell$ and $\bar{B} \to X\ell^-\bar{\nu}_\ell$ is possible if one considers weak interactions which connect the final states $X\ell^+\nu_\ell$ with intermediate hadronic states. That is, a small nonvanishing asymmetry A_{sl} may be obtained by considering an interference between the dominant tree amplitude for $B \to X\ell^+\nu_\ell$ and an amplitude which is second order in weak interactions. A similar interference has been shown to imply a tiny CP asymmetry in inclusive semileptonic rare top quark decays, $t \to d\ell^+\nu_\ell$ [12,13].

A very crude upper bound on this asymmetry is

$$|A_{sl}| < \frac{1}{4\pi}\frac{G_F}{\sqrt{2}}(m_b - m_c)^2 \sim \text{few} \times 10^{-6}. \qquad (14)$$

This estimate includes a suppression by a loop factor $1/4\pi$ and a phase space factor $(m_b - m_c)^2$. Further suppression of the asymmetry may be due to a small weak phase difference between the two interfering amplitudes and due to a possible extra dynamical suppression of the second-order amplitude. We will show below that, indeed, such suppression factors exist in the CKM framework. The above upper limit is one order of magnitude smaller than the estimate (3) for a CP asymmetry in B_s–$\bar{B}_s$ mixing, and two orders of magnitude below the asymmetry in B^0–$\bar{B}^0$ mixing.

We will now calculate the CP asymmetry in CKM favored $B \to X_{(C=-1)}\ell^+\nu_\ell$ decays dominated by a tree amplitude proportional to V_{cb}^* describing a quark transition $\bar{b} \to \bar{c}\ell^+\nu_\ell$. In order to produce an asymmetry, the second amplitude, leading to the same final states, must involve a CKM factor with a different weak phase. A second-order amplitude fulfilling these two requirements consists of a product of a penguin amplitude for $\bar{b} \to \bar{c}c\bar{s}$ involving $V_{tb}^*V_{ts}$ (see discussion below) and a tree amplitude for $c\bar{s} \to \ell^+\nu_\ell$ involving V_{cs}^*. Two diagrams, describing the tree amplitude for $b \to c\ell^-\bar{\nu}_\ell$ and the second-order amplitude for this transition, are shown in Figs. 1 and 2, respectively. A relative CP-conserving phase of 90° between the two amplitudes follows by taking the absorptive (i.e., imaginary) part of the second-order amplitude. The absorptive part is described by a discontinuity cut crossing the $\bar{c}s$ lines in the second-order diagram, which amounts to summing over corresponding on-shell intermediate states.

In order to calculate the asymmetry we write down expressions for an effective Hamiltonian associated with each of the three four-
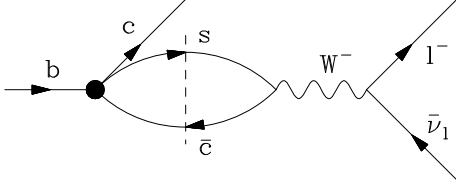
Fig. 2. Second-order diagram for $b \to c\ell^-\bar{\nu}_\ell$ representing $\bar{B} \to X_{(C=1)}\ell^-\bar{\nu}_\ell$.

fermion vertices appearing in the two diagrams in Figs. 1 and 2. The tree diagram is obtained from

$$H_{\text{eff}}^{b \to c\ell^-\bar{\nu}_\ell} = \frac{G_F}{\sqrt{2}} V_{cb}[\bar{c}\gamma^\mu(1-\gamma_5)b][\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell]. \quad (15)$$

The Hamiltonian related to the first vertex in the second-order diagram is [14],

$$H_{\text{eff}}^{b \to c\bar{c}s(\text{pen})} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^*(c_3 O_3 + c_4 O_4 + c_5 O_5 + c_6 O_6), \quad (16)$$

$$c_3(m_b) = 0.012,$$

$$O_3 = [\bar{s}_\alpha \gamma^\mu(1-\gamma_5)b_\alpha][\bar{c}_\beta \gamma_\mu(1-\gamma_5)c_\beta],$$

$$c_4(m_b) = -0.033,$$

$$O_4 = [\bar{s}_\alpha \gamma^\mu(1-\gamma_5)b_\beta][\bar{c}_\beta \gamma_\mu(1-\gamma_5)c_\alpha],$$

$$c_5(m_b) = 0.0096,$$

$$O_5 = [\bar{s}_\alpha \gamma^\mu(1-\gamma_5)b_\alpha][\bar{c}_\beta \gamma_\mu(1+\gamma_5)c_\beta],$$

$$c_6(m_b) = -0.040,$$

$$O_6 = [\bar{s}_\alpha \gamma^\mu(1-\gamma_5)b_\beta][\bar{c}_\beta \gamma_\mu(1+\gamma_5)c_\alpha]. \quad (17)$$

Here α, β are color indices. The Wilson coefficients c_i ($i = 3\text{–}6$) have been calculated in the next-to-leading logarithmic approximation (NLL). The second vertex in this diagram is described by

$$H_{\text{eff}}^{\bar{c}s \to \ell^-\bar{\nu}_\ell} = \frac{G_F}{\sqrt{2}} V_{cs}[\bar{c}\gamma^\mu(1-\gamma_5)s][\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell]. \quad (18)$$

We are interested in the imaginary part of the amplitude involving the $\bar{c}s$ loop illustrated in Fig. 2. Contributions of the $(V-A)(V+A)$ operators O_5 and O_6 are all proportional to m_s and some are also proportional to m_ℓ. [See Eqs. (20) and (21).] These contributions will be neglected. We consider the dominant terms from O_3 and O_4. After Fierz rearrangement of these terms the second-order loop amplitude is given by

$$M_1 = \frac{G_F^2}{2}(c_3 + N_C c_4) V_{tb} V_{ts}^* V_{cs}$$
$$\times [\bar{c}\gamma_\mu(1-\gamma_5)b] T^{\mu\nu}[\bar{\ell}\gamma_\nu(1-\gamma_5)\nu_\ell], \quad (19)$$

where

$$T^{\mu\nu} \equiv -\int \frac{d^4k}{(2\pi)^4} \text{Tr}\left[\gamma^\mu(1-\gamma_5)\frac{i(\slashed{k}+\slashed{q}+m_s)}{(k+q)^2-m_s^2}\right.$$
$$\left. \times \gamma^\nu(1-\gamma_5)\frac{i(\slashed{k}+m_c)}{k^2-m_c^2}\right]. \quad (20)$$

The minus sign takes account of the closed fermion loop. The most general form of $T^{\mu\nu}$ is

$$T^{\mu\nu} = T_1 g^{\mu\nu} + T_2 q^\mu q^\nu. \quad (21)$$

We shall be interested in the T_1 part, neglecting the T_2 contribution to M_1 which is proportional to m_ℓ. Contracting $T^{\mu\nu}$ with $g_{\mu\nu}$ and $q_\mu q_\nu$ we obtain

$$g_{\mu\nu}T^{\mu\nu} = 4T_1 + q^2 T_2, \qquad q_\mu q_\nu T^{\mu\nu} = q^2 T_1 + (q^2)^2 T_2, \quad (22)$$

so

$$T_1 = \frac{1}{3}\left(g_{\mu\nu}T^{\mu\nu} - \frac{q_\mu q_\nu}{q^2}T^{\mu\nu}\right). \quad (23)$$

Performing the appropriate traces, one finds

$$T_1 = \frac{1}{6}\int \frac{d^4k}{\pi^4}\left(-\frac{2(q\cdot k)^2}{q^2} - k^2 - 3k\cdot q\right)$$
$$\times [(k+q)^2 - m_s^2]^{-1}[k^2 - m_c^2]^{-1}. \quad (24)$$

To take twice the absorptive part [15], we put the internal propagators on the mass shell, replacing

$$[(k+q)^2 - m_s^2]^{-1} \quad \Rightarrow \quad -2\pi i\delta[(k+q)^2 - m_s^2],$$
$$[k^2 - m_c^2]^{-1} \quad \Rightarrow \quad -2\pi i\delta[k^2 - m_c^2]. \quad (25)$$

On-shell, we have $k^2 = m_c^2$ and $(k+q)^2 = m_s^2$, implying $2k \cdot q = m_s^2 - m_c^2 - q^2$. Simplifying by neglecting m_s, we find the expression in the large brackets in Eq. (24) reduces to $(1/2q^2)(2q^4 - m_c^2 q^2 - m_c^4)$. The delta functions reduce the loop integral to an integral over two-body phase space:

$$\int d^4k\, \delta[(k+q)^2 - m_s^2]\delta[k^2 - m_c^2]$$
$$= \int d^4k \int d^4p\, \delta^4(q+k-p)\delta(p^2 - m_s^2)\delta(k^2 - m_c^2)$$
$$= \int \frac{d^3k}{2E_k}\int \frac{d^3p}{2E_p}\delta^4(q+k-p) = (2\pi)^2 \int d^2(ps), \quad (26)$$

where the two-body phase space integral in the limit $m_s = 0$ is

$$\int d^2(ps) = \frac{q^2 - m_c^2}{8\pi q^2}.$$

We define an n-body phase space:

$$\int d^n(ps) \equiv (2\pi)^4\left[\prod_i^n \int \frac{d^3p_i}{2E_i(2\pi)^3}\right]\delta^4(P_{\text{final}} - P_{\text{initial}}). \quad (27)$$

Putting the pieces together, we find

$$T_{1(\text{abs})} = -\frac{q^2 - m_c^2}{12\pi q^2}\left(\frac{2q^4 - m_c^2 q^2 - m_c^4}{q^2}\right)$$
$$= -\frac{(q^2 - m_c^2)^2}{12\pi (q^2)^2}(2q^2 + m_c^2), \quad (28)$$

$$M_{1(\text{abs})} = \frac{G_F^2}{2}(c_3 + N_C c_4) V_{tb} V_{ts}^* V_{cs} T_{1(\text{abs})}(q^2)[\bar{c}\gamma^\mu(1-\gamma_5)b]$$
$$\times [\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell]. \quad (29)$$

Given the tree level amplitude M_0 in Eq. (15), the expression for the semileptonic asymmetry then becomes, performing the three-body phase space integrals,

$$A_{sl} = \frac{2\int d^3(ps)\,\text{Im}(M_0 M_{1(\text{abs})}^\dagger)}{\int d^3(ps)\,|M_0|^2}$$
$$= -(c_3 + N_C c_4)\frac{\text{Im}[V_{tb}^* V_{ts} V_{cb} V_{cs}^*]}{|V_{cb}|^2}\frac{G_F}{\sqrt{2}}\frac{m_b^2}{6\pi}R, \quad (30)$$

where

Table 1

Range of constituent-quark masses providing adequate descriptions on charmonium and bottomonium spectra [16].

m_b (GeV)	m_c (GeV)	$r_c = m_c^2/m_b^2$	R	$m_b^2 R$ (GeV^2)
4.5	1.082	0.058	0.296	5.99
4.75	1.359	0.082	0.207	4.67
5.0	1.626	0.106	0.136	3.39

$$R \equiv \frac{\int dz \, F(z) G(z)}{\int dz \, F(z)}, \tag{31}$$

$$F(z) \equiv w(z)(z w_1 + w_2 w_3),$$

$$G(z) \equiv (1 - r_c/z)^2 (2z + r_c). \tag{32}$$

Here $r_c \equiv m_c^2/m_b^2$, $z \equiv q^2/m_b^2$, $w_1 \equiv 1 + r_c - z$, $w_2 \equiv 1 - r_c + z$, $w_3 \equiv 1 - r_c - z$, and $w(z) \equiv (w_2^2 - 4z)^{1/2}$. The upper limit of integration in both integrals in the numerator and denominator of R is $z_{\max} = (1 - \sqrt{r_c})^2$. The lower limit of the integral in the numerator is $z_{\min} = r_c$ (in the limit that the strange quark mass may be neglected), while the lower limit in the denominator is zero in the limit of vanishing lepton mass. Thus the denominator obtains the well-known expression for the kinematic factor in $b \to c \ell \bar{\nu}_\ell$,

$$2 \int_0^{(1 - \sqrt{r_c})^2} dz \, F(z) = 1 - 8 r_c + 8 r_c^3 - r_c^4 + 12 r_c^2 \ln(1/r_c). \tag{33}$$

For m_c and m_b we use constituent masses, which we expect to simulate QCD effects to a certain degree. These masses were found in Ref. [16] to reproduce charmonium and bottomonium spectra for a rather wide range as long as $m_b - m_c$ lay within a narrower range. The masses considered there are summarized in Table 1, together with the corresponding values of r_c, R, and $R m_b^2$ (in which the dependence of R on m_b is partially compensated). The uncertainty on $R m_b^2$ is approximately 28%, which we carry in our final estimate of the asymmetry.

Using CKM fits [17],

$$\frac{\text{Im}(V_{tb}^\star V_{ts} V_{cb} V_{cs}^\star)}{|V_{cb}|^2} \approx \text{Arg}(V_{tb}^\star V_{ts} V_{cb} V_{cs}^\star) \equiv -\beta_s = -0.018, \tag{34}$$

we then find

$$A_{s\ell} = (-3.2 \pm 0.9) \times 10^{-9}. \tag{35}$$

4. Direct asymmetry beyond the Standard Model

A crude estimate for a maximum asymmetry based on dimensional arguments was given in Eq. (14). It applies to a generic new physics contribution to the asymmetry occurring at one-loop order. Here we wish to be more concrete by considering a specific model leading to CP-violation in $B \to X \ell^+ \nu_\ell$ at one loop, without involving suppression factors which occur in the CKM framework.

An example that falls into this category is a left–right-symmetric model [18], in which the interaction of two charged vector-bosons W_1 and W_2 is given by

$$\mathcal{L}_W^{LR} = \frac{g}{\sqrt{2}} \bar{u}_i \left(\cos \xi \, V_{ij}^L \gamma^\mu P_L - e^{i\omega} \sin \xi \, V_{ij}^R \gamma^\mu P_R \right) d_j W_{1\mu}$$
$$+ \frac{g}{\sqrt{2}} \bar{u}_i \left(e^{-i\omega} \sin \xi \, V_{ij}^L \gamma^\mu P_L + \cos \xi \, V_{ij}^R \gamma^\mu P_R \right) d_j W_{2\mu}$$
$$+ \text{H.c.} \tag{36}$$

Here $P_{L,R} \equiv (1 \pm \gamma_5)/2$ while $V^{L,R}$ are CKM-like matrices for left (right)-handed quark fields. The angle ξ is a (small) mixing angle between the two charged vector-bosons W_1 and W_2 and ω is a

new CP phase related to this mixing. The light mass eigenstate is identified with the Standard Model (SM) gauge boson W_L, $W_1 \sim W_L$ with $M_1 = M_W$.

The interaction (36) contributes to the asymmetry in $B \to X \ell^+ \nu_\ell$ at one-loop order. We consider a W_1 exchange diagram as in Fig. 1, with self-energy insertions on the W_1 line of $\bar{c}s$ and $\bar{u}d$ quark loops and $\ell' \nu_{\bar{\ell}'}$ ($\ell' = [e, \mu] \neq \ell$) and $\tau \bar{\nu}_\tau$ leptonic loops. Elastic weak rescattering from an intermediate $\ell \bar{\nu}$ state is not included for consistency with CPT [19]. Loop amplitudes involving W_2 exchange or mixed W_1–W_2 exchanges are expected to be smaller because a suppression factor $\sin \xi$ is replaced in these amplitudes by M_1^2/M_2^2. One obtains $M_2 > 1.6$ TeV if $V_L = V_R$ [20] (a later estimate gave 1.4 TeV [21]) but this bound can be relaxed if the left-hand- and right-hand-quark mixings are different [21]. For the case $V_L = V_R$, which we consider below, the W_1 exchange diagram with self-energy insertions dominates over these other contributions.

The absorptive part of the dominant one-loop amplitude is

$$M_{1(\text{abs})}^{LR} = -\frac{G_F^2}{2} N_C \sin \xi \cos^3 \xi \, V_{cb}^R e^{i\omega} \cdot T_{1(\text{abs})}^{LR}(q^2)$$
$$\cdot \left[\bar{c} \gamma^\mu (1 + \gamma_5) b \right] \left[\bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell \right], \tag{37}$$

with

$$T_{1(\text{abs})}^{LR}(q^2) = |V_{cs}^L|^2 T_{1(\text{abs})}^{cs} + |V_{ud}^L|^2 T_{1(\text{abs})}^{ud}$$
$$+ (1/N_C)\left(T_{1(\text{abs})}^{\ell' \nu_{\ell'}} + T_{1(\text{abs})}^{\tau \nu_\tau} \right). \tag{38}$$

Here $T_{1(\text{abs})}^{cs}(q^2)$ is the same as in the SM, $T_{1(\text{abs})}^{cs} \equiv T_{1(\text{abs})}$, while $T_{1(\text{abs})}^{ud}$, $T_{1(\text{abs})}^{\ell' \nu_{\ell'}}$ and $T_{1(\text{abs})}^{\tau \nu_\tau}$ correspond to a $\bar{u}d$ loop and to the two leptonic loops. Using the notations of Eq. (32) and the approximation $m_u = m_d = m_{\ell'} \simeq 0$, $m_\tau \simeq m_c$, one has

$$T_{1(\text{abs})}^{\tau \nu_\tau}(z) \simeq T_{1(\text{abs})}^{cs}(z) = -\frac{m_b^2}{12\pi} G(z),$$

$$T_{1(\text{abs})}^{\ell' \nu_{\ell'}} \simeq T_{1(\text{abs})}^{ud}(z) = -\frac{m_b^2}{6\pi} z. \tag{39}$$

Comparing the contribution of this amplitude to the asymmetry with the asymmetry calculated in the SM, we obtain a ratio

$$\frac{A_{s\ell}^{LR}}{A_{s\ell}^{SM}} = 8 \sin \xi \cos^3 \xi \, \frac{m_c}{m_b} \frac{N_C}{c_3 + N_C c_4} \frac{I^{LR}}{I^{SM}} \frac{R^{LR}}{R^{SM}}. \tag{40}$$

This ratio involves two enhancement factors following from a suppression which occurs in the Standard Model but not in its left–right-symmetric extension. The first factor, $N_C/(c_3 + N_C c_4) = -34$, originates in a loop suppression of the Wilson coefficients for penguin operators in (16). A second potential enhancement is due to the ratio of weak phase factors I^{LR}/I^{SM}, where

$$I^{SM} \equiv \frac{\text{Im}[V_{tb}^{L*} V_{ts}^L V_{cb}^L V_{cs}^{L*}]}{|V_{cb}^L|^2}, \qquad I^{LR} \equiv \frac{\text{Im}[V_{cb}^L V_{cb}^{R*} e^{-i\omega}]}{|V_{cb}^L|^2}. \tag{41}$$

While this factor is -0.018 in the SM, it may be of order one in the LR model if $V_{cb}^R = V_{cb}^L$ and if ω is large. The last ratio in (40) depends on quark couplings and on phase space. Neglecting u, d and s quark masses and setting $|V_{cs}^L|^2 = |V_{ud}^L|^2 = 1$, it is given by

$$\frac{R^{LR}}{R^{SM}} = \frac{\int dz \, z w(z) G(z) + 2 \int dz \, z^2 w(z)}{\int dz \, F(z) G(z)}. \tag{42}$$

The upper limit of integration in the three integrals is $z_{\max} = (1 - \sqrt{r_c})^2$ as in (31). The lower limit of the first integral in the numerator and the one in the denominator is $z_{\min} = r_c$, while that
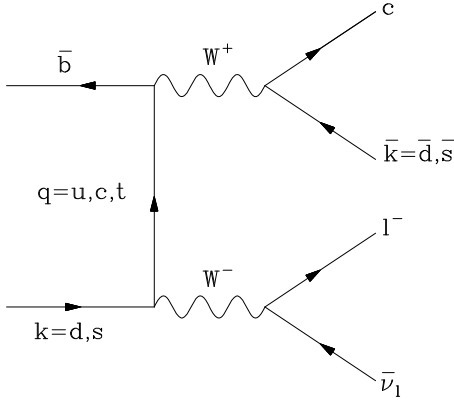
Fig. 3. Second-order quark diagrams contributing to $B_k^0 \to X\ell^- \bar{\nu}_\ell$.

of the second integral in the numerator is zero. Taking $r_c = 0.082$ as a central value in Table 1, one finds $R^{LR}/R^{SM} = 0.93$.

Combining all factors and assuming that CP-violation in semi-leptonic B decays is dominated by the phase ω, one obtains

$$\left| \frac{A_{s\ell}^{LR}}{A_{s\ell}^{SM}} \right| \simeq 4 \times 10^3 \frac{|V_{cb}^R|}{|V_{cb}^L|} |\sin\xi \sin\omega|. \tag{43}$$

A recent study of phenomenological constraints on right-handed quark currents obtains an upper bound on a $b \to c$ right-handed coupling of several percent relative to a left-handed coupling [22], $|(V_{cb}^R/V_{cb}^L) \tan\xi \cos\omega| = (2.5 \pm 2.5) \times 10^{-2}$. A comparable upper bound may be obtained on $|(V_{cb}^R/V_{cb}^L) \tan\xi \sin\omega|$ from a recent measurement of CP asymmetry in $B^+ \to J/\psi K^+$ [23], $A_{\rm CP}(B^+ \to J/\psi K^+) = [-0.76 \pm 0.50({\rm stat}) \pm 0.22({\rm syst})] \times 10^{-2}$. This bound requires assuming that a final state interaction phase difference between two interfering $B \to J/\psi K$ hadronic amplitudes, for tree-level $(V-A)(V-A)$ and $(V+A)(V-A)$ $b \to c\bar{c}s$ transitions, is not small. Thus, the asymmetry in the left–right-symmetric model may be at most two orders of magnitude larger than in the Standard Model.

5. Wrong-sign leptons without neutral $B_{(s)}$ mixing

In Section 2 we have assumed $A(\bar{B}_k^0 \to X\ell^+\nu_\ell) = A(B_k^0 \to X\ell^-\bar{\nu}_\ell) = 0$, neglecting second-order-weak contributions to these two processes which occur in the CKM framework leading to "wrong-sign" leptons without B_k^0–$\bar{B}_k^0$ mixing. Interference between these second-order contributions and first-order tree amplitudes for $\bar{B}_k^0 \to X\ell^-\bar{\nu}_\ell$ and $B_k^0 \to X\ell^+\nu_\ell$ leads to additional time-dependent terms in Eqs. (6) of the forms $e^{-\Gamma_k t} \sinh(\Delta\Gamma_k t/2)$ and $e^{-\Gamma_k t} \sin(\Delta m_k t)$. Second-order amplitudes for $B_k^0 \to X\ell^-\bar{\nu}_\ell$ have been discussed in Ref. [24] without estimating their magnitudes. For completeness, as we have studied tiny CP asymmetries from second-order amplitudes, we will estimate the ratio of second-order amplitudes for "wrong-sign" leptons and first-order amplitudes for "right-sign" leptons, showing that this ratio is negligibly small.

Second-order amplitudes for $B_k^0 \to X\ell^-\bar{\nu}_\ell$ are described by diagrams plotted in Fig. 3, in which both the $\bar{b}$ quark and the spectator k quark undergo weak decays into $\bar{q}c\bar{k}$ and $q\ell^-\bar{\nu}_\ell$, respectively, by exchanging $q = u, c, t$ quarks. These second-order amplitudes, involving CKM factors $V_{qb}^* V_{qd} V_{cd}$ and $V_{qb}^* V_{qs} V_{cs}$ in B^0 and B_s decays, lead to final hadronic states with quark structures $X = c\bar{d}$ and $X = c\bar{s}$, respectively, as in first-order amplitudes for $\bar{B}^0$ and $\bar{B}_s$ semileptonic decays.

Let us denote the second-order-weak amplitude for B_k^0 decay by $A_\ell^k \equiv A(B_k^0 \to X\ell^-\bar{\nu}_\ell)$. We wish to estimate the ratios of semileptonic rates

$$R_k = \left| \frac{A_\ell^k}{\bar{A}_\ell^k} \right|^2 \frac{\Phi_2}{\Phi_1}, \tag{44}$$

where $\Phi_{1,2}$ are the appropriate phase-space factors for the first-order and second-order processes. By neglecting the effect of the spectator quark in $\bar{A}_\ell^k$ we are treating the first-order process as leading to a three fermion final state, while the second-order diagram illustrated in Fig. 3 involves four fermions in the final state. A naive dimensional analysis then leads to

$$\frac{\Phi_2}{\Phi_1} = \frac{(m_b - m_c)^2}{16\pi^2}. \tag{45}$$

The ratio $A_\ell^k/\bar{A}_\ell^k$ must involve a factor of $G_F f_B$ which has a suitable dimension. The momentum passing through the propagator of the fermion $q = u, c, t$ is of order m_b, and kinematic factors of the same order will cancel it for $q = u, c$, while the contribution of $q = t$ is highly suppressed by the heavy t-quark mass. The corresponding CKM factors are $V_{ub}^* V_{us} V_{cs} \sim \mathcal{O}(\lambda^4)$ $(q = u)$, $V_{cb}^* V_{cs} V_{cs} \sim \mathcal{O}(\lambda^2)$ $(q = c)$ and $V_{ub}^* V_{ud} V_{cd} \sim \mathcal{O}(\lambda^4)$ $(q = u)$, $V_{cb}^* V_{cd} V_{cd} \sim \mathcal{O}(\lambda^4)$ $(q = c)$ for B_s and B^0 decays, respectively, where $\lambda = 0.23$. Thus, the c quark dominates A_ℓ^s with a CKM factor comparable to V_{cb} governing $\bar{A}_\ell^s$, while the contributions of the u and c quarks in A_ℓ^d are comparable to each other and are suppressed by $\lambda^2 \simeq 1/20$ relative to the CKM factor in $\bar{A}_\ell^d$. Taking $m_b - m_c = 3.4$ GeV (see Table 1 above for values of m_b and m_c) and $f_B = 230$ MeV, one then finds

$$R_s \sim \frac{(m_b - m_c)^2}{16\pi^2} (f_B G_F)^2 \simeq 5.3 \times 10^{-13}, \tag{46}$$

$$R_d \sim (1/20)^2 R_s \simeq 1.3 \times 10^{-15}. \tag{47}$$

The corresponding ratios of square roots, $\sim 0.7 \times 10^{-6}$ and $\sim 0.4 \times 10^{-7}$, characterize coefficients of additional time-dependent terms of forms $e^{-\Gamma_k t} \sinh(\Delta\Gamma_k t/2)$ and $e^{-\Gamma_k t} \sin(\Delta m_k t)$ in Eqs. (6) for B_s and B^0 which may be safely neglected.

6. Conclusion

Inclusive semileptonic B and B_s decays are shown to have a small non-zero direct CP asymmetry in the Standard Model as a result of interference of first-order- and second-order-weak processes. This stands in contrast with statements about the vanishing of this asymmetry made in two textbooks on CP violation [9, 10]. Taking a range of effective quark masses and estimating the asymmetry in $b \to c\ell\bar{\nu}_\ell$ as due to weak rescattering from the intermediate state in $b \underset{\rm peng}{\to} c\bar{c}s$, we have found

$$A_{sl} = (-3.2 \pm 0.9) \times 10^{-9}. \tag{48}$$

A dimensional argument leads to a model-independent upper bound on A_{sl} which is three orders of magnitude larger, while an extension of the Standard Model to a left–right-symmetric variant can increase the above asymmetry by at most two orders of magnitude. These values are far smaller than a value of about -1% recently reported by the D0 Collaboration [1,2], which may still be associated with a new source of CP violation in neutral B meson mixing.

As a by-product of second-order-weak amplitudes, we have estimated their effect on wrong-sign leptons in direct decays of neutral B mesons and found it to be much below any reasonable sensitivity.

Acknowledgements

References

[1] V.M. Abazov, et al., D0 Collaboration, Phys. Rev. D 82 (2010) 032001, arXiv:1005.2757 [hep-ex].
[2] V.M. Abazov, et al., D0 Collaboration, Phys. Rev. Lett. 105 (2010) 081801, arXiv:1007.0395 [hep-ex].
[3] CDF note 9015, Measurement of CP asymmetry in semileptonic B decays, http://www-cdf.fnal.gov/physics/new/bottom/070816.blessed-acp-bsemil.
[4] A. Lenz, U. Nierste, JHEP 0706 (2007) 072, arXiv:hep-ph/0612167.
[5] M. Gronau, J.L. Rosner, Phys. Rev. D 82 (2010) 077301, arXiv:1007.4728 [hep-ph].
[6] See e.g. T.E. Browder, T. Gershon, D. Pirjol, A. Soni, J. Zupan, Rev. Mod. Phys. 81 (2009) 1887, arXiv:0802.3201 [hep-ph].
[7] G.C. Branco, L. Lavoura, J.P. Silva, CP Violation, Int. Ser. Monogr. Phys., vol. 103, Oxford Science Publications, 1999.
[8] I.I.Y. Bigi, A.I. Sanda, CP Violation, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol., vol. 9, Cambridge University Press, 2000.
[9] Ref. [7], p. 58.
[10] Ref. [8], p. 186 mentions the vanishing of A_{sl} in the Standard Model.
[11] We think that a first statement about the vanishing by CPT of an inclusive semileptonic asymmetry in D decays was made by Abraham Pais and Sam Treiman: A. Pais, S.B. Treiman, Phys. Rev. D 12 (1975) 2744;
A. Pais, S.B. Treiman, Phys. Rev. D 16 (1977) 2390, Erratum.
[12] J.M. Soares, Phys. Rev. Lett. 68 (1992) 2102.
[13] G. Eilam, J.L. Hewett, A. Soni, Phys. Rev. Lett. 68 (1992) 2103.
[14] G. Buchalla, A.J. Buras, M.E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125, arXiv:hep-ph/9512380.
[15] M.E. Peskin, D.V. Schroeder, An Introduction to Quantum Field Theory, Addison–Wesley, 1995, p. 232ff.
[16] C. Quigg, J.L. Rosner, Phys. Rev. D 23 (1981) 2625.
[17] J. Charles, et al., CKMfitter Group, Eur. Phys. J. C 41 (2005) 1, arXiv:hep-ph/0406184, updated results are available at http://ckmfitter.in2p3.fr.
[18] J.C. Pati, A. Salam, Phys. Rev. Lett. 32 (1974) 1083;
R.N. Mohapatra, J.C. Pati, Phys. Rev. D 11 (1975) 566;
R.N. Mohapatra, J.C. Pati, Phys. Rev. D 11 (1975) 2558;
G. Senjanovic, R.N. Mohapatra, Phys. Rev. D 12 (1975) 1502;
R.N. Mohapatra, F.E. Paige, D.P. Sidhu, Phys. Rev. D 17 (1978) 2462;
G. Senjanovic, Nucl. Phys. B 153 (1979) 334.
[19] J.M. Gerard, W.S. Hou, Phys. Rev. Lett. 62 (1989) 855;
L. Wolfenstein, Phys. Rev. D 43 (1991) 151;
J.M. Soares, Ref. [12].
[20] G. Beall, M. Bander, A. Soni, Phys. Rev. Lett. 48 (1982) 848.
[21] P. Langacker, S. Uma Sankar, Phys. Rev. D 40 (1989) 1569.
[22] A.J. Buras, K. Gemmler, G. Isidori, arXiv:1007.1993 [hep-ph].
[23] K. Sakai, et al., Belle Collaboration, arXiv:1008.4993 [hep-ex].
[24] G.V. Dass, K.V.L. Sarma, Phys. Rev. D 54 (1996) 5880, arXiv:hep-ph/9607274;
G.V. Dass, K.V.L. Sarma, arXiv:hep-ph/9610466.