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Bounds on fourth generation induced lepton flavour violating double insertions in supersymmetry

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ABSTRACT

We derive bounds on leptonic double mass insertions of the type $\delta^l_{i4}\delta^l_{4j}$ in four generational MSSM, using the present limits on $l_i \to l_j + \gamma$. Two main features distinguish the rates of these processes in MSSM4 from MSSM3: (a) $\tan\beta$ is restricted to be very small $\lesssim 3$ and (b) the large masses for the fourth generation leptons. In spite of small $\tan\beta$, there is an enhancement in amplitudes with LLRR ($\delta^{ll}_{i4}\delta^{rr}_{4j}$) type insertions which pick up the mass of the fourth generation lepton, $m_{\tau'}$. We find these bounds to be at least two orders of magnitude more stringent than those in MSSM3.

1. In the recent times, there has been a renewed interest in the idea of the fourth generation of Standard Model fermions. While additional generations have been proposed quite a while ago [1,2], the present exploration [3–21] of the fourth generation is more timely and in tune with the on-going searches at LHC [4] as well at the Tevatron [22,23]. The presence of fourth generation can enhance the production rates of the Higgs at the Tevatron and thus ruling out a significantly larger mass range [5] compared to the three generation SM.

In addition to the direct searches at Colliders [4,22–24], the fourth generation could be probed indirectly in processes where the fourth generation can contribute through loop effects. The fourth generation contributions to the S and T parameters would push them out of the experimentally allowed (3σ) range. A heavy higgs, together with *almost* degenerate masses for the fourth generation has been proposed by Kribs et al. [6] to over come this problem (see also [8]).

The presence of the fourth generation would also modify the CKM matrix thus leading to strong effects in B and K physics. One crucial factor is the value of V_{tb}. The present Tevatron limits [10] on V_{tb} would allow it to be as small as ≈ 0.7 at 3σ. Such large deviations can lead to significant effects in B-physics [12]. Unitarity of the CKM4 together with electroweak precision measurements put significant constraints on the allowed forms of the four generational CKM matrix. Similarly, flavour observables in the K and D physics sector would also constraint the mixing and the masses of the fourth generation quarks [14].

In the leptonic sector, the effects of fourth generation are quite different compared to the Standard Model with massive neutrinos. The fourth generation neutrino is necessarily greater than 45 GeV to escape the LEP limit on the invisible decay width of Z. As with the CKM, the PMNS matrix is now 4×4 whose form determines the couplings of the fourth generation neutrino. These are strongly constrained from the deviations of the Fermi coupling constant, lepton universality tests as well as rare lepton decays [15,16].

Supersymmetric extensions of the four generation can be motivated as a solution to the little hierarchy problem. The fourth generation with new additional Yukawa couplings much larger than the top (especially t' and b') can easily enhance the 1-loop correction to the light higgs mass by a factor of 2 or more. In fact $\tan\beta$ of $\mathcal{O}(1)$ could now easily be allowed [17]. The relevant Higgs production cross sections and decay branching fractions have been studied in [18]. Another reason why the combination of supersymmetry and four generations is interesting is that first order phase transition relevant for electroweak baryogenesis is now possible without introducing an additional singlet [19].

Such large Yukawa couplings however elude traditional SUSY-GUT model building with four generations. Perturbativity of the Yukawa couplings puts strong UV cut-offs on these models, which can utmost be of $\mathcal{O}(100\ \text{TeV})$ [20]. Thus perturbative gauge coupling unification is not possible unless additional matter is added [21].[1] In a similar manner, with four generations, it would be hard to realise traditional supersymmetric breaking methods like mSUGRA, minimal AMSB, etc., with soft terms defined at the high scale and renormalisation group evolution determining the soft

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[1] Even in the Standard Model, contrary to the several statements in the literature, perturbative unification of the gauge couplings at 2-loop is not possible, with the present limits on the fourth generation masses.

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spectrum at the weak scale. While minimal gauge mediation is already ruled out, General Gauge Mediation and variations of it are more suited for the case of MSSM with four generations [20,25].

Similar to the Standard Model with four generations (SM4), one would expect MSSM with four generations (MSSM4) would also contribute to the flavour processes. However unlike in the SM4, in MSSM4, flavour violation is determined by the *mis-match* between flavour states of SM particles and their super-partners (the super-CKM or super-MNS basis). In fact, it has been known that large flavour violating terms within the super-partners are strongly constrained by various flavour violating experiments [26]. One more feature that would make flavour studies within MSSM4 worthwhile is that $\tan\beta$ is restricted to be very small $\lesssim 3$. Thus, large $\tan\beta$ enhancement which is typical most MSSM flavour violating processes, especially the ones which involve dipole operators, is absent within the case of MSSM4. Secondly, the large masses of the fourth generation could lead to enhancement of amplitudes within the context of some dipole operators. Taken together, we think the interplay between two factors make it worthwhile to explore flavour processes within MSSM4.

In the present work, we explore flavour violating constrains in MSSM4. We concentrate on the leptonic sector. Typically, the leptonic sector provides an unambiguous constraint on the flavour violating entries compared to the hadronic sector where the bounds are dependent on the parameterisation of the CKM matrix as well as the uncertainties in the hadronic matrix elements.

Before proceeding further, a couple of comments are in order. The fourth generation neutrino with a mass $m_{\nu_{\tau'}} > 45$ GeV could be a Majorana particle or a Dirac particle. While in the SM, Lepton Flavor Violation (LFV) processes don't significantly get modified due to this, the construction of models in each case could be quite different. In most cases, there could be additional particles at low scale [27]. In supersymmetric theories, lepton flavour violation is typically proportional to the scale of supersymmetry breaking. While there could be significant model dependence in construction of the neutrino mass matrices, the flavour violation in the supersymmetry breaking soft sector it selves could be a major contributing factor. In the present work, we will assume all the dominant source of LFV in MSSM4 comes from the soft sector, which is model independent. In should be noted that in realistic models, in addition to the flavour violation from the soft sector, the Standard Model contribution with four leptonic generations and any additional contribution pertaining to the model should be taken in to account.

2. As is well known, in MSSM, the dominant source of flavour violation is from the soft terms. Thus, in a similar manner to MSSM3, there are sources of lepton flavour violation in flavour violating soft terms and are independent of the neutrino masses. To this extent there could up to twelve new flavour violating entries in the soft lagrangian in MSSM4. These are given as

$$\mathcal{L}_{\text{soft}} = \Delta_{ll}^{i4}\tilde{l}_i^\star\tilde{l}_4 + \Delta_{rr}^{i4}\tilde{r}_i^\star\tilde{r}_4 + \Delta_{lr}^{i4}\tilde{l}_i^\star\tilde{r}_4 + \cdots \tag{1}$$

where $\tilde{l}$ denotes the leptonic doublets (left-handed), $\tilde{r}$ are the leptonic singlets and $i = \{1, 2, 3\}$ for the standard three generations. While the presence of these terms would definitely give rise flavour violating decays for the fourth generation fermions, they would also contribute to flavour violating processes in the first three generations. This happens when two fourth generation flavour violating couplings combine to form a flavour violating entry within the first three generations. For illustration purposes, let us consider the case of $\mu \to e + \gamma$. The diagram with fourth generation mass insertions (of the (ll) type) is shown in Fig. 1. In general

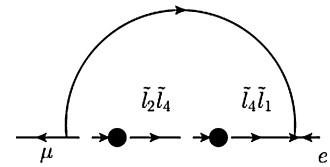


Fig. 1. Double insertions. A schematic diagram showing the double insertions of a fourth generation leading to flavour violation in 1–2 sector. The photon line is suppressed.

this contribution would add to the contribution generated by the flavour violation already present in the soft potential Δ_{ll}^{12}.

Defining $\delta_{ll}^{ij} = \Delta_{ll}^{ij}/m_{\tilde{l}}^2$, we can write the total flavour violating δ as

$$\delta_{ll}^{ij} = \delta_{ll}^{ij(3)} + \delta_{ll}^{ij(4)} \tag{2}$$

where

$$\delta_{ll}^{ij(4)} = \delta_{ll}^{i4}\delta_{ll}^{4j} \tag{3}$$

and $\delta_{ll}^{ij(3)}$ is the which is independent of the presence of the fourth generation. These single mass insertions are divided in to four types: ll, rr, lr and rl depending on the chirality of the corresponding fermion; it also represents the location of the flavour violating entry in the slepton mass matrix represented schematically as

$$\mathcal{M}_{\tilde{f}} = \begin{pmatrix} m_{\tilde{l}\tilde{l}}^2 & m_{\tilde{l}\tilde{r}}^2 \\ m_{\tilde{r}\tilde{l}}^2 & m_{\tilde{r}\tilde{r}}^2 \end{pmatrix} \tag{4}$$

The possible combinations of double insertions which give the effective single flavour violating insertions are:

$$\begin{aligned} &l_i l_j = l_i l_4 l_4 l_j \quad \| \quad r_i r_j = r_i r_4 r_4 r_j \\ &l_i r_j = l_i l_4 r_4 r_j; \quad l_i r_4 r_4 r_i \quad \| \quad r_i l_j = r_i l_4 l_4 l_j; \quad r_i r_4 l_4 l_j \end{aligned} \tag{5}$$

Finally, let us note that δ_{ij}^3 can be thought of as being generated by integrating out the fourth generation sleptons.

In the present work, we will derive bounds on the double insertions due to the $\delta_{ij}^{(4)}$. In the presence of non-fourth generation flavour violation the bounds would only become stronger, unless of course one considers fine tuned cancellations between the two contributions. Thus from now on, we set $\delta_{ij}^{(3)} = 0$ and derive the bounds on δ_{ij}, where we have suppressed the superscript (4). We will use the mass insertion approximations to compute the bounds as done for MSSM3 [29].

Before we list the amplitudes for each of these mass insertions, let us make a few comments on the supersymmetric spectrum one considers to evaluate these bounds. For the fourth generation MSSM, as of now, there is no concrete model of supersymmetry breaking. The classic models of supersymmetry breaking like minimal supergravity, gauge mediated supersymmetry breaking, Anomaly mediated supersymmetry breaking, etc., which are well established in three generational MSSM cannot be generalised to four generations in their present form [20]. In all probability [25], the SUSY breaking model could be a strongly coupled sector with a low mediation scale in a similar view to the general gauge mediation scheme of Seiberg and co-workers [32]. In the following, we will consider a model independent approach and evaluate the bounds in generic low energy MSSM. Thus, our approach will be similar to the approach taken by Gabbiani et al. [26]. Accordingly, we will quote our bounds in terms of the slepton mass $m_{\tilde{l}(\tilde{r})}$ and ratios of the parameters which we are defined as $x_{l(r)} = M_1^2/m_{\tilde{l}(\tilde{r})}^2$, $y_{l(r)} = \mu^2/m_{\tilde{l}(\tilde{r})}^2$ and $z_{l(r)} = M_2^2/m_{\tilde{l}(\tilde{r})}^2$. We also

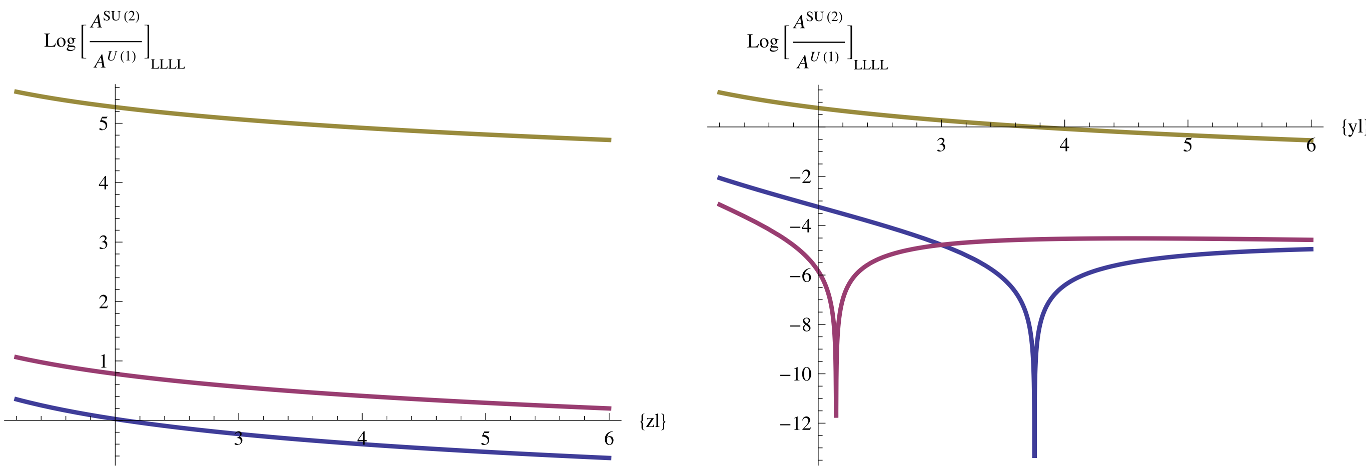


Fig. 2. Comparison of SU(2) vs. U(1) contribution against z_l, y_l for LLLL case. Rest of the parameter values correspond to representative points S1, S2 and S3, shown by blue, pink and brown lines respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

Table 1
The parameter space points in terms of ratios w.r.t. the (left-handed) slepton mass. $x_l = M_1^2/m_{\tilde{l}}^2$, $y_l = \mu^2/m_{\tilde{l}}^2$, $z_l = M_2^2/m_{\tilde{l}}^2$, and $t_{rl} = m_{\tilde{r}}^2/m_{\tilde{l}}^2$.

	x_l	y_l	z_l	t_{rl}
S1	0.1	0.3	2.5	0.7
S2	0.3	0.1	5.5	0.4
S3	0.003	0.01	0.5	0.9
T1	0.05	0.3	0.5	0.09
T2	0.06	0.03	0.6	0.09
T3	0.07	0.01	0.7	0.09

fix the ratio $t_{rl} = m_{\tilde{r}}^2/m_{\tilde{l}}^2$. Thus, once the left-handed slepton mass and its ratios are given, the right-handed slepton mass and its ratios also get fixed. A crucial distinction of MSSM4 and MSSM3 is the restriction on $\tan\beta$. With the fourth generation masses being very large, tree level perturbativity restricts $\tan\beta$ to be:

$$\tan\beta \lesssim \left(2\pi (v/m_{b'})^2 - 1\right)^{1/2} \tag{6}$$

For a bottom-prime mass, $m_{b'} \approx 300$ GeV, we have $\tan\beta \approx 2$. This upper bound on $\tan\beta$ is very generic to MSSM4 and is independent of supersymmetric breaking. It holds as long as one does not change the particle spectrum. Here we will present our results for few representative points in parameter space. In the following we will consider value of $\tan\beta$ to be 2.[2] The values chosen for rest of the parameters are given in Table 1. The parameter space points S1, S2, S3 are similar to those one has in mSUGRA/CMSSM models with universal scalar and gaugino masses at the high scale. (While S1 represents the case with $m_0 \approx M_{1/2}$, S2 has $M_{1/2} \gg m_0$ and S3 represents $m_0 \gg M_{1/2}$.) The points T1, T2 and T3 are motivated by the general gauge mediation framework of Seiberg and co-workers [32]. The three possible choices for the ratio Λ_G/Λ_S (here Λ_G, Λ_S refers to the gaugino and scalar mass scale respectively) fixes *xl* and *zl* in this case. μ is left to be a free parameter with ratios 0.3, 0.03 and 0.01 for T1, T2 and T3 respectively.

While choosing the points above, we have not taken in to consideration the relic density constraints on neutralino dark matter from the WMAP experiment. The leptonic flavour violating rates would be different compared to those at the points chosen above. A particularly interesting case in the three generations is that of the co-annihilation region where the $\tilde{\tau}_1$, has a mass very close to that of the lightest neutralino. In MSSM4, it is quite probable that in large regions of the parameter space, $\tilde{\tau}_1'$ is the NLSP. This is especially true if mSUGRA like boundary conditions could be realised in this model. In such regions the relation $m_{\tilde{\tau}'} \approx M_1$ is roughly satisfied.

3. The contributions from double insertions of the type $\delta_{ik}\delta_{kj}$ leading to flavour violating $i \to j$ processes have already been studied in literature [28,29] for the case of $\mu \to e + \gamma$ where the third generation flavour violation contributes. Here we generalise them to the four generation case. We list below the amplitudes for the various possible combinations of mass insertions from the fourth generation one by one.

The amplitude associated with $l_i l_4 l_4 l_j$ has contributions both from chargino as well as neutralino sector. These contributions are typically listed as SU(2) and U(1) contributions in the literature [28,29,31]:

$$\left(A_{l2}^{ij}\right)_{\mathrm{SU}(2)} = \tilde{\alpha}_2 \delta_{ll}^{i4}\delta_{ll}^{4j}\left[\frac{I_{1n}(z_l)+I_{1c}(z_l)}{m_{\tilde{l}}^2} + \frac{\mu M_2 \tan\beta}{(M_2^2-\mu^2)}\left(\frac{I_{2n}(z_l,y_l)+I_{2c}(z_l,y_l)}{m_{\tilde{l}}^2}\right)\right] \tag{7}$$

$$\left(A_{l2}^{ij}\right)_{\mathrm{U}(1)} = \tilde{\alpha}_1 \delta_{ll}^{i4}\delta_{ll}^{4j}\left[\frac{I_{1n}(x_l)}{m_{\tilde{l}}^2} + \mu M_1 \tan\beta\left(-\frac{I_{2n}(x_l,y_l)}{m_{\tilde{l}}^2(M_1^2-\mu^2)} + \frac{1}{(m_{\tilde{r}}^2-m_{\tilde{l}}^2)}\left(\frac{2I_{2n}(x_l)}{m_{\tilde{l}}^2} + \frac{2f_{2n}(x_l)}{(m_{\tilde{r}}^2-m_{\tilde{l}}^2)} + \frac{m_{\tilde{l}}^4}{(m_{\tilde{r}}^2-m_{\tilde{l}}^2)^2}\left(\frac{f_{3n}(x_r)}{m_{\tilde{r}}^2} - \frac{f_{3n}(x_l)}{m_{\tilde{l}}^2}\right)\right)\right)\right] \tag{8}$$

Given the larger value of $\tilde{\alpha}_2$ one would expect that the SU(2) contribution to dominate over the U(1) contribution. The various loop functions appearing in amplitudes are listed in Appendix A.

In Fig. 2 we have shown the comparison of SU(2) vs. U(1) amplitudes for different parameters sets taken from Table 1. It is evident from figures that in some regions of parameter space SU(2) is dominant while in others U(1) has larger contribution. This dominance is stable under the variation of parameters z_l and y_l unless there is cancellation between loop functions. The dips in curves are due to these cancellations.

Given that $\tan\beta$ is confined to low values in MSSM4, one would expect that there is no large enhancement associated with diagrams with chirality flips either in the vertex or on the internal

[2] Note that such low values are not ruled out by the light higgs mass constraint in MSSM4.

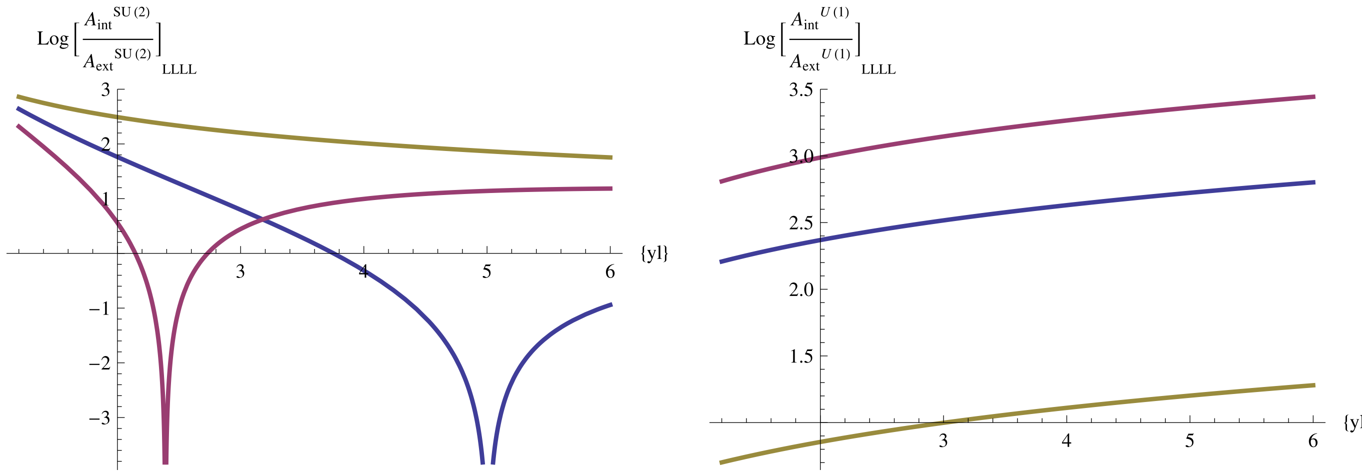


Fig. 3. Comparison of internal vs. external flip contributions for SU(2) and U(1) respectively in LLLL case. Rest of the parameter values correspond to representative points S1, S2 and S3, shown by blue, pink and brown lines respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

line; both these amplitudes being proportional to $\mu\tan\beta$. In the allowed regions of $\tan\beta$, the amplitudes of external chirality flip diagrams can become comparable in magnitude with those of internal flip ones. This is evident from Fig. 3, where we have shown the ratios of the internal contribution to the external contribution for SU(2) and U(1) separately. As can be seen from figure while the internal chirality flip diagrams still dominates, there are regions in parameter space where the external amplitudes become comparable or dominate as can be seen in U(1) contribution for point S3. Of course there could also be regions where there are cancellations within the internal amplitudes as can be seen in SU(2) amplitudes for points S2 and S3. Overall we see that for S3, not only U(1) amplitudes dominate but also external flip contributions dominate for small values of y_l.

In the following (Tables 2–7) we will present the bounds on the double mass insertions for the spectrum points $S_i(m_L = 200\ \text{GeV})$ and $T_i(m_L = 500\ \text{GeV})$. The Branching Fraction (BR) for $l_i \to l_j\gamma$ in terms of the amplitudes is given by

$$\frac{BR(l_i \to l_j\gamma)}{BR(l_i \to l_j\nu_i\bar{\nu}_j)} = \frac{48\pi^3\alpha}{G_F^2}\left(\left|A_L^{ij}\right|^2 + \left|A_R^{ij}\right|^2\right)$$

where α is the fine structure constant and G_F is the Fermi constant.

The present experimental on the limits of the various branching fractions are given as

$$\text{Br}(\mu \to e\gamma) = 1.2 \times 10^{-11} \quad [33]$$

$$\text{Br}(\tau \to \mu\gamma) = 4.4 \times 10^{-8} \quad [34]$$

$$\text{Br}(\tau \to e\gamma) = 3.3 \times 10^{-8} \quad [34]$$

The involved branching ratios of leptonic τ decays are [35]

$$\text{Br}(\tau \to \nu_\tau\mu\bar{\nu}_\mu) = (17.36 \pm 0.05)\%$$

$$\text{Br}(\tau \to \nu_\tau e\bar{\nu}_e) = (17.84 \pm 0.05)\%$$

4. The chirality flip associated with the fourth generation lepton mass however, makes its appearance in amplitudes with double mass insertions of the type $l_i l_4 r_4 r_j$ where there would be a chirality flipping $m^2_{\tilde{l}_4\tilde{r}_4} = m_{\tau'}\mu\tan\beta$ mass insertion. Here the amplitude gets enhanced by a $m_{\tau'}/m_i$ factor associated with the mass of the decaying lepton. This factor which could be quite large could significantly strengthen the bounds by an order of magnitude or more, depending on the mass of the fourth generation lepton (τ') chosen. The amplitude for this mass insertion is given by:

Table 2
Bounds on $((\delta_{ll})_{i4}(\delta_{ll})_{4j})$.

MI	S1	S2	S3
21	0.00114	0.00105	0.00037
32	0.16588	0.15336	0.05469
31	0.14171	0.13102	0.04672
MI	T1	T2	T3
21	0.00237	0.00215	0.00242
32	0.34525	0.31317	0.35238
31	0.29495	0.26754	0.30103

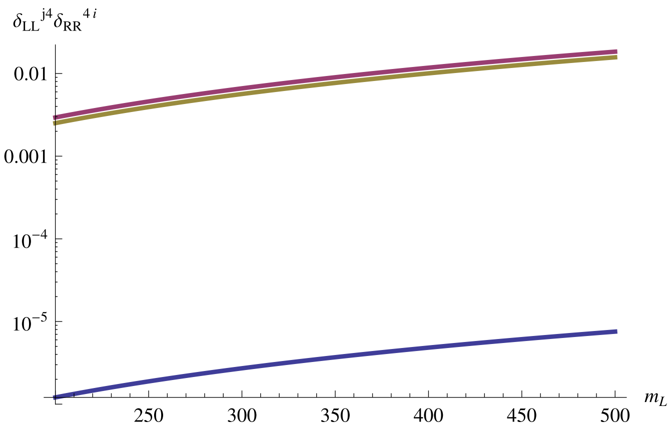
$$A_{l3}^{ij} = -2\tilde{\alpha}_1\frac{m_{\tau'}}{m_i}\mu M_1\tan\beta\,\delta_{ll}^{i4}\delta_{rr}^{4j}\frac{m_{\tilde{l}}^2 m_{\tilde{r}}^2}{(m_{\tilde{l}}^2 - m_{\tilde{r}}^2)^2}\left(\frac{f_{2n}(x_l)}{m_{\tilde{l}}^4} + \frac{f_{2n}(x_r)}{m_{\tilde{r}}^4} + \frac{1}{(m_{\tilde{r}}^2 - m_{\tilde{l}}^2)}\left[\frac{f_{3n}(x_r)}{m_{\tilde{r}}^2} - \frac{f_{3n}(x_l)}{m_{\tilde{l}}^2}\right]\right) \tag{9}$$

For present work we have chosen $m_{\tau'} = 100$ GeV consistent with present limits from direct searches [35]. In Fig. 4 we have shown the variation of LLRR bound w.r.t. slepton mass, m_L. The bound scales inversely with increasing value of (square of) m_L. The bound also becomes weaker as move to higher values of m_R as evident from second part of Fig. 4 with different t_{rl} values. As mentioned previously the bounds on LLRR are sensitive to $m_{\tau'}$ and thus they have stronger constraints compared to other double insertions. In Tables 3, 4 we present a comparison of bounds in MSSM4 and MSSM3. It is clear MSSM4 bounds are much stronger, by atleast couple of orders of magnitude.

The amplitudes associated with *rlll* double insertions is given as follows. The corresponding bounds are presented in Table 5,

$$A_{l1}^{ij} = \tilde{\alpha}_1\frac{M_1}{m_i}\delta_{rl}^{i4}\delta_{ll}^{4j}\frac{m_{\tilde{l}}^2 m_{\tilde{r}}^2}{(m_{\tilde{l}}^2 - m_{\tilde{r}}^2)} \times\left[\frac{2f_{2n}(x_l)}{m_{\tilde{l}}^4} + \frac{1}{(m_{\tilde{l}}^2 - m_{\tilde{r}}^2)}\left(\frac{f_{3n}(x_l)}{m_{\tilde{l}}^2} - \frac{f_{3n}(x_r)}{m_{\tilde{r}}^2}\right)\right] \tag{10}$$

The amplitude associated with $(lr)(rr)$ is given by the above expression with $(l \leftrightarrow r)$ (for corresponding bounds see Table 6).



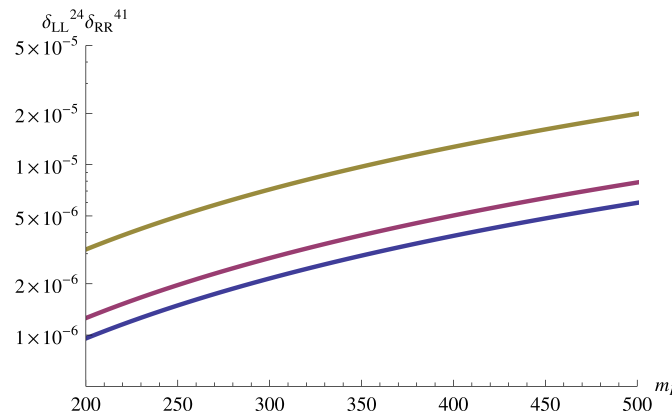


Fig. 4. Variation of $\delta_{ll}\delta_{rr}$ bound w.r.t. slepton mass i.e. m_L. Left figure corresponds to bound in 21 (blue), 32 (pink) and 31 (brown) sector while the right figure is with different t_{rl} values (blue, pink and brown corresponds to t_{rl} values of 0.01, 0.1 and 0.5 respectively) for 21 case. Merging of pink and brown line is due to nearly equal bound in 23 and 31 sector. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

Table 3
Bounds on $((\delta_{ll})_{i4}(\delta_{rr})_{4j})$ for $m_{\tau'} = 100$ GeV from MSSM4. It varies linearly with inverse of $m_{\tau'}$.

MI	S1	S2	S3
21	3.97×10^{-6}	6.46×10^{-6}	6.75×10^{-5}
32	0.00969	0.01576	0.16447
31	0.00828	0.01346	0.14051
MI	T1	T2	T3
21	7.54×10^{-6}	2.49×10^{-5}	4.51×10^{-5}
32	0.01837	0.06076	0.11004
31	0.01570	0.05191	0.09401

Table 4
Bounds on $((\delta_{ll})_{i3}(\delta_{rr})_{3j})$ from MSSM3. Hyphen (–) sign indicates the unphysical bound larger than unity.

MI	S1	S2	S3
21	2.23×10^{-4}	3.64×10^{-4}	0.00379
32	0.54556	0.88706	–
31	0.46607	0.75781	–
MI	T1	T2	T3
21	4.24×10^{-4}	1.40×10^{-3}	2.54×10^{-3}
32	1.03418	–	–
31	0.88349	–	–

Finally, the amplitude associated with RRRR double mass insertions is given by

$$A_{r2}^{ij} = \tilde{\alpha}_1 \delta_{rr}^{i4}\delta_{rr}^{4j}\Bigg[4\frac{I_{1n}(x_r)}{m_{\tilde{r}}^2} + \mu\tan\beta M_1\Bigg(2\frac{I_{2n}(x_r, y_r)}{m_{\tilde{r}}^2(M_1^2-\mu^2)} + \frac{1}{(m_{\tilde{r}}^2-m_{\tilde{l}}^2)}\Bigg\{-2\frac{I_{2n}(x_r)}{m_{\tilde{r}}^2} + 2\frac{f_{2n}(x_r)}{(m_{\tilde{r}}^2-m_{\tilde{l}}^2)} + \frac{m_{\tilde{r}}^4}{(m_{\tilde{r}}^2-m_{\tilde{l}}^2)^2}\left(\frac{f_{3n}(x_r)}{m_{\tilde{r}}^2} - \frac{f_{3n}(x_l)}{m_{\tilde{l}}^2}\right)\Bigg\}\Bigg)\Bigg] \quad (11)$$

The corresponding bounds on double insertions are given in Table 7. As one can see like MSSM3 the constraints on these parameters are very weak in this case.

Table 5
Bounds on $((\delta_{rl})_{i4}(\delta_{ll})_{4j})$.

MI	S1	S2	S3
21	2.57×10^{-6}	3.29×10^{-6}	7.36×10^{-6}
32	0.00626	0.00801	0.01794
31	0.00535	0.00685	0.01532
MI	T1	T2	T3
21	1.14×10^{-5}	1.15×10^{-5}	1.16×10^{-5}
32	0.02787	0.02804	0.02834
31	0.02381	0.02390	0.02421

Table 6
Bounds on $((\delta_{lr})_{i4}(\delta_{rr})_{4j})$.

MI	S1	S2	S3
21	1.89×10^{-6}	1.67×10^{-6}	6.63×10^{-6}
32	0.00462	0.00408	0.01617
31	0.00395	0.00348	0.01381
MI	T1	T2	T3
21	1.56×10^{-6}	1.62×10^{-6}	1.69×10^{-6}
32	0.00381	0.00397	0.00413
31	0.00326	0.00339	0.00353

Table 7
Bounds on $((\delta_{rr})_{i4}(\delta_{rr})_{4j})$.

MI	S1	S2	S3
21	0.00113	0.00081	0.00139
32	0.16521	0.1189	0.20335
31	0.14113	0.10157	0.17372
MI	T1	T2	T3
21	0.00191	0.00104	0.00106
32	0.27888	0.15229	0.15447
31	0.23825	0.13010	0.13196

5. Double insertions are an effective way of constraining four generation flavour violating entries in supersymmetric theories. The importance of these insertions has already been stressed in the works of Hisano and Nomura [28] and Paradisi [29]. In the present work, we have used this approach to constraint fourth generation flavour violating entries from the existing lepton flavour violating decays. While most chiral combinations of these entries like LLLL or RRRR, etc., have bounds similar to that of the single insertions, LLRR insertions are special as they pick up the mass of the fourth generation lepton leading to enhanced amplitudes. The resultant bounds are stringent by at least an order of magnitude and could reach up to three orders of magnitude stronger constraints compared to the existing ones. Of course, please note that these are just conservative bounds in the limit the single insertions are negligible; in their presence the bounds are further stringent.

In the present work, we have considered constraints only from the lepton flavour violating decays considering dipole transitions from gauge interactions. In addition to these processes, the double insertions could play a role in EDMs also [30]. The large mass of the fourth generation particle can lead to enhanced contributions to the EDMs. Similarly, Higgs mediated diagrams [36–38] could have transitions with double insertions. The LLRR insertion as in the present case could have enhanced contribution due to the large fourth generation mass insertion compared to its third generation counterpart, however they may be suppressed due to the low $\tan\beta$ requirement of MSSM4. The interplay between these two effects need to be explored.

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Appendix A. Loop functions

In this appendix we will give the explicit form of loop functions appearing in amplitudes:

$$f_{2n}(x) = \frac{-5x^2 + 4x + 1 + 2x(x+2)\ln x}{4(1-x)^4}$$

$$f_{3n}(x) = \frac{1 + 2x\ln x - x^2}{2(1-x)^3}$$

$$I_{1n}(x) = \frac{3x^4 + 44x^3 - 36x^2 - 12x + 1 - 12x^2(2x+3)\ln x}{24(1-x)^6}$$

$$I_{2n}(x) = \frac{x^3 + 9x^2 - 9x - 1 - 6x(x+1)\ln x}{4(1-x)^5}$$

$$I_{1c}(x) = \frac{10x^3 + 9x^2 - 18x - 1 - 3x(3 + 6x + x^2)\ln x}{6(1-x)^6}$$

$$I_{2c}(x) = \frac{3x^2 - 3 - (x^2 + 4x + 1)\ln x}{(1-x)^5}$$

$$I_{2(c,n)}(x, y) = I_{2(c,n)}(x) - I_{2(c,n)}(y) \tag{12}$$

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