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# The ${}^{14}N(p, \gamma){}^{15}O$ reaction, solar neutrinos and the age of the globular clusters

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#### Abstract

We discuss implications of a new measurement of  ${}^{14}N(p,\gamma){}^{15}O$  concerning solar neutrinos, solar models and globular cluster dating. Predictions for the gallium and chlorine experiments are reduced by 2 and 0.1 SNU, respectively. Predictions for helioseismic observables are unchanged within the present uncertainties. The ages of globular clusters deduced from the turn-off luminosity are increased by about 0.7 Gyr.

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### 1. Introduction

The last few years have seen a remarkable progress in our understanding hydrogen core burning in stars: the solution of the solar neutrino puzzle provided by the SNO [1] and KamLAND [2] experiments has allowed a precise determination of the <sup>8</sup>B neutrino flux, in agreement with the theoretical prediction of Standard Solar Model (SSM) calculations. The predicted signals for gallium and chlorine detectors are also in good agreement with SSMs, once the survival probability of electron neutrinos is calculated according to the Large Mixing Angle (LMA) solution, see, e.g., [3].<sup>1</sup> In this way, the nuclear energy source of the Sun, proposed by Eddington [5], Bethe [6] and von Weizsäcker [7] long ago, has been checked with observation: the production rate of nuclear energy in the Sun, as deduced from neutrino observations, agrees with the observed photon luminosity to within about 20% [3,8].

On the laboratory side, recent experiments have provided refined measurements of nuclear cross sections relevant to hydrogen burning: the  ${}^{3}\text{He}({}^{3}\text{He}, 2p){}^{4}\text{He}$ astrophysical *S*-factor has been measured well in the solar Gamow peak with 6% accuracy [9] and that of  ${}^{7}\text{Be}(p, \gamma){}^{8}\text{B}$  is presently known with an accuracy of

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<sup>&</sup>lt;sup>1</sup> Actually, there is a slight ( $\simeq 2\sigma$ ) tension between prediction and measurement for the chlorine experiment [3,4].

about 5% as a result of several recent measurements, see [8] and references therein.

Although providing precise information concerning the *pp* chain, neutrino observations and laboratory experiments shed little light on the role of the CNO bicycle in the Sun. This cycle, producing a tiny ( $\approx 1.5\%$ according to SSM) contribution to the solar energy output, is most important in more advanced burning phases, where it produces a large fraction of the stellar luminosity by hydrogen burning in shells as soon as hydrogen is exhausted at the stellar center.

The ages of globular clusters, the oldest systems in the Galaxy, are determined by locating the turning point on the Hertzsprung–Russell (HR) diagram, i.e., the point which signals hydrogen exhaustion at the stellar center. The efficiency of the CNO cycle is thus expected to be relevant for globular cluster dating.

No experiment aimed at a direct determination of CNO neutrinos from the Sun has been performed so far. Gallium and chlorine experiments are sensitive to neutrinos from the CNO cycle, however these reactions provide an indistinguishable contribution to the total signal that is dominated by neutrinos from the pp chain. A combined analysis of all solar and reactor neutrino experiments only provides an upper bound for the CNO contribution to solar luminosity,  $L(\text{CNO})/L_0 < 7.3\%$  at  $3\sigma$  [10].

As is well known, the key reaction for deciding the efficiency of the CNO cycle (see Fig. 1) is the radiative proton capture  ${}^{14}N(p,\gamma){}^{15}O$ . It has been measured by several groups in the last fifty years, but only the measurements of Schröder et al. [11] extend over a wide energy range, above  $E_{\rm cm} = 200$  keV. For the extrapolation to lower energies, the effect of a subthreshold resonance at  $E_{\rm r} = -504$  keV is important. The zero energy astrophysical *S*-factor recommended by the NACRE compilation [12]

$$S_{\text{NACRE}} = (3.2 \pm 0.8) \text{ keV b}$$
 (1)

is mainly based on data from [11]. This value, which we shall refer to as the SSM value, is essentially the same as the estimate in the compilation by Adelberger et al. [13],  $S = (3.5^{+0.4}_{-1.6})$  keV b.

Angulo and Descouvemont [14] reconsidered the data of [11] within the framework of an R-matrix model. The S-factor which is extracted from their analysis is essentially halved with respect to that of NACRE. Thus, whereas all the S-factors for the pp



Fig. 1. The CNO bi-cycle.

chain have been determined with accuracy of 10% or better, until recently the key cross section for the CNO cycle had an uncertainty of a factor of two.

A new experiment, performed at the underground Gran Sasso Laboratory by the LUNA Collaboration, has just presented its first results [15]. The new value of the astrophysical *S*-factor,

$$S_{\text{LUNA}} = (1.7 \pm 0.2) \text{ keV b},$$
 (2)

is in good agreement with that found in Ref. [14].

The present Letter addresses the following questions:

- (i) what is the impact on solar neutrinos produced by change of a factor two in the astrophysical *S*-factor of  ${}^{14}N(p,\gamma){}^{15}O$ ?
- (ii) what are the consequences for the estimated globular cluster ages?

# 2. The Sun and ${}^{14}N(p, \gamma){}^{15}O$

As a first approximation, one expects that the produced neutrino fluxes<sup>2</sup> of CNO neutrinos are proportional to the astrophysical *S*-factor, since the production rates depend linearly on *S*. In contrast, the density of reacting nuclei and temperature are weakly dependent on it, being essentially fixed by the solar mass,

 $<sup>^{2}</sup>$  Here and in the following we refer to "produced neutrino fluxes" and "produced neutrino signals" to indicate predictions in the absence of oscillation.



Fig. 2. Produced O, N and *pp* fluxes. Crosses denote solar model results and the straight lines are linear fits. We use  $S_{\text{NACRE}} = 3.2 \text{ keV b}$ . The arrow corresponds to the new value  $S_{\text{LUNA}} = 1.7 \text{ keV b}$ .

composition and age and by the main mechanism the *pp* chain—for energy production.

A decrease by a factor two in S will thus produce a corresponding decrease of the produced CNO fluxes. Since the total produced neutrino flux is fixed by the observed solar luminosity, this must produce an increase of neutrinos from the pp chain.

This qualitative picture is confirmed by the solar models we have built with FRANEC [16] for different values of *S*, see Figs. 2 and 3. We find a linear dependence of the CNO produced neutrino flux which is essentially compensated by the variation of  $\Phi(pp)$ , the beryllium flux staying practically unchanged. The boron flux slightly decreases with decreasing *S*.

The sensitivity of the produced neutrino fluxes to changes of the physical and chemical inputs of SSM is usually parametrized in terms of power laws [8,17, 18]:

$$\Phi(i) = \Phi_{\text{NACRE}}(i) \left(\frac{S}{S_{\text{NACRE}}}\right)^{\alpha_i}.$$
(3)



Fig. 3. Produced B, pep and Be fluxes. Same notation as in Fig. 2.

Table 1Slope of the flux dependence on S

Source	a <sub>i</sub>
рр	-0.013
рер	-0.018
Be	-0.003
В	+0.018
Ν	+0.875
0	+1.008

These laws are valid for small changes of the input with respect to the adopted NACRE value. In our case it is preferable to use a linear parametrization:

$$\Phi(i) = \Phi_{\text{NACRE}}(i) \left[ 1 + a_i \frac{(S - S_{\text{NACRE}})}{S_{\text{NACRE}}} \right].$$
(4)

The values of  $a_i$ , obtained from least square fitting, are presented in Table 1.

The produced signal rates  $R_{\rm pr}$  in radiochemical experiments are defined as:

$$R_{\rm pr} = \sum_{i} \sigma(i) \Phi(i), \tag{5}$$

where  $\sigma(i)$  is the capture cross section of the *i*th neutrinos in the detector. Their dependence on S is shown

Table 2 Produced neutrino fluxes and signals with  $S_{NACRE} = 3.2$  keV b. All other inputs as in [18]

Source	Flux $[10^9 \text{ cm}^{-2} \text{ s}^{-1}]$	Cl [SNU]	Ga [SNU]
pp	59.99	0	70.3
рер	0.142	0.227	2.89
Be	4.52	1.08	32.4
В	$5.21 \times 10^{-3}$	5.94	12.5
Ν	0.515	0.0875	3.13
0	0.437	0.297	4.97
Total		7.64	126.3

Table 3 Produced neutrino fluxes and signals when  $S_{\text{NACRE}} \rightarrow S_{\text{LUNA}}$ 

Source	Flux $[10^9 \text{ cm}^{-2} \text{ s}^{-1}]$	Cl [SNU]	Ga [SNU]
рр	60.33	0	70.7
pep	0.143	0.229	2.92
Be	4.53	1.09	32.5
В	$5.17 \times 10^{-3}$	5.90	12.4
Ν	0.305	0.0518	1.84
0	0.226	0.154	2.57
Total		7.43	123.2

in Tables 2 and 3 and Fig. 4. One sees that the linear S-dependence of the fluxes translates into a similar behaviour of the signals. For gallium detectors, the reduction of S requires CNO neutrinos to be replaced with a similar number of lower energy pp neutrinos. Since the cross section decreases when energy decreases, the resulting signal is smaller. Note that ppneutrinos are below threshold for chlorine detectors, which are sensitive to B, Be and CNO neutrinos. For this reason, a decrease of CNO neutrinos results in a signal decrease.

These effects are enhanced when considering the effective signal rates, which include neutrino oscillations,

$$R = \sum_{i} \sigma(i) \Phi(i) P_{ee}(i), \tag{6}$$

where  $P_{ee}(i)$  is the average survival probability of *i*th neutrinos. The LMA solution predicts that  $P_{ee}$  decreases with energy, so the contribution of lower energy neutrinos is larger in the effective signal than in the produced signal.



Fig. 4. Produced neutrino signals in radiochemical experiments. Same notation as in Fig. 2.

For gallium and chlorine experiments the change  $S_{\text{NACRE}} \rightarrow S_{\text{LUNA}}$  results in a signal reduction  $\Delta R(\text{Ga}) = 2$  SNU and  $\Delta R(\text{Cl}) = 0.1$  SNU. This alleviates, but does not eliminate, the slight discrepancy between the chlorine result and the LMA + SSM prediction noticed in [4] and [3].

Complementary to neutrinos, helioseismic observations provide us with a detailed view of the solar interior. By means of helioseismology one can reconstruct the sound speed profile inside the Sun, see, e.g., [19]. SSM calculations, including gravitational settling and elemental diffusion, agree well with helioseismic data for metal abundance  $(Z/X)_{\rm ph} \simeq 0.023-$ 0.026, see Figs. 5 and 6. However, recent downward revisions of the solar photospheric C, N and O abundances imply  $(Z/X)_{\rm ph} \simeq 0.018$ , [21]. These lower abundances have a profound impact on solar structure and evolution and the corresponding models look in conflict with helioseismic results, see [22]. The origin and the implications of this discrepancy is presently debated.



Fig. 5. Comparison between SSM calculation ( $u_{SSM}$ ) from [16,20] and the helioseismic determination ( $u_{sun}$ ) for the squared isothermal sound speed  $u = P/\rho$ . The dark (light) shaded area corresponds to the 1 $\sigma$  (3 $\sigma$ ) uncertainty on helioseismic determination [19].



Fig. 6. Relative change (model-SSM)/SSM of the squared isothermal sound speed  $u = P/\rho$  as a function of the radial coordinate, for the indicated values of  $S/S_{\text{NACRE}}$ . The dark (light) shaded area corresponds to the  $1\sigma$  ( $3\sigma$ ) uncertainty on helioseismic determination [19].

Since the efficiency of the CNO cycle is important for energy generation in the innermost part of the Sun, the core sound speed should be sensitive to *S*. On the other hand, models with  $S > 5S_{\text{NACRE}}$  are excluded at  $\ge 3\sigma$ . In other words, the helioseismic limit to the energy generation rate by the CNO cycle in the Sun is  $L(\text{CNO})/L_0 < 7.5\%$ , comparable to the bound obtained in [3] from solar neutrino experiments.

## 3. Globular clusters and ${}^{14}N(p, \gamma){}^{15}O$

The ages of globular clusters are extremely important for understanding the galactic evolution and to give a firm lower limit to the Galaxy formation epoch.

The effect of the  ${}^{14}N(p, \gamma){}^{15}O$  cross section on the evolutionary characteristics of population II stars has been analyzed in [23]. To assess the impact of the LUNA result, we repeat and extend these calculations using a version of the FRANEC code containing updated physical inputs, see [25,26], and including microscopic diffusion of He and heavy elements. The energy transport by convection is treated within a mixing length approach [27] and its efficiency is calibrated so as to reproduce the Red Giant Branch (RGB) color of globular clusters with different chemical composition. The resulting models reproduce the color–magnitude diagram of well observed globular clusters as M68, M3 and M13 [26].

The present calculations have been made for two chemical compositions (Z = 0.0002, Y = 0.230 and Z = 0.001, Y = 0.232) which are representative of the globular cluster population.

The main parameter providing the age of a stellar cluster is the luminosity at turn-off,  $L_{\text{TO}}$ , in the HR diagram, see, e.g., [28]. Stellar evolution theory predicts the behaviour of  $L_{\text{TO}}$  as a function of cluster age *t* depending on the physical inputs adopted in the calculations, in particular on the assumed value of *S*.

In Fig. 7 we show the 12 Gyr isochrones obtained for different values of the  ${}^{14}N(p,\gamma){}^{15}O$  astrophysical factor, with the same chemical composition (Z =0.001, Y = 0.232). By decreasing S,  $L_{TO}$  increases. The Turn-Off (TO) marks the exhaustion of central hydrogen and the onset of the hydrogen burning in shells, which occurs mainly through CNO cycle. If the CNO efficiency is reduced, this onset is delayed and



Fig. 7. Isochrone dependence on S. The calculated luminosity L, in units of the solar luminosity  $L_0$ , is presented as a function of the effective temperature  $T_e$  in Kelvin.



Fig. 8. Luminosity at turn-off  $L_{\text{TO}}$  as a function of the cluster age *t*. Points are the results of evolutionary calculations, continuous curves correspond to linear fits in the log  $L_{\text{TO}}$ -log *t* plane.

TO occurs at a later time and with a larger luminosity [23].

The dependence is shown more quantitatively in Fig. 8 for the higher metallicity composition (the low metallicity case looks similar). By halving *S* the same value of  $L_{\text{TO}}$  corresponds to an age increase<sup>3</sup>  $\Delta t \simeq 0.7$  Gyr.

This approach assumes that  $L_{TO}$  can be fixed from observations, independently of *S*. Actually the determination of  $L_{TO}$  requires the knowledge of the cluster distance modulus, which is often obtained by using RR Lyrae stars in the Horizontal Branch (HB)<sup>4</sup> as standard candles. In this case the relevant observable is the ratio of  $L_{TO}$  to the HB luminosity,  $L_{HB}$ , which is independent of the cluster distance. A frequently used variable for determining the cluster age is defined as:

$$\log L_{\rm HB-TO} = \log(L_{\rm HB}/L_{\rm TO}). \tag{7}$$

As discussed in [23], variations of *S* also affect the HB stars, which are powered by He burning in the core and by H burning in a surrounding shell, mainly through the CNO cycle. A decrease of *S* has two competing effects: it decreases the CNO cycle efficiency, decreasing  $L_{\text{HB}}$ , and at the same time it produces an increase of the helium core mass at He ignition, increasing  $L_{\text{HB}}$ . Helium burning starts when temperature in the helium core reaches values near  $10^8$  K. The rise of temperature in the core is driven by the growth of the core mass. A smaller *S* leads to a less efficient hydrogen burning in the Red Giant phase and, thus, to a slower growth of the core. This translates into a less efficient heating of the core and thus into a delayed He ignition with a larger He core mass [36].

The net effect depends on the cluster metallicity. For low metallicities (Z = 0.0002) we find that a decrease of *S* by a factor two leads to an increase  $\Delta \log L_{\rm HB} \sim 0.01$ . For moderately metal-rich HB stars (Z = 0.001), where CNO burning is more important, the same variation produces instead a decrease  $\Delta \log L_{\rm HB}$  of about the same amount. This means that when using log  $L_{\rm HB-TO}$  as an age indicator, the LUNA result leads to an increase of the estimated age which depends on the cluster metallicity: we obtain  $\Delta t \sim 0.5$  Gyr for Z = 0.0002 and  $\Delta t \sim 1$  Gyr for Z = 0.001.

The determination of globular clusters ages is presently affected by several uncertainties resulting from the chemical composition, from the adopted physical inputs and from the efficiency of various

<sup>&</sup>lt;sup>3</sup> A similar conclusion has been obtained by Straniero et al. in Ref. [24].

<sup>&</sup>lt;sup>4</sup> For the sake of precision, our candles are provided by the HB lower envelope (Zero Age Horizontal Branch, ZAHB) in the RR Lyrae region.

physical mechanisms (e.g., microscopic diffusion). Additional uncertainties arise from the comparison between theoretical and observed luminosities, see, e.g., [29–32].

A precise determination of the overall uncertainty is thus difficult. The cluster age determined from the absolute value of  $L_{\rm TO}$  (i.e., assuming that the cluster distance is known in an independent way) has an uncertainty of ~ 1.5 Gyr [25,29,32]. If  $\log L_{\rm HB-TO}$  is used as an age indicator, the uncertainty is ~ 2.0 Gyr, see [25,32]. The increased the globular cluster ages following from the LUNA result are thus within the error bar of the present determinations. Nevertheless the new and more precise value of *S* will be important when better astrophysical inputs will be available.

We conclude this section by discussing the effect on another interesting evolutionary feature of globular clusters: the so-called RGB bump, a region of the HR cluster diagram with higher star density. The RGB bump corresponds the momentanous decrease of the stellar luminosity in RGB which marks the encounter of the CNO H-burning shell with the chemical discontinuity produced by the first dredge-up, see, e.g., [37–39]. We find that a reduction of *S* by a factor two leads to an increase of the bump luminosity of about  $\Delta \log L_{\text{bump}} \sim 0.02$ , which is well within the estimated theoretical and observational uncertainties on this quantity, see, e.g., [33–35].

# 4. Concluding remarks

We summarize here the main points of this Letter:

- The LUNA result on the astrophysical *S*-factor for  ${}^{14}N(p, \gamma){}^{15}O$  implies that SSM + LMA predictions for the gallium and chlorine experiments are reduced by 2 and 0.1 SNU, respectively. This alleviates the slight tension between theory and chlorine result.
- The new *S* value does not change significantly helioseismic observables.
- On the other hand, helioseismology excludes a CNO contribution to solar luminosity larger than 7.5%.
- The age of globular clusters is increased by a quantity 0.5–1 Gyr, depending on the method for

determining the turn-off luminosity and on the cluster metallicity.

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