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Propagation of SH waves in a visco-elastic layer overlying an inhomogeneous isotropic half-space

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Abstract This paper is concerned with the propagation of SH-waves in an inhomogeneous viscoelastic layer overlying an inhomogeneous isotropic half-space. For the study of viscoelastic layer *Kelvin–Voigt* type model has been considered. The exponential type inhomogeneity is considered in viscoelastic layer with different parameters for viscoelastic part and for density of medium whereas for the half-space, it is quadratic type with single inhomogeneity parameter for rigidity and density. The solutions are obtained analytically for both the layer and half-space. Dispersion relation is obtained and subjected to continuity conditions at interfaces of viscoelastic layer and half-space and the upper boundary of layer as free surface. The effects of the inhomogeneity parameter are studied for both viscoelastic layer and inhomogeneous isotropic half-space. The numerical results are shown by plotting the graph between phase velocity and wave number for different values of inhomogeneity parameter and for the variation of depth of the layer.

1. Introduction

The problems of propagation of SH-waves in an isotropic and inhomogeneous elastic half-space are of great practical importance. They are not only helpful in investigating the internal structure of the Earth but also very helpful in exploration of natural resources buried inside the Earth like oil and gases and other hydrocarbon and minerals, etc. Earth is also highly inhomogeneous and some materials also exhibit viscoelastic properties. Materials such as coal-tar, salt, and sediments that are buried beneath the Earth surface can be modelled as viscoelastic materials. The general theory of viscoelasticity describes the liner behaviour of both elastic and anelastic materials and provides the basis for describing the attenuation of seismic waves due to anelasticity. When seismic waves propagate underground then these are not only influenced by anisotropy of the medium but are also influenced by the intrinsic viscosity of the medium [1]. Das and Sengupta [2] discussed the surface wave propagation in general viscoelastic media of higher order. They considered the general theory of surface waves in higher order viscoelastic solid containing time rate of strain and investigated the particular surface waves of Rayleigh, Love and Stoneley type. Borocherdt [3] discussed an excellent exposition to motion of seismic waves in arbitrary linear viscoelastic media under different geometries.

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Chattopadhyay et al. [4,5] treated the propagation of SH-waves in a viscoelastic medium with irregularity in different geometries. Recently, Sethi et al. [6] and Kakar et al. [7] attempted to study the nature of torsional wave propagation in viscoelastic media; however, they have taken the real wave number, while the hypothesis of real value of wave number is not valid for viscoelastic medium.

The earth crust is highly anisotropic and inhomogeneous in nature. The characteristics of medium such as elastic moduli, density, thermal conductivities are not homogeneous throughout the medium. So, by considering different types of depth dependent inhomogeneity, the study can be made more close to the real scenario of seismic wave propagation. Authors such as Wilson [8], Deresiewicz [9], Dutta [10], Pal and Acharya [11], Pal et al. [12], Kumar and Pal [13] considered the exponential type inhomogeneity for the study of seismic wave propagation in their problems while Chakrabarty and De [14], Gazetas [15], Dey et al. [16], Kumar et al. [17] considered quadratic inhomogeneity. Recently authors such as Kundu et al. [18,19] and Zorammuana and Singh [20] studied SH-wave propagation in inhomogeneous media.

This problem is considered on the basis that the layered material inside possesses stress and they are not uniform throughout the space domain. The stresses of material change over time and depth. So, the layer of the problem is considered the viscoelastic material in which stress is time dependent and also depth dependent whereas in the half-space the stresses are only depth dependent. Also, the density of the material is not homogeneous throughout the medium. The depth-dependent densities are chosen for both layer and half-space. This study investigates the effects of various parameters namely inhomogeneity parameter, layer width, wave number and phase velocity. The numerical results are shown by plotting the graph between phase velocity and wave number for different values of inhomogeneity parameter and depth of layer.

2. Formulation of the problem

Consider an inhomogeneous viscoelastic layer of finite thickness h and an inhomogeneous isotropic half-space. The interface of these two media is taken at $z = 0$ whereas the free surface is at $z = -h$. Here z-axis is taken positive along vertically downwards direction and x-axis is assumed in the direction of wave propagation with velocity c. The geometrical configuration is depicted in Fig. 1. For the layer, the inhomogeneity parameter for viscoelastic coefficients is α and density is β whereas for half-space inhomogeneity parameter for rigidity and density is γ. For SH-type of waves the displacement and body forces do not depend on y and if (u, v, w) be the displacement at any point $P(x, y, z)$ into the medium then

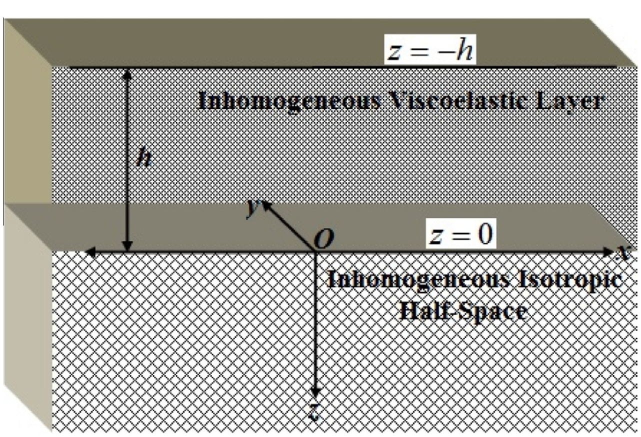

Figure 1 Geometry of the problem.

$$u \equiv w = 0, \quad v = v(x, z, t) \text{ and } \frac{\partial}{\partial y} \equiv 0 \tag{1}$$

3. Solution of the problem

3.1. Inhomogeneous viscoelastic layer

The general equation of motion for the SH-waves propagation without body force for the layer is given by,

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} = \rho_1 \frac{\partial^2 v_1}{\partial t^2} \tag{2}$$

where ρ_1 is the density of the medium, τ_{xy} and τ_{yz} are stress components and v_1 is the displacement component in y direction in the layer. The stress–strain relations as considered by Flugge [21] for viscoelastic layer of *Kelvin–Voigt* type model and inhomogeneity for both viscoelastic coefficients and density parameter are taken as considered by Wilson [8]

$$\tau_{yz} = e^{-\alpha z} D_\mu \frac{\partial v_1}{\partial z}, \quad \tau_{xy} = e^{-\alpha z} D_\mu \frac{\partial v_1}{\partial x}, \quad \rho_1 = \rho_0 e^{\beta z} \tag{3}$$

where $D_\mu = \sum_{j=0}^{n} \mu_j \frac{\partial^j}{\partial t^j}$ and α, β are inhomogeneity parameters whose unit is inverse of length.

Substituting Eq. (3) in Eq. (2) we have

$$D_\mu \left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2} - \alpha \frac{\partial v_1}{\partial z} \right) = \rho_0 e^{(\alpha+\beta)z} \frac{\partial^2 v_1}{\partial t^2} \tag{4}$$

Assuming the solution as $v_1(x, z, t) = V_1(z) e^{ik(x-ct)}$ and substituting in Eq. (4) we have,

$$\frac{d^2 V_1}{dz^2} - \alpha \frac{dV_1}{dz} - k^2 \left(1 - \frac{c^2}{c_1^2} e^{(\alpha+\beta)z} \right) V_1 = 0 \tag{5}$$

where $c_1 = \sqrt{\frac{D'_\mu}{\rho_0}}$ and $D'_\mu = \sum_{j=0}^{n} \mu_j (-ikc)^j$.

To eliminate the first order derivative $\frac{dV_1}{dz}$, we substitute $V_1(z) = e^{\alpha z/2} \phi_1(z)$ in Eq. (5) we get,

$$\frac{d^2 \phi_1}{dz^2} - \left(\frac{\alpha^2 + 4k^2}{4} - \frac{k^2 c^2}{c_1^2} e^{(\alpha+\beta)z} \right) \phi_1 = 0 \tag{6}$$

Considering new variable $\chi(z) = e^{(\alpha+\beta)z}$, Eq. (6) becomes

$$\chi^2 \frac{d^2 \phi_1}{d\chi^2} + \chi \frac{d\phi_1}{d\chi} + (m^2 \chi - l^2) \phi_1 = 0 \tag{7}$$

where $l = \frac{\sqrt{\alpha^2 + 4k^2}}{2(\alpha+\beta)}$ and $m = \frac{kc}{c_1(\alpha+\beta)}$.

Eq. (7) is well known Bessel's equation and its solution is given by

$$V_1(z) = e^{\alpha z/2} \left[C J_l(m\sqrt{\chi}) + D Y_l(m\sqrt{\chi}) \right] \tag{8}$$

where C, D are constants, $J_l(m\sqrt{\chi})$ and $Y_l(m\sqrt{\chi})$ are Bessel's function of order l of first and second kind respectively.

Hence, the displacement and stress component in inhomogeneous viscoelastic layer are given by

$$v_1(x,z,t) = V_1(z)e^{ik(x-ct)} = e^{\alpha z/2}\left[CJ_l\left(me^{\frac{(\alpha+\beta)z}{2}}\right) + DY_l\left(me^{\frac{(\alpha+\beta)z}{2}}\right)\right]e^{ik(x-ct)} \tag{9}$$

$$(\tau_{yz})_I = e^{-\alpha z}D_\mu \frac{\partial v_1}{\partial z}, \quad (\tau_{xy})_I = e^{-\alpha z}D_\mu \frac{\partial v_1}{\partial x} \tag{10}$$

3.2. Inhomogeneous half-space

The equation of motion for SH-wave in the inhomogeneous isotropic half-space is given as

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} = \rho_2 \frac{\partial^2 v_2}{\partial t^2} \tag{11}$$

where ρ_2 is the density of the medium, τ_{xy} and τ_{yz} are stress components and v_2 is the displacement component in y direction in half-space. The stress–strain relation and density for the inhomogeneous isotropic half-space are considered following Dey et al. [16]

$$\tau_{xy} = \mu_0'(1+\gamma z)^2 \frac{\partial v_2}{\partial x}, \quad \tau_{yz} = \mu_0'(1+\gamma z)^2 \frac{\partial v_2}{\partial z},$$
$$\rho_2 = \rho_0'(1+\gamma z)^2 \tag{12}$$

where γ is the inhomogeneity parameter and its unit is inverse of length.

Now substituting Eq. (12) in Eq. (11) we get,

$$\frac{\partial^2 v_2}{\partial x^2} + \frac{2\gamma}{1+\gamma z}\frac{\partial v_2}{\partial z} + \frac{\partial^2 v_2}{\partial z^2} = \frac{1}{c_2^2}\frac{\partial^2 v_1}{\partial t^2} \tag{13}$$

where $c_2 = \sqrt{\frac{\mu_0'}{\rho_0'}}$.

Assuming the solution of Eq. (13) as $v_2(x,z,t) = V_2(z)e^{ik(x-ct)}$ and substituting we have

$$\frac{d^2 V_2}{dz^2} + \frac{2\gamma}{1+\gamma z}\frac{dV_2}{dz} - k^2 s_2^2 V_2 = 0 \tag{14}$$

where $s_2 = \sqrt{1 - \frac{c^2}{c_2^2}}$.

To eliminate first order derivative $\frac{dV_2}{dz}$, we substitute $V_2 = \frac{\phi_2(z)}{1+\gamma z}$ in Eq. (14), we have

$$\frac{d^2\phi_2}{dz^2} - k^2 s_2^2 \phi_2 = 0 \tag{15}$$

The solution of Eq. (15) is given by

$$\phi_2(z) = Ae^{-ks_2 z} + Be^{ks_2 z} \tag{16}$$

where A, B are constants.

The displacement component is bounded, as $z \to \infty$ the second term of Eq. (16) makes the solution unbounded. Therefore, the approximate solution of Eq. (16) is given as

$$\phi_2(z) = Ae^{-ks_2 z} \tag{17}$$

Hence, the displacement and stress component in the half-space are given by

$$v_2(x,z,t) = V_2(z)e^{ik(x-ct)} = \frac{Ae^{-ks_2 z}}{1+\gamma z}e^{ik(x-ct)} \tag{18}$$

$$(\tau_{xy})_{II} = \mu_2 \frac{\partial v_2}{\partial x}, \quad (\tau_{yz})_{II} = \mu_2 \frac{\partial v_2}{\partial z} \tag{19}$$

4. Boundary conditions

We assume that layer is in contact with free surface at $z = -h$ and the interface of layer and half-space is in welded contact. Mathematically, these boundary conditions can be expressed as follows:

$$(\tau_{yz})_I = 0, \quad \text{at } z = -h \tag{20}$$

$$(\tau_{yz})_I = (\tau_{yz})_{II} \quad \text{at } z = 0 \tag{21}$$

$$v_1 = v_2 \quad \text{at } z = 0 \tag{22}$$

Using boundary conditions (20)–(22) and Eqs. (9), (10), (18) and (19), we get

$$[\alpha J_l(P) + PQ\{J_{l-1}(P) - J_{l+1}(P)\}]C + [\alpha Y_l(P) + PQ\{Y_{l-1}(P) - Y_{l+1}(P)\}]D = 0 \tag{23}$$

$$-A + CJ_l(m) + DY_l(m) = 0 \tag{24}$$

$$2\mu_0'(\gamma + ks_2)A + D_\mu'[\alpha J_l(m) + mQ\{J_{l-1}(m) - J_{l+1}(m)\}]C + D_\mu'[\alpha Y_l(m) + mQ\{Y_{l-1}(m) - Y_{l+1}(m)\}]D = 0 \tag{25}$$

where P, Q are given by

$$P = me^{-Qh}, \quad Q = \left(\frac{\alpha+\beta}{2}\right) \tag{26}$$

Eliminating A, C and D from Eqs. (23), (24) and (25), we get the dispersion relation for SH-type wave propagation in viscoelastic layer overlying inhomogeneous isotropic half-space subjected to free surface as upper boundary condition in third order determinant form

$$|a_{ij}| = 0, \quad \forall i,j = 1,2,3 \tag{27}$$

where a_{ij} the entries of third-order determinants are as follows

$$a_{11} = 0, \quad a_{12} = \alpha J_l(P) + PQ\{J_{l-1}(P) - J_{l+1}(P)\},$$
$$a_{13} = \alpha Y_l(P) + PQ\{Y_{l-1}(P) - Y_{l+1}(P)\}$$
$$a_{21} = -1, \quad a_{22} = J_l(m), \quad a_{23} = Y_l(m)$$
$$a_{31} = 2\mu_0'(\gamma + ks_2), \quad a_{32} = D_\mu'[\alpha J_l(m) + mQ\{J_{l-1}(m) - J_{l+1}(m)\}],$$
$$a_{33} = D_\mu'[\alpha Y_l(m) + mQ\{Y_{l-1}(m) - Y_{l+1}(m)\}]$$

5. Particular cases

Case I: When $h \to -\infty$, the problem reduces to propagation of SH-wave in inhomogeneous viscoelastic half-space overlying inhomogeneous isotropic half-space. Then the displacement component for half-space from Eq. (9) can be written as

$$v_1(x,z,t) = De^{\alpha z/2}Y_l\left(me^{\frac{(\alpha+\beta)z}{2}}\right)e^{ik(x-ct)} \tag{28}$$

Since, the first term of solution get unbounded as $z \to -\infty$. Hence following the boundary condition (21) and (22), the dispersion relation for propagation of SH-wave in inhomogeneous viscoelastic half-space overlying inhomogeneous isotropic half-space can be obtained as

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$$\mu_0'(\gamma + ks_2) = \frac{D_\mu'}{2}\left[mQ\left\{\frac{Y_{l+1}(m) - Y_{l-1}(m)}{Y_l(m)}\right\} - \alpha\right] \tag{29}$$

Case II: When $h \to 0$, the problem reduces to propagation of SH-wave in inhomogeneous isotropic half-space. So, the dispersion relation for SH-wave in inhomogeneous half-space with stress free surface at interface at the vacuum i.e. $(\tau_{yz})_{II} = 0$ can be obtained from Eqs. (19) and (20) as

$$c = c_2\sqrt{1 - \frac{\gamma^2}{k^2}} \tag{30}$$

Case III: When $\alpha \to 0$, $\beta \to 0$ and $\gamma \to 0$, the problem reduces to SH-wave propagation in homogeneous viscoelastic layer overlying homogeneous isotropic half-space. The dispersion relation can be obtained as from Eq. (27)

$$\tan\left(kh\sqrt{\frac{c^2}{c_1^2} - 1}\right) = \frac{\mu_0'\sqrt{1 - \frac{c^2}{c_2^2}}}{D_\mu'\sqrt{\frac{c^2}{c_1^2} - 1}} \tag{31}$$

Case IV: When $\alpha \to 0$, $\beta \to 0$, $\gamma \to 0$ and $D_\mu' = \mu_0$, the problem reduces to SH-wave propagation in homogeneous isotropic layer overlying homogeneous isotropic half-space. The dispersion relation can be obtained as from Eq. (27)

$$\tan\left(kh\sqrt{\frac{c^2}{c_1^2} - 1}\right) = \frac{\mu_0'\sqrt{1 - \frac{c^2}{c_2^2}}}{\mu_0\sqrt{\frac{c^2}{c_1^2} - 1}} \tag{32}$$

which is the well-known classical equation of the Love wave in the elastic homogeneous isotropic layer overlying a homogeneous isotropic elastic half-space.

6. Numerical results and discussion

In order to show the effect of inhomogeneity parameter and dependency of phase velocity on wave number, we have taken data for inhomogeneous isotropic half-space from Babuska and Cara [22]

$\mu_0' = 218$ GPa, $\rho_0' = 4400\ \text{kg/m}^3$

For viscoelastic layer we have taken the following data from Kumar et al. [17]

$\mu_0 = 8.16$ GPa, $\mu_1 = 0.82$ GPa, $\mu_2 = 0.2$ GPa,
$\rho_1 = 3320\ \text{kg/m}^3$

The graphs are plotted separately for both real and imaginary parts for phase velocity against wave number. In Figs. 2 and 3, the graphs are plotted for real part of phase velocity i.e. $Re(c)$ and imaginary part of phase velocity i.e. $Im(c)$ against non-dimensional wave number k for different values of non-dimensional inhomogeneity parameter $\alpha(= 0.10, 0.15, 0.20)$ $\beta = 0.1$, $\gamma = 0.1$ and $h = 3$ km. It is observed that the phase velocity decreases with increasing wave number for both real and imaginary parts of the phase velocity. As we increase the inhomogeneity parameter the magnitude in phase velocity decreases but the nature of the curves is same. For increasing values of wave number and inhomogeneity parameter, phase velocity decreases. In Figs. 4 and 5, the graphs are plotted for real and imaginary parts of the phase velocity separately for different values of inhomogeneity parameter $\beta(= 0.10, 0.15, 0.20)$ $\alpha = 0.1$, $\gamma = 0.1$ and $h = 3$ km. From these figures, we observe

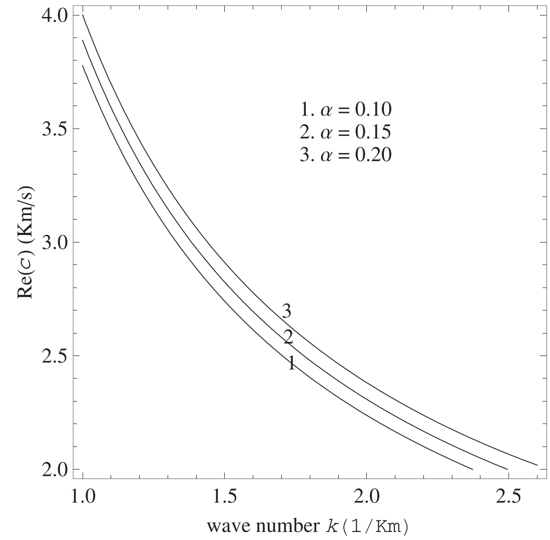

Figure 2 Variation of $Re(c)$ against wave number k for fixed value of β and h.

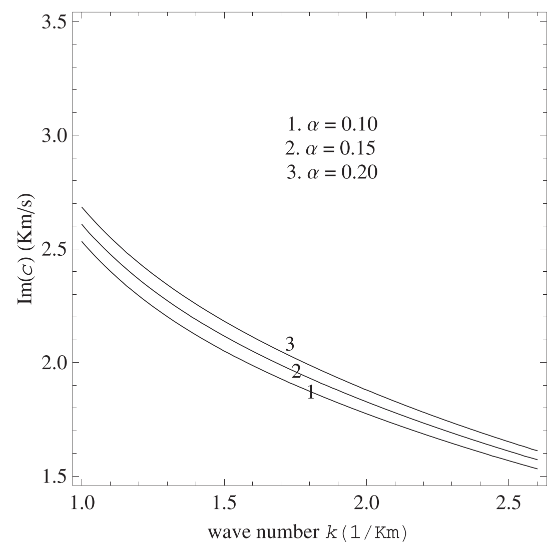

Figure 3 Variation of $Im(c)$ against wave number k for fixed value of β and h.

that the nature of the graphs is same as in Figs. 2 and 3. In Figs. 6 and 7 we plot the graph for different values of layer depth i.e. $h = (2.5\text{ km}, 3\text{ km}, 3.5\text{ km})$, and fixed value of inhomogeneity parameter $\alpha = 0.1$, $\beta = 0.1$ and $\gamma = 0.1$. Here we see that as we increase wave number phase velocity decreases for both the cases of real and imaginary part of the phase velocity but one thing differs from above figure that phase velocity increases with increasing depth. Due to less involvement of inhomogeneity parameter γ in the dispersion relation, it has very less effect on the phase velocity.

7. Conclusions

The propagation of SH-type surface waves is investigated in the inhomogeneous viscoelastic layer and inhomogeneous isotropic half-space. For the layer, viscoelastic coefficients

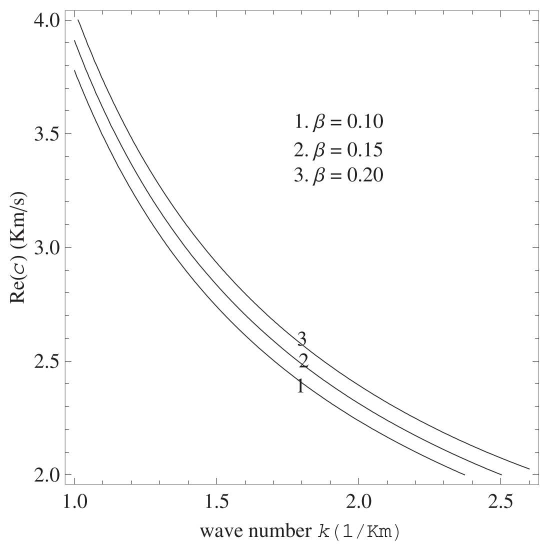

Figure 4 Variation of $Re(c)$ against wave number k for fixed value of α and h.

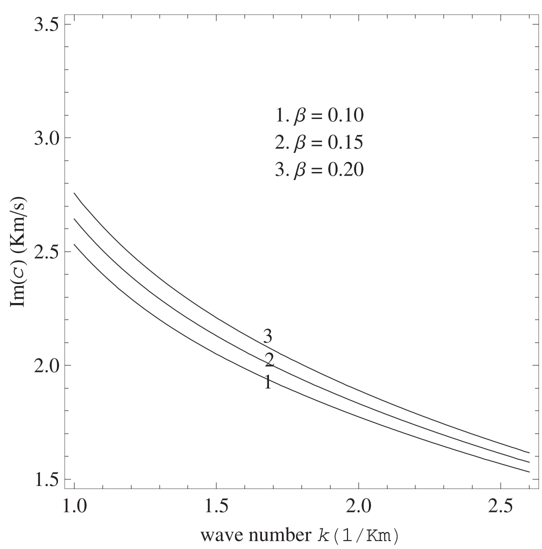

Figure 5 Variation of $Im(c)$ against wave number k for fixed value of α and h.

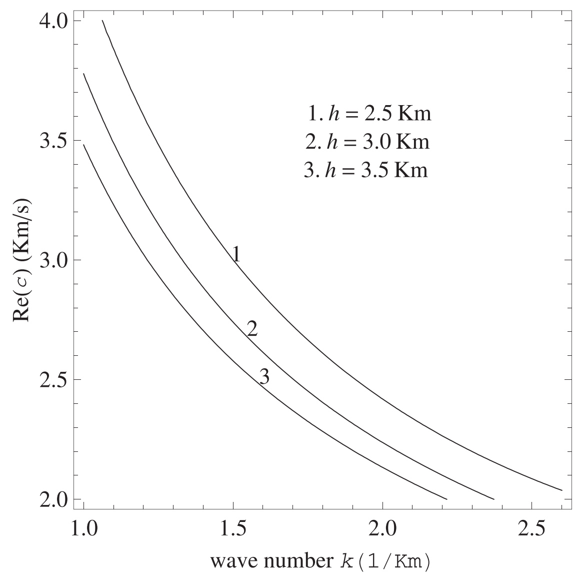

Figure 6 Variation of $Re(c)$ against wave number k for fixed value of α and β.

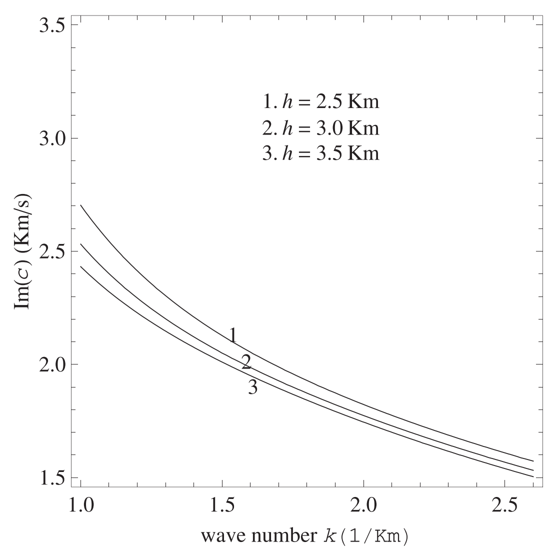

Figure 7 Variation of $Im(c)$ against wave number k for fixed value of α and β.

are taken as exponentially decaying function with depth whereas density is taken exponentially increasing with depth. The rigidity and density of isotropic half-space are taken as quadratically increasing function with depth. The solutions of both layer and half-space are obtained by variable separable technique, the solution in the layer leading to well known Bessel's function type. The dispersion relation is obtained subjected to continuity condition at interface and upper contact of layer as free surface. From the dispersion relation, it can be observed that there is sufficient involvement of the inhomogeneity parameters and depth of layer in dispersion relation to affect the propagation of SH-waves. The numerical results are obtained for particular model and the results are shown in the figures. The figures reveal the fact that the phase velocity for both real and imaginary parts decreases as the wave number increases but the magnitude of phase velocity decreases for increasing value of inhomogeneity parameter while it increases for increasing value of depth of the layer. This discloses the fact that heterogeneity of material and depth of layers have great impact on the velocity of seismic waves. By observing the records of seismographs for these phase velocities of seismic waves enables in predicting the nature of materials inside the earth. However, it is not easy to solve the problem for any type of inhomogeneity analytically and in nature the inhomogeneity of earth is random.

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