HISTORIA MATHEMATICA 20 (1993), 320–323

REVIEW

Edited by KAREN HUNGER PARSHALL

All books, monographs, journal articles, and other publications (including films and other multisensory materials) relating to the history of mathematics are abstracted in the Abstracts Department. The Reviews Department prints extended reviews of selected publications.

Materials for review, except books, should be sent to the editor of the Abstracts Department, Prof. David E. Zitarelli, Department of Mathematics, Temple University, Philadelphia, PA 19122, U.S.A. Books for review should be sent to the Book Review Editor, Prof. Karen Hunger Parshall, Department of Mathematics, Math/Astro Building, University of Virginia, Charlottesville, VA 22903-3199.

Most reviews are solicited. However, colleagues wishing to review a book are invited to make known their wishes. Comments on books, articles, or reviews should be submitted to the Correspondence Department. We also welcome retrospective reviews of older books. Colleagues interested in writing such reviews should consult first with the editor to avoid duplication.

Revolutions in Mathematics. Edited by Donald Gillies. Oxford (Clarendon Press). 1992. 353 pp. £55.

REVIEWED BY DAVID E. ROWE

Fachbereich 17—Mathematik, Universität Mainz, 6500 Mainz 1, Germany

Does mathematical knowledge build on itself continually over time, or does its historical fabric reveal abrupt and irregular breaks in an otherwise smooth and flowing pattern? Does it make sense to speak of revolutions in the history of mathematics, and, if so, what kinds of revolutions seem to occur? The present volume, a collection of essays written by a dozen historians and philosophers of mathematics, addresses this controversial issue from a variety of different perspectives. In a certain sense, it may also be seen as a welcome effort on the part of the editor, Donald Gillies, to reengage a debate and the historians who initiated it in the mid 1970s.

Gillies's introduction discusses the pertinent prehistory and the general plan of the book. This is followed by reprints of earlier essays written by Michael Crowe, Herbert Mehrtens, and Joseph Dauben. In the wake of Thomas Kuhn's influential study, *The Structure of Scientific Revolutions* [Kuhn 1962, 1970]—a work which itself sparked a revolution in the historiography of the natural sciences—these three historians staked out their own positions regarding the viability of Kuhn's model within the field of mathematics. Crowe rejected the idea of revolutionary change outright; Mehrtens found Kuhn's approach suggestive insofar as it empha-

0315-0860/93 $5.00

sized the social dimension of the creation of knowledge, but argued against its wider efficacy; Dauben largely distanced himself from the Kuhnian model, suggesting a different kind of analogy between periods of upheaval in mathematics and revolutionary events that transpire in the political arena.

The consensus of opinion, as presented here and in the new essays that follow (including more recent reflections by Crowe, Mehrtens, and Dauben), suggests that revolutions do, indeed, occur in the history of mathematics, but that these often cannot be understood in terms of the "paradigm shifts," "disciplinary matrices," or "exemplars" so central to Kuhn's theory. Thus, while Caroline Dunmore prefers to view abrupt changes in mathematics as taking place at a "metalevel," neither she nor any of the other contributors appears willing to subscribe to Crowe's "tenth law," according to which "revolutions never occur in mathematics." Nevertheless, Dunmore's views are far more consonant with Crowe's than are those of most of the other contributors.

This is particularly clear in the case of Luciano Boi. In examining the profound shift in conceptions of space that unfolded over the latter half of the nineteenth century, Boi explicitly rejects Crowe's previously conventional view, but he also argues against any attempt (like Kuhn's) to explain dramatic shifts in mathematical outlook by appealing to sociological factors operating within the research community. Boi's criticism of Crowe's position is directed toward a central point of contention in this debate. By denying revolutionary change *in* mathematics, Crowe seeks to differentiate between a stable body of mathematical knowledge proper and incidental factors such as the form in which mathematical ideas are represented. Thus, he contends that the content of mathematics remains essentially unaffected even amid revolutionary changes in "mathematical nomenclature, symbolism, metamathematics" (p. 19), and other related forms of expression. These factors may impinge upon mathematical developments, but Crowe regards them as lying "outside" (p. 19) mathematics proper.

Herbert Mehrtens addressed this issue in his 1976 paper (reprinted here) on "Kuhn's Theories and Mathematics," and in doing so he already pointed to the Achilles heel of Crowe's argument. For as tempting as it may seem to distinguish form from content in mathematics, there seem to be no practical criteria for making such a distinction and utilizing it to illuminate concrete historical situations. Any attempt to circumscribe the "substance" of mathematics and to set this off against some other (presumably less essential) concomitants of mathematical knowledge (for example, the language or symbolism used by particular authors in a given era) quickly leads to a quagmire of historical difficulties. (This point was forcefully made in connection with ancient mathematics by Sabetai Unguru in [Unguru 1975].)

While the present volume testifies to a new-found convergence of interests uniting historians and philosophers of mathematics, it also betrays a problem that continues to plague both disciplines, namely, the tendency to dramatize foundational issues and debates while overlooking the vast terrain that actually dominated the attention of leading researchers. Donald Gillies's introductory remark that the volume "deals with most of the major episodes in mathematical history from Descartes in the 1630s to Robinson in the 1960s" hinges, I suppose, on what is meant by a "major episode" (p. 8). Still, such a statement certainly suggests an extremely contracted view of the mathematical landscape over this lengthy and exceedingly rich period. This point can perhaps best be illustrated through some passing remarks on the individual contributions.

Several articles in the book represent case studies that seek to shed light on its general theme. Paolo Mancosu, for example, considers arguments for and against considering Descartes's *Géométrie* a revolutionary contribution to the history of mathematics, ultimately finding the negative side of the equation weightier. Emily Grosholz, on the other hand, marshals considerable evidence supporting the view that Leibniz was a truly revolutionary figure. Both studies are solid enough, but their narrow focus leads to an inevitable artificiality. Just as one cannot hope to frame an argument about the general nature of revolutions in mathematics on the basis of a few case studies (even with such pivotal figures as Descartes and Leibniz), so is it illusory to expect that one can gain any real insight into the revolutionary (or nonrevolutionary) character of a mathematician's work without first addressing the global issues involved. Thus, neither of these contributions offers any significant new findings with regard to the overall structure of mathematical revolutions.

An interesting attempt to analyze the "fine structure" of a particular revolution comes from Giulio Giorello. By examining the repercussions in Britain of the debates over the foundations of the calculus, Giorello invokes the idea of competing "paradigms of legimitation" (pp. 139–140). Whereas Kuhn emphasized the sense of loss and disorientation that normally accompanies such crises, Giorello stresses the creative role of a transitional figure like Colin Maclaurin, whose work mediated between an older geometrical paradigm and one grounded on modern analytical concepts. The latter "revolution in rigor" inaugurated by Cauchy receives brief attention in Dauben's "Revolutions Revisited," where it is treated as a prelude to a discussion of the impact of Abraham Robinson's nonstandard analysis.

Non-Euclidean geometry—a theme that has proven to be a perennial favorite for historians and philosophers of mathematics alike—is accorded its traditional place of honor in an article by Yuxin Zheng. Quoting Morris Kline with approval ("The creation of non-Euclidean geometry was the most consequential and revolutionary step in mathematics since Greek times") (p. 169), Zheng covers this well-trodden trail without turning up any new surprises.

The contributions of Herbert Breger and Donald Gillies also deal with particular aspects of the foundations of mathematics. Breger considers the little-known work of Paul Finsler on set theory and gives a thoughtful analysis of why Finsler's theory failed to find significant support. His argument, as noted by Gillies, echoes Giorello's claims about the role of "paradigms of legitimacy." Gillies own discussion of "The Fregean Revolution in Logic" follows an eclectic approach that has much in common with Kuhn's but also with Dauben's orientation.

Finally, the article by Caroline Dunmore, "Meta-level Revolutions in Mathematics," delves into nineteenth-century algebra a bit, but most of the discussion hovers well above the historical ground. Moreover, once again one encounters the philosophers' penchant for extrapolating from an analysis of a highly restricted set of mathematical results to a sweeping characterization of the mathematical enterprise as a whole. Here, as elsewhere throughout the volume, one wonders about the "disciplinary matrix" that produced the mathematics under discussion.

The most striking exception in this regard is Jeremy Gray's paper on "The Nineteenth-Century Revolution in Mathematical Ontology," which points the way to some promising new terrain. Drawing on developments in algebraic number theory and geometry, Gray suggests that the fundamental concepts employed by research mathematicians in these fields underwent a subtle, but radical shift over the course of the century. Rather than looking inward to account for these changes, however, he suggests that we view them as part of a large-scale transformation that transcended not only the particular disciplines involved but even the field of mathematics itself.

One of the strengths of Kuhn's theory resides in its unity. It offers an account of how revolutionary changes can explode within a discipline, yet, just as important, it provides a framework for understanding the standard phenomenon, "normal science," that is, the norms that guide scientific research during periods of stable development. To address the role of "revolutions" (Kuhnian or otherwise) in the history of mathematics, it would seem essential to have a fairly clear idea of what constituted "normal" research-level mathematics in a given era. Unfortunately, this side of the matter has received but scant attention in the historical literature. One can only hope that the present undertaking will at least inspire others to reconsider the overall development of mathematical knowledge both during its phases of relative normalcy and during those periods when new or rival ideas disrupted the scene. Part of the challenge this poses for historians of mathematics would be to identify the dominant fields of research and their practitioners, particularly during the nineteenth and early twentieth centuries when the discipline operated in a variety of new social, institutional, and professional contexts. Without a clearer picture of mainstream developments in modern mathematical research and the factors that sustained them, general discussions about the growth and development of mathematical knowledge are unlikely to lead to major new breakthroughs.

REFERENCES

Kitcher, P. S. 1983. *The nature of mathematical knowledge*. Oxford: Oxford Univ. Press.

Kuhn, T. S. 1962. *The structure of scientific revolutions*, 1st ed. Chicago: Univ. of Chicago Press.

——— 1970. *The structure of scientific revolutions*, 2nd ed. Chicago, Univ. of Chicago Press.

Lakatos, I., & Musgrave, A., Eds. 1970. *Criticism and the growth of knowledge*. Cambridge: Cambridge Univ. Press.

Mehrtens, H. 1990. *Moderne—Sprache—Mathematik*. Frankfurt: Suhrkamp.

Unguru, S. 1975. On the need to rewrite the history of Greek mathematics. *Archive for History of Exact Sciences* **15,** 67–114.