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ORIGINAL ARTICLE

MHD flow over exponential radiating stretching sheet using homotopy analysis method

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Abstract An analytical solution for MHD boundary layer flow of a viscous incompressible fluid over an exponentially stretching sheet is developed in this study. The effect of thermal radiation is included in the energy equation. Through suitable similarity transformations, the governing equations are transformed into a system of nonlinear ordinary differential equations. Homotopy analysis method (HAM) has been used to get accurate and complete analytic solution. This study reveals that the governing parameters, namely, the magnetic and the radiation parameters have major effects on the flow field, skin friction coefficient, and the heat transfer rate. The magnetic field enhances the dimensionless temperature inside the thermal boundary layer whereas reduces the dimensionless velocity inside the hydrodynamic boundary layer. Heat transfer rate becomes low with magnetic and radiation parameters while the friction factor is increased with magnetic field. Moreover, a comparative study between the previously published and the present results in special cases is conducted and an excellent agreement is found between them.

1. Introduction

The boundary layer flow and heat transfer of a viscous fluid over flat surfaces have been investigated in several technological processes such as hot rolling, metal extrusion, continuous stretching of plastic films and glass-fiber, polymer extrusion, wires drawing and metal spinning. Various researchers are engaged in this rich area. Sakiadis (1961) is the pioneer for investigation of boundary layer flow over a stretched surface moving with a constant velocity and developed the boundary layer equations for axisymmetric flows in two-dimensions. Erickson et al. (1966) extended the work of Sakiadis (1961) with addition of suction and injection at a stretched surface moving with constant velocity and investigated the effects on flow and heat transfer. Numerous physical phenomena related to stretched sheet moving with constant velocity under various thermal conditions have been investigated by Carragher and Crane (1982), Grubka and Bobba (1985) and Magyari and Keller (2000).

A good effort has been made to gain insight information regarding the stretching flow problem in various situations. Such situations include considerations of porous surfaces, MHD fluids, heat and mass transfer, slip effects etc. Mukhopadhyay (2013) examined slip effects on MHD

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boundary layer flow over an exponentially stretching sheet. The thermal boundary layer of a power law fluid over a stretching surface was studied by Ali (1995). A new dimension is added to this investigation by Elbashbeshy (2001) who examined the flow and heat transfer characteristics over an exponentially stretching permeable surface. Magyari and Keller (1999) investigated the steady boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution. Some recent attempts in this direction are described in El-Aziz (2009), Ishak (2011), Cortell (2012), Fang (2004), Rashad (2007), Khan and Pop (2011).

The study of magneto-hydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. The process of fusing of metals in an electrical furnace by applying a magnetic field and the process of cooling of the wall inside a nuclear reactor containment vessel are good examples of such fields (Ibrahim et al., 2013). Some important applications of radiative heat transfer include MHD accelerators, high temperature plasmas, power generation systems and cooling of nuclear reactors. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of pertinent equipment (Seddeek, 2003). In controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet, applied magnetic field may play an important role (Turkyilmazoglu, 2012). Kumaran et al. (2009) investigated that magnetic field makes the streamlines steeper which results in the velocity boundary layer being thinner. The heat transfer analysis of boundary layer flow with radiation is also important in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry and other industrial areas. Raptis et al. (2004) reported the effect of thermal radiation on the MHD flow of a viscous fluid past a semi-infinite stationary plate.

The present paper provides an analytical solution of MHD boundary layer flow over an exponentially stretching sheet in the presence of radiation, which has not been considered before. The governing equations for the MHD boundary layer flow have been simplified with some suitable transformations and then solved analytically via HAM technique (Liao, 2005; Abbasbandy, 2007; Sajid and Hayat, 2008; Rashidi et al., 2013; Rashidi et al., 2014). The convergence of the series solution has been discussed by plotting ħ-curves. The effects of controlling parameters on MHD flow and heat transfer characteristics are discussed and shown graphically.

The paper is structured as: The problem formulation and quantities of physical interest are presented in Sections 2 and 3. HAM solution for the proposed problem is incorporated in Section 4. In Section 5 we have provided the convergence of the HAM solution. Sections 6 and 7 are reserved for the results, discussion and concluding remarks respectively.

2. Problem formulation

We consider a steady, two dimensional flow of an incompressible, viscous and electrically conducting fluid caused by a stretching sheet. Assume that the plate with surface temperature T_w is placed in a quiescent fluid of uniform ambient temperature T_∞ as shown in Fig. 1. A variable magnetic field $B(x)$

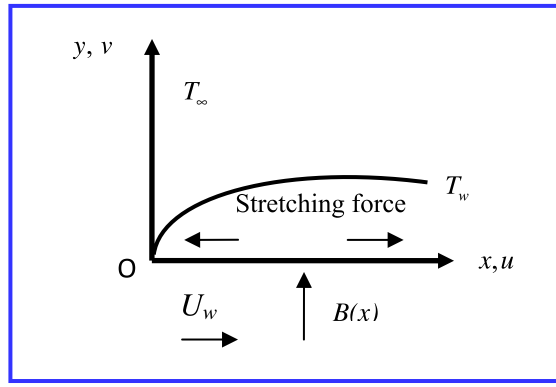

Figure 1 Flow configuration and coordinate system.

is applied normally to the sheet surface while the induced magnetic field is negligible, which can be justified for MHD flow at small magnetic Reynolds number. Under boundary layer approximations, the flow and heat transfer with radiation effects are governed by the following dimensional form of equations

$$\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + \mathrm{v}\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

where u and v are the components of the velocity in the x-, y-directions respectively, ν is the kinematic viscosity, α is thermal diffusivity, T is the fluid temperature in the boundary layer, ρ is fluid density, q_r is the radiative heat flux, c_p is the specific heat at constant pressure.

By the use of Rosseland approximation for radiation, we have

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}, \tag{4}$$

where σ is Stefan–Boltzman constant, and k^* is the absorption coefficient.

We assume the temperature difference within the flow such that T^4 may be expanded in a Taylor series about T_∞, (the free stream temperature) and neglecting terms of higher order, we have

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4.$$

Therefore, Eq. (4) becomes

$$u\frac{\partial T}{\partial x} + \mathrm{v}\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y^2}. \tag{5}$$

The hydrodynamic boundary conditions are (Ishak, 2011)

$$u = U_w(x), \quad \mathrm{v} = 0 \text{ at } y = 0, \tag{6}$$

$$u \to 0 \text{ as } y \to \infty. \tag{7}$$

where $U_w(x) = U_0 e^{x/L}$ is the stretching velocity, U_0 is the reference velocity, L is the characteristic length.

The thermal boundary conditions are

$$T = T_w = T_\infty + T_0 e^{\frac{x}{2L}} \text{ at } y = 0 \text{ and } T \to T_\infty \text{ as } y \to \infty, \tag{8}$$

where T_w is the variable temperature at the sheet with T_0 being a constant.

It is assumed that the magnetic field $B(x)$ is of the form: $B(x) = B_0 e^{\left(\frac{x}{2L}\right)}$, where B_0 is a constant magnetic field.

The continuity Eq. (1) is satisfied by introducing a stream function ψ such that:

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}.$$

For nondimensionalized form of momentum and energy equations as well as boundary conditions, the following transformations are introduced (Sajid and Hayat, 2008):

$$\eta = y\sqrt{\frac{U_0}{2vL}} e^{\frac{x}{2L}}, \quad u = U_0 e^{\frac{x}{L}} f'(\eta),$$
$$v = -\sqrt{\frac{vU_0}{2L}} e^{\frac{x}{2L}} \{f(\eta) + \eta f'(\eta)\}, \quad T = T_\infty + T_0 e^{\frac{x}{2L}} \theta(\eta). \tag{9}$$

where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature and prime denote differentiation with respect to η.

Using Eq. (9), the momentum and energy equations can be reduced into ordinary differential equations:

$$\begin{cases} f''' + ff'' - 2f'^2 - Mf' = 0, \\ \left(1 + \frac{4}{3}R\right)\theta'' + Pr(f\theta' - f'\theta) = 0. \end{cases} \tag{10}$$

The transformed boundary conditions of the problem are:

$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) = 0,\ \theta(0) = 1,\ \theta(\infty) = 0, \tag{11}$$

where $M = \frac{2\sigma B_0^2 L}{\rho U_0}$ is the magnetic parameter, $\Pr = \frac{v}{\alpha}$ is Prandtl number $R = \frac{4\sigma T_\infty^3}{k^* k}$ is the radiation-conduction parameter.

3. Physical quantities

Quantities of physical interest are the local friction factor, C_{fx} and the local Nusselt number, Nu_x. Physically, C_{fx} represents the wall shear stress, Nu_x defines the heat transfer rate.

$\frac{1}{\sqrt{2}} C_{fx}\sqrt{Re_x} = f''(0)$, $\quad \frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0)$, where $Re_x = \frac{U_w x}{v}$ is the local Reynolds number.

4. HAM solutions

In the view of boundary conditions Eq. (11), the dimensionless velocity $f(\eta)$ and temperature $\theta(\eta)$ can be expressed by the set of base functions

$$\{\eta^k \exp(-n\eta) | k \geqslant 0, n \geqslant 0\}, \tag{12}$$

in the form of following series

$$\begin{cases} f(\eta) = a_{0,0}^0 + \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_{m,n}^k \eta^k \exp(-n\eta), \\ \theta(\eta) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} b_{m,n}^k \eta^k \exp(-n\eta), \end{cases} \tag{13}$$

where $a_{m,n}^k$, $b_{m,n}^k$ are the coefficients. We follow the *rule of solution expression* for determining the initial approximations, auxiliary linear operators, and the auxiliary functions. According to the rule of solution expression, we choose the initial guesses $f_0(\eta)$,$\theta_0(\eta)$ based on boundary condition (11) and linear operators L_1 and L_2 in the following way

$$f_0(\eta) = 1 - e^{-\eta}, \quad \theta_0(\eta) = e^{-\eta}, \tag{14}$$

$$L_1(f) = \frac{d^3 f}{d\eta^3} - \frac{df}{d\eta},\ L_2(\theta) = \frac{d^2\theta}{d\eta^2} - \theta. \tag{15}$$

The operators L_1, L_2 have the following properties:

$$L_1(C_1 + C_2 e^{-\eta} + C_3 e^{\eta}) = 0, L_2(C_4 e^{-\eta} + C_5 e^{\eta}) = 0, \tag{16}$$

where $C_i (i = 1 - 5)$ are arbitrary constants. Let $q \in [0,1]$ represent an embedding parameter and $\hbar \neq 0$ be the auxiliary parameter to adjust the convergence rate of the perturbation series. Then we construct the following zeroth order deformation of the problem as

$$(1-q)\ L_1[\hat{f}(\eta;q) - f_0(\eta)] = q\hbar_f\ N_1[\hat{f}(\eta;q)], \tag{17}$$
$$(1-q)\ L_2[\hat{\theta}(\eta;q) - \theta_0(\eta)] = q\hbar_\theta\ N_2[\hat{\theta}(\eta;q), \hat{f}(\eta;q)], \tag{18}$$

subject to the conditions

$$\hat{f}(0;q) = 0, \quad \hat{f}'(0;q) = 1, \quad \hat{f}'(\infty;q) = 0, \tag{19}$$
$$\hat{\theta}(0;q) = 1, \quad \hat{\theta}(\infty;q) = 0, \tag{20}$$

where the non-linear operators are defined as

$$N_1 = \frac{\partial^3 \hat{f}(\eta;q)}{\partial\eta^3} + \hat{f}(\eta;q)\frac{\partial^2 \hat{f}(\eta;q)}{\partial\eta^2} - 2\left(\frac{\partial \hat{f}(\eta;q)}{\partial\eta}\right)^2 - M\frac{\partial \hat{f}(\eta;q)}{\partial\eta}, \tag{21}$$

$$N_2 = \left(1 + \frac{4R}{3}\right)\frac{\partial^2 \hat{\theta}(\eta;q)}{\partial\eta^2} + \Pr\left[\hat{f}(\eta;q)\frac{\partial\hat{\theta}(\eta;q)}{\partial\eta} - \frac{\partial \hat{f}(\eta;q)}{\partial\eta}\hat{\theta}(\eta;q)\right]. \tag{22}$$

For $q = 0$ and $q = 1$ we have

$$\hat{f}(q;\eta;0) = f_0(q;\eta), \hat{f}(q;\eta;1) = f(q;\eta), \tag{23}$$
$$\hat{\theta}(q;\eta;0) = \theta_0(q;\eta), \hat{\theta}(q;\eta;1) = \theta(q;\eta). \tag{24}$$

Defining

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m f(\eta;q)}{\partial\eta^m}\bigg|_{q=0}, \quad \theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \theta(\eta;q)}{\partial\eta^m}\bigg|_{q=0}, \tag{25}$$

and expanding $\hat{f}(q;\eta), \hat{\theta}(q;\eta)$ by means of Taylor's theorem with respect to q, we obtain

$$\hat{f}(q;\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) q^m, \tag{26}$$

$$\hat{\theta}(q;\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) q^m. \tag{27}$$

The auxiliary parameters are properly chosen so that series (26) and (27) converges at $q = 1$ and thus

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \tag{28}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta). \tag{29}$$

The resulting problems at the mth-order deformation are

$$L_1[f_m(\eta) - X_m f_{m-1}(\eta)] = \hbar_f\ R_m^f(\eta), \tag{30}$$
$$L_2[\theta_m(\eta) - X_m \theta_{m-1}(\eta)] = \hbar_\theta\ R_m^\theta(\eta), \tag{31}$$
$$f_m(0) = 0,\ f_m'(0) = 0,\ f_m'(\infty) = 0, \theta_m(0) = 0,\ \theta_m(\infty) = 0, \tag{32}$$

$$R_m^f = f_{m-1}'''(\eta) + \sum_{k=0}^{m-1}\left(f_k f_{m-1-k}'' - 2f_k' f_{m-1-k}'\right) - Mf_{m-1}', \tag{33}$$

$$R_m^\theta = \left(1 + \frac{4R}{3}\right)\theta_{m-1}''(\eta) + Pr\sum_{k=0}^{m-1}\left(f_k\theta_{m-1-k}' - f_{m-1-k}'\theta_k\right), \tag{34}$$

$$\mathcal{X}_m = \begin{cases} 0, m \leqslant 1, \\ 1, m > 1. \end{cases} \tag{35}$$

The general solution of Eqs. (30) and (31) is

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 e^{-\eta} + C_3 e^{\eta}, \quad (36)$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_4 e^{-\eta} + C_5 e^{\eta}, \quad (37)$$

where $f_m^*(\eta)$ and $\theta_m^*(\eta)$ are the particular solutions and the constants are to be determined by the boundary condition Eq. (32).

5. Convergence of homotopy solution

The convergence rate of approximation of the HAM solution strongly depends on the values of non-zero auxiliary parameter ħ, which is mentioned by Liao (2005). As, Eqs. (36) and (37) involve $\hbar_f$ and $\hbar_\theta$, so we can adjust the convergence of our HAM solution. To compute the range of admissible values of $\hbar_f$ and $\hbar_\theta$, we display the ħ-curves of the function $f''(0)$ and $\theta'(0)$ for different order of approximations. Fig. 2 depicts that range for the admissible values of $\hbar_f$ and $\hbar_\theta$ which are $-0.8 \leqslant \hbar_f \leqslant -0.3, -0.9 \leqslant \hbar_\theta \leqslant -0.2$. We observed that the series presented in Eqs. (28) and (29) converge in the whole region of η when $\hbar_f = \hbar_\theta = -0.62$. Figs. 3 and 4 show the ħ-curves for the dimensionless velocity and temperature for various values of controlling parameters. Convergence of the series solution up to 50th order of approximations for $\hbar_f = \hbar_\theta = -0.62$ is presented in Table 1. It is found from Table 1 that the convergence is achieved up to 32^{nd} order of approximation. In order to check the accuracy of the method, we have shown the residual errors in Fig. 5.

4. Results and discussion

The approximate analytical solutions are obtained using homotopy analysis method (HAM) for different values of the governing parameters, namely, the magnetic parameter (M), The Prandtl number (Pr), and the radiation parameter (R). Effects of M, Pr, and R on the steady MHD boundary layer flow, and heat transfer over exponential stretching sheet are discussed in detail. Figs. 6–10 have been plotted to illustrate the effect of controlling parameters on the flow field and heat transfer characteristics. To ensure the HAM accuracy, the values of $-\theta'(0)$ are compared with the previous published data in Table 2 and are found in excellent agreement. Thus, we are very much confident that the present HAM results are accurate.

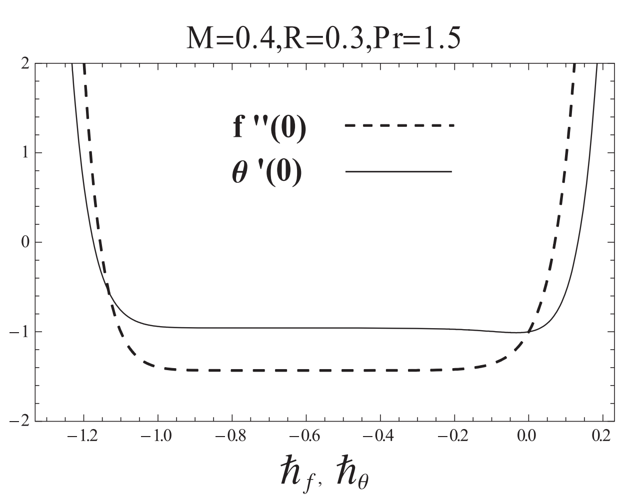

Figure 2 Combined h-curves for $f''(0)$ and $\theta'(0)$ at 10th order of approximations.

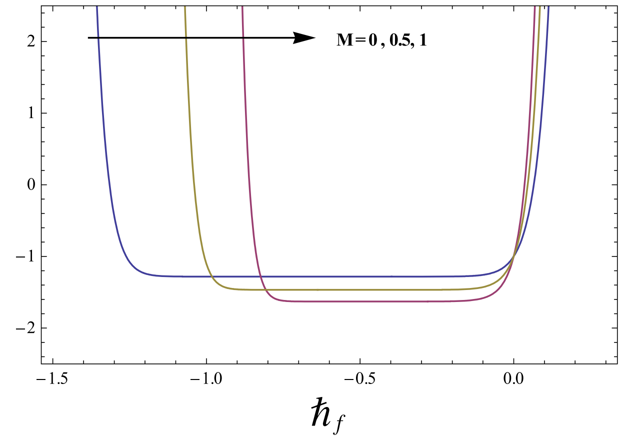

Figure 3 The $h-$ curves of $f''(0)$ obtained by 20th order approximation of HAM for different values of M when $R = 1$, Pr $= 1$

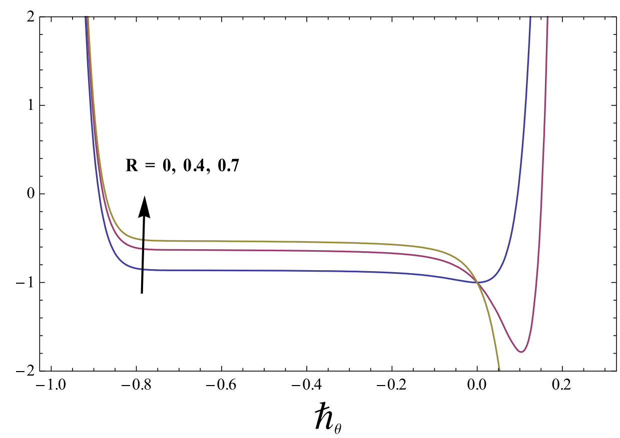

Figure 4 The $h-$ curves of $\theta'(0)$ obtained by 20th order approximation of HAM for different values of R when $M = 1$, Pr $= 1$

Table 1 Convergence of HAM solutions for different order of approximations when $M = 0.4$, $R = 0.3$, Pr $= 1.5$ and $\hbar_f = \hbar_\theta = -0.62$.

Order of approximation	$f''(0)$	$-\theta'(0)$
1	−1.33067	1.03000
5	−1.43025	0.96504
10	−1.43157	0.95827
15	−1.43157	0.95728
25	−1.43157	0.95699
32	−1.43157	0.95697
35	−1.43157	0.95697
40	−1.43157	0.95697
50	−1.43157	0.95697

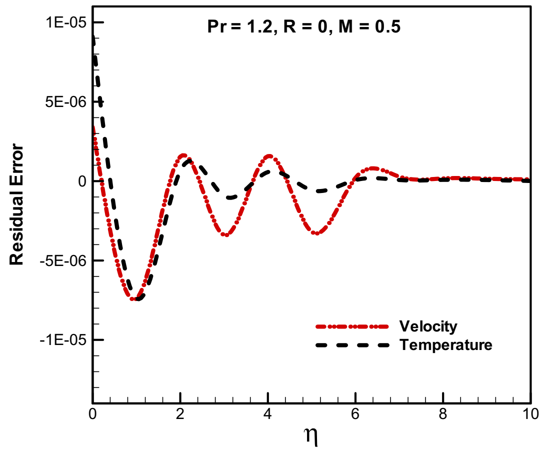

Figure 5 Residual error for dimensionless velocity and temperature.

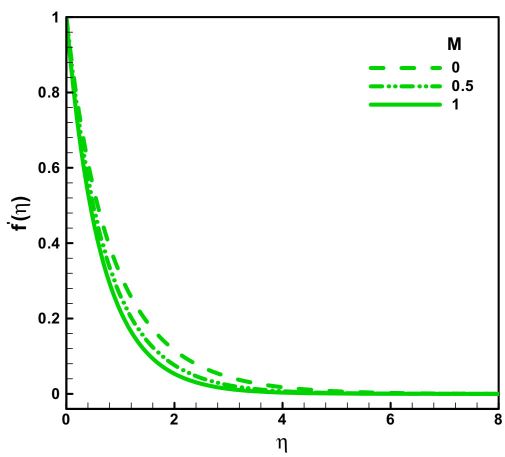

Figure 6 Effect of magnetic parameter on dimensionless velocity.

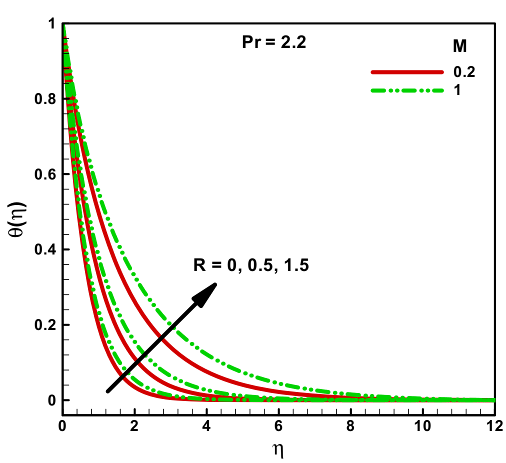

Figure 7 Effects of radiation and magnetic parameters on dimensionless temperature.

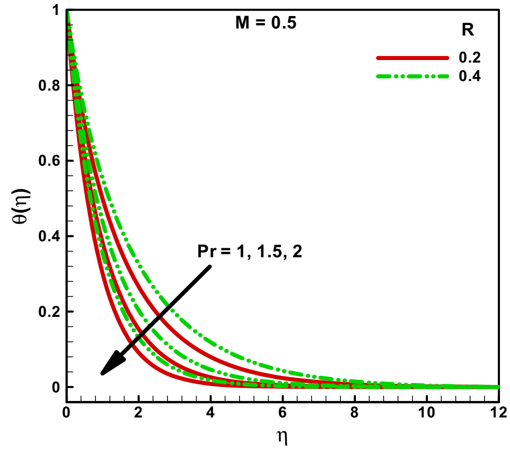

Figure 8 Effects of Prandtl number and radiation parameter on dimensionless temperature.

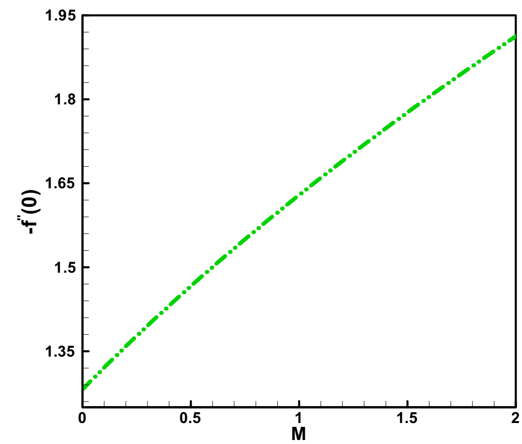

Figure 9 Variation of skin friction coefficient against magnetic parameter.

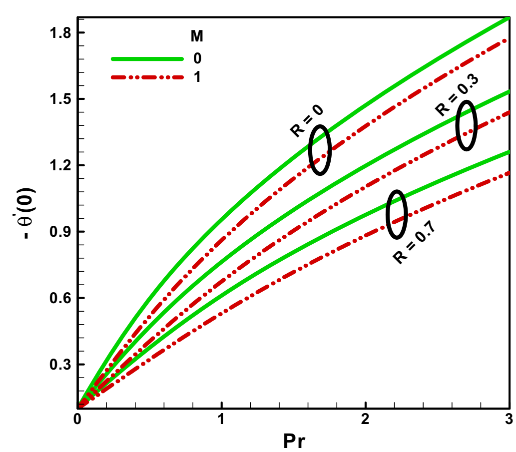

Figure 10 Variation of heat transfer rate against Prandtl number, magnetic and radiation parameters.

Table 2 Comparison of $-\theta'(0)$ for several values of magnetic, radiation parameters and Prandtl number.

R	M	Pr	Magyari and Keller (1999)	El-Aziz (2009)	Ishak (2011)	Mukhopadhyay (2013)	HAM solution
0	0	1	0.9548	0.9548	0.9548	0.9547	0.95478
		2	–	–	1.4715	1.4714	1.47151
		3	1.8691	1.8691	1.8691	1.8691	1.86909
		5	2.5001	2.5001	2.5001	2.5001	2.50012
		10	3.6604	3.6604	3.6604	3.6603	3.66039
1	0	1	–	–	0.5312	0.5312	0.53121
0	1		–	–	0.8611	0.8610	0.86113
0.5	0	2	–	1.0735	–	1.0734	1.07352
		3	–	1.3807	–	1.3807	1.38075
1			–	1.1214	–	1.1213	1.12142
	1	1	–	–	0.4505	–	0.45052

We now continue to discuss the results obtained by using HAM method. The dimensionless velocity $f'(\eta)$, and temperature θ (η) for various values of magnetic parameter M are shown in Figs. 6 and 7. The dimensionless velocity reduces with an increase in magnetic parameter, as the magnetic field opposes the transport phenomena. Physically, an increase in magnetic parameter M leads to an increase in the Lorentz force. While the dimensionless temperature increases with M. The Lorentz force has the tendency to increase the temperature and consequently, the thermal boundary layer thickness becomes thicker for stronger magnetic field. It is also noticed from Fig. 7 that the dimensionless temperature increases with an increase in radiation parameter R. The variation of dimensionless temperature with various Prandtl numbers is given in Fig. 8 which shows that increasing values of Prandtl number imply the decrease in the thermal boundary layer thickness which increases the heat transfer rate.

We now discuss the variations of the physical quantities of engineering importance, that is, the local skin friction coefficient C_f , and the local Nusselt number Nu_x. The values of local skin friction coefficient $-f''(0)$ verses M are displayed in Fig. 9. It is noticed that for stronger magnetic field the value of $-f''(0)$ increases monotonically. From physical viewpoint, it can be noticed that the Lorentz force increases the values of local skin friction coefficient. The variation of the Nusselt number $-\theta'(0)$ is presented for various values of M, Pr, and R in Fig. 10. The heat transfer rate decreases with an increase in both radiation and magnetic parameters. The Nusselt number increases with an increase in the Prandtl number. This is due to the fact that a higher Prandtl number reduces the thermal boundary layer thickness and increases the surface heat transfer rate. Also high Prandtl number implies more viscous fluid which tends to retard the motion. But Prandtl number has no effects on skin-friction coefficient as the momentum boundary layer equation is independent of θ.

5. Conclusion

Homotopy analysis method (HAM) is employed to investigate the effects of radiation on MHD boundary layer flow over exponential stretching sheet. The HAM results are in good agreement with those reported in open literature. The solutions for dimensionless velocity, temperature, skin friction coefficient, and the dimensionless heat transfer rate for various values of governing parameters were obtained and have been illustrated in graphical form. The findings of the analysis can be summarized as follows.

a) Due to stronger magnetic field the dimensionless velocity decreases and temperature increases.
b) The dimensionless temperature is enhanced with radiation.
c) Due to magnetic filed the skin friction coefficient increases monotonically.
d) Increasing the Prandtl number results in reduction of thermal boundary layer thickness. Consequently, the dimensionless heat transfer rate increases with Pr.

Acknowledgements

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