Analysis of Process Damping in Milling

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Abstract

Regenerative chatter has been identified as one of the major limitations on achieving higher material removal rates in milling. There are numerous studies in the literature in which various time and frequency domain models have been developed and successfully employed to predict stability boundaries for different milling operations. However, many of these studies have neglected the dynamics of tool–workpiece interaction that describes process damping. As this interaction becomes prominent at low speeds, it enables substantially higher chatter-free depths that would be of great importance especially for difficult-to-cut alloys. In this study, an alternative process damping model has been presented for flat-end milling on the basis of an equivalent viscous damping approach. Parameters included in the process damping model are cutting speed, vibration amplitude, wear land width and clearance angle. Computed stability diagrams show an agreement with cutting tests which were carried out on Ti6Al4V. The model demonstrates strong dependence to vibration amplitude. Also, it is shown that the spindle’s low-frequency vibration mode governs the dynamics of process damping at low speeds.

1. Introduction

In milling, productivity is generally associated with material removal rate (MRR) which is defined as the product of radial depth, axial depth, number of flutes, spindle speed and feed rate. The ultimate objective of a process planner is to maximise the MRR without violating constraints such as tool life, surface quality and form errors. Unfortunately, it is not feasible to achieve this by going through trial and error process. On the other hand, investigation of process dynamics offers valuable solutions on the scientific basis and enables substantial improvements in MRR.

From the perspective of dynamics, instability in machining or so-called as chatter has been one of the major limitations on productivity. There have been various studies in literature focused on understanding, detection and prediction of chatter vibrations in milling. Developed methods are successful to an extent in obtaining maximum chatter-free depths when cutting at higher speeds however experimental data indicate that simulations tend to under predict stability limits as cutting speed decreases. This is caused by the process damping phenomenon that suppresses chatter and is mostly observed at low speeds. Including the process damping into dynamic milling model is critical for machining of difficult-to-cut materials such as titanium and nickel alloys since cutting speed is inherently limited due to tool wear for these alloys. If the maximum depth of cut allowed by the process is correctly determined at low speeds, the productivity lost due to inability of cutting faster can be regained, and this is much appreciated by the machining industry.

Sisson and Kegg [1] were one of the earliest who noticed through experimental studies that the main cause of the increase in damping at low speeds is due to the contact between cut surface and tool flank face. They also discovered that the use of worn cutting edge and reground flank would improve the process stability. These findings were confirmed afterwards by the significant experimental effort conducted by several researchers [2, 3] to introduce process damping into
analytical stability models within the complex part of dynamic cutting force coefficients. Although this calibration method would facilitate the modeling, Tsusty [3] argued that there were inconsistencies in results even when a standardized set-up had been used. These were mainly attributed to the difficulty in instrumentation. Later on, Tsusty and Ismail [4] established a relationship between surface waviness and process damping which has been used for years as the most practical measure of the process damping performance. As given in Eq. (1) this relationship states that as cutting speed decreases or vibration frequency increases, cut surface becomes steeper and this in turn enlarges the contact area with tool flank and leads to more energy dissipation. On the assumption of harmonic tool vibration, \( L_c \) is the wavelength of the cut surface, \( f \) is the vibration frequency and \( V_c \) is the cutting speed here.

\[
L_c = \frac{V_c}{f} \tag{1}
\]

Wu [5] developed an indentation force model in which energy loss due to process damping is described by the ploughing forces acting in the tool-workpiece interference. Applying an empirical constant, normal component of the ploughing force is related to the material displaced under tool flank. For tangential component, an average coefficient of friction based on Coulomb Law was assumed between tool and cut surface. Elbestawi et al. [6] adapted Wu’s approach into 2-DOF dynamic milling model in order to simulate the ploughing forces and their contribution to stability. Despite that time-domain simulations and experimental results point out a good agreement, authors did not present any direct solution which could be used to predict stability limits.

On the basis of Wu’s indentation force model, Ahmadi and Ismail replaced [7] process damping coefficients by linear viscous dampers in the way described in [8] and included them into multi-frequency and semi-discrete solutions of dynamic milling on the assumption of low amplitude vibration.

Tunc and Budak, in connection with their previous research [9-11], recently proposed an inverse stability method for milling in which average process damping coefficients are determined experimentally by deducting structural damping from the total damping [12]. These coefficients are then calibrated iteratively in the forward stability algorithm with respect to cutting conditions and tool geometry.

In this paper, using the equivalent viscous damping approach, process damping in milling is modeled as a function of surface wavelength and included into analytical stability solution. Fundamentals of the approach and parameters used in the model are explained in Section 2. Inclusion of process damping into dynamic milling model and computation procedure of the stability lobes are described in Section 3. Experiment set-up involving impact testing and slotting trials performed on Ti6Al4V are described in Section 4. Discussions on the comparison of computed stability lobes to experimental results are made in Section 4 and 5 respectively.

2. Representation of Process Damping

2.1. Indentation Force Model

It is believed that the primary mechanism of process damping is the ploughing force that occurs due to tool-workpiece contact. The indentation force model developed by Wu [5] relates the components of ploughing force to the volume of material compressed by tool flank into cut surface. If the width of cut \( b \) is assumed to be constant, the indentation volume \( V_i \) can be expressed as,

\[
V_i(t) = bA_i(t) \tag{2}
\]

where \( A_i \) is the indentation area. In order to derive the normal component of ploughing force \( F_n \), the indentation volume is calibrated as follows, using an indentation coefficient \( K_i \) which incorporates material properties of tool and workpiece.

\[
F_n(t) = K_i V_i(t) \tag{3}
\]

Assuming an average coefficient of friction \( \mu \), the tangential component \( F_s \) is then expressed as below.

\[
F_s(t) = \mu F_n(t) \tag{4}
\]

It can be seen from equations (2-4) that the essential term behind the ploughing mechanism is the indentation area. It can be computed either analytically (by approximating the geometry of indentation) or numerically as described in the following section.

2.2. Computation of Indentation Area

It is a well-known fact that the tool vibrates at a frequency close but not equal to one of its dominant vibration modes under the onset of chatter. In addition, Sims and Turner [13] reported that when process damping becomes prominent over the regenerative vibration, the tool oscillates in a limit cycle. It is therefore reasonable to assume that the tool vibration is harmonic and for one vibratory cycle it can be defined in Cartesian coordinates as in Eq. (5). Here, \( A_i \) is the vibration amplitude, \( L_c \) is the wavelength.

\[
y(x) = A_i \sin\left(\frac{2\pi x}{L_c}\right) \tag{5}
\]

It should be noted that as the vibrating tool leaves its imprints onto surface being cut, Eq. (5) also describes the toolpath. As illustrated in Fig. 1, within the boundaries in which slope of the path \( dy/dx \) is negative, the tool is said to indent the workpiece surface and ploughing forces acting on the tool flank start dissipating the vibratory energy. As the path is sinusoidal, boundaries can be easily found as \( L_c/4 \) and \( 3L_c/4 \).
In order to compute the indentation area numerically, it is required to discretize this path. For this purpose, the wavelength is first divided into \( N \) equidistant sections. \( N \) indicates the step size. The horizontal distance between each section \( dL \) can be found as below.

\[
dL = \frac{L}{N} \quad (6)
\]

Subsequently, tool is positioned at \( y\left(\frac{Lc}{4}\right) \) and a loop begins in which tool is moved at each step by \( dL \) towards the positive x direction. In addition, a second inner loop is also run at each step to locate the coordinates of workpiece surface vector \( \mathbf{w} \) and tool flank vector \( \mathbf{f} \) by following the algorithm such;

\[
w_{i,c} = y_i \quad \text{if } c \leq k; \quad k = \text{round}(VB/dL)
\]

\[
f_{i,c} = y_i + \tan(\gamma)(c-k)dL
\]

where, \( i \) is the first loop’s count, \( c \) is the second loop’s count, \( VB \) is the wear land width and \( \gamma \) is the clearance angle.

Afterwards, the distance between the surface wave and tool flank vectors \( d \) is checked to ensure that it is positive. As long as it is positive the discrete area between each section \( dS \) is calculated using trapezoidal rule.

\[
dS_{i,c} = \left(\frac{d_{i,c} + d_{i,c-1}}{2}\right)dL \quad (8)
\]

Once all discrete areas have been calculated, the total indentation area at step \( i \) can be found as below.

\[
S_i = \sum_{c=1}^{m} dS_{i,c} \quad (9)
\]

Index \( m \) in Eq. (9) is the step number where the distance \( d \) between two vectors is no longer positive which means tool does not indent the surface anymore.

2.3. Equivalent Viscous Damping Approach

Complex damping mechanisms are often modeled through a linear viscous damping element which is expected to dissipate same amount of energy in one vibration cycle. In order to apply this method to process damping, the amplitude and wavelength of the vibration cycle should be known before. If the same assumptions given in Sec. 2.2 are used, the cyclic energy loss in normal direction can be written as below.

\[
E^d_{eq}(x) = \int_0^L F^d dy. \quad (10)
\]

Substituting Eq. (3) into Eq. (10) and setting \( dy=(dy/dx)dx \) leads to

\[
E^d_{eq}(x) = \frac{2\pi K hA}{L_v} \int_0^L A(x) \cos(\frac{2\pi}{L_v}x) dx. \quad (11)
\]

If a linear viscous damper \( C_{eq}^d \), which acts equivalently to ploughing forces, is introduced to the system, the energy dissipated by this damper in the same cycle \( E_{eq}^d \) is,

\[
E_{eq}^d(x) = \int_0^\infty C_{eq}^d \frac{dy}{dt} dy. \quad (12)
\]

Changing the variables in the integration as

\[
\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt} \quad \text{and } \frac{dy}{dx} = \frac{dy}{dt} \quad \text{gives}
\]

\[
E_{eq}^d(x) = C_{eq}^d \frac{4\pi A^2}{L_v} \frac{1}{L_v} \cos^2(\frac{2\pi}{L_v}x) dx. \quad (13)
\]

where \( v \) is the tool’s surface speed in the x direction. Equating Eq. (11) and Eq. (13) yields to,
In Eq. (14), $C_d$ is the geometric damping factor which includes the properties of tool geometry (wear land width and clearance angle), vibration amplitude and wavelength. As the surface becomes steeper, the amount of material that tool flank could compress into the cut surface increases. Similarly, a smaller clearance angle means longer contact between tool and workpiece. On the other hand, (c) and (d) show that vibration amplitude and wear land width affect the process damping in exactly the opposite way. This is also as expected considering the fact that as tool wears out or vibration amplitude escalates, ploughing forces are amplified.

**Ahmadi and Ismail [7]** used this energy analysis method in the same way to obtain the geometric damping factor. However, in their dynamic milling model, this factor was approximated to a quadratic function of wear land width assuming that vibration amplitude is small. In the next section, an alternative implementation will be described.

### 3. Milling Stability Including Process Damping

According to regenerative chatter theory, the equations of motions in 2-DOF flat-end milling can be written as in Eq. (15). It should be noted that the total damping $c^T$ includes the structural damping $c^s$ and the process damping $c^*T$ in x and y directions.

$$m_x \ddot{x} + c^s_x \dot{x} + k_x x = F_x; \quad c^s_x = c^1_x + c^2_x$$

$$m_y \ddot{y} + c^s_y \dot{y} + k_y y = F_y; \quad c^s_y = c^1_y + c^2_y$$

$$m_{x,y} \ddot{X} + c^T_{x,y} \dot{X} + k_{x,y} X = F_{x,y}; \quad c^T_{x,y} = c^1_{x,y} + c^2_{x,y}$$

where,

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \frac{1}{2} b K c \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}; \quad \Delta x = (x - x_{eq}), \Delta y = (y - y_{eq})$$

In Eq. (15), $b$ is the axial depth of cut, $K$ is the time-varying directional force coefficient matrix, $x_{eq}, y_{eq}$ are the present tool vibrations, $x_{eq}, y_{eq}$ are the vibrations at previous tooth period $T$. If Eq. (16) is expressed in the frequency-domain and the manipulations described by Altintas and Budak [14] are carried out, the eigenvalue which is necessary to obtain the stability boundaries can be found as

$$b_{lm} = \frac{-2\pi \lambda_r (1 + k^2)}{NK}, \quad \rho = \frac{\lambda_l}{\lambda_r} = \frac{\sin \omega_T T}{1 - \cos \omega_T T}$$

where $\omega_T$ is the chatter frequency. Spindle speed $\Omega$ corresponding to stable depth of cut can be found from Eq. (17).

$$\psi = \tan^{-1} \kappa, \quad \psi = \pi - 2\psi$$

$$T = \frac{k \pi + \psi}{\omega_T}, \quad \Omega = \frac{60}{NT}$$

where $k$ is the integer lobe number. The procedure to incorporate the process damping into conventional frequency-domain solution of milling is described as follows:

1. Set the wear land width, clearance angle and maximum allowable vibration amplitude.
2. Compute $C_d$ for a range of wavelength $L_c$. Fit an exponential curve to obtained dataset in the form given in Eq. (19). Determine coefficients $k_0$, $k_1$ and $k_2$ from the fitted curve in order to express the geometric damping factor as a function of $L_c$.
3. Select a chatter frequency around the dominant mode and set $\omega_T = 0$.
4. Calculate $b_{lm}$ and $\Omega$ from Eq. (17) and Eq. (18) without the effect of process damping.
5. Find $\nu$ from Eq. (20) and $L_c$ from Eq (1). $D$ is the tool diameter.

$$v = \frac{\pi D \Omega}{60}$$

6. Substitute $L_c$ into $C_d$ function derived in Step 2 and find the corresponding geometric damping factor. Then, calculate $C^*_{eq}$ for $b_{lm}$, $v$ and $L_c$ determined in Step 4 and Step 5 respectively.
7. Update $c^1_x$ and $c^2_x$ with $c^1_x = c^1_{x,eq}$ and $c^2_x = \mu c^1_{x,eq}$. This adds the effect of process damping into total damping.
8. Using updated damping coefficients, recalculate the process damping stable $b_{lm}$ and $\Omega$ as in Step 4.
4. Experimental Studies

All experimental studies were performed on the HAAS VF-6 3-axis milling centre using 16 mm diameter, four fluted, constant pitch, solid carbide end mill with 8 deg primary and 10 deg secondary clearance angles. Tool was clamped in a mechanical holder through a chuck-collet adaptor and stick-out length was set to 50 mm. A prismatic Ti6Al4V block was bolted-down on a Kistler 9257A 3-axis table dynamometer which was then clamped on the machine bed. A Shure PG81 microphone was attached to the spindle housing through a magnetic base and it was directed into cutting zone as close as possible. Tool run-out was measured by a dial-gauge to ensure that it was below 10 microns. In order to design the experiment matrix for cutting trials and estimate stability lobes later on, modal parameters of the machine tool were required to be found. For this purpose, an impact test was performed both on the tool-tip and workpiece-dynamometer assembly using a PCB-086C01 small accelerometer (5.75 mV) and PCB-352C23 medium size steel-tip impact hammer (11.7 mV). Both sensor data was collected by NI-USB 9162 4-channel data acquisition module. Curve fitted results are given in Fig. 4.

From Fig. 4, it can be said that tool-tip dynamics is symmetric in x and y directions. For low speed milling, first two modes are most critical ones to be considered. All modes coming from the workpiece-dynamometer assembly can be neglected as it seems to be much more rigid compared to tool.

Fig. 4. Frequency response functions of tool-tip and workpiece-dynamometer assembly.

After finding modal parameters of the machine tool, a set of slot-milling trials were carried out on the given set-up. Feed per tooth was constant in all trials as 0.075 mm/tooth. Cutting forces in x and y directions were measured by the dynamometer, sound pressure was acquired by the microphone and after each trial, top and side surfaces of the workpiece were visually examined. Experiment set-up is given in Fig. 5.

Fig. 5. Description of experiment set-up.

Fig. 6 shows two cases given to explain the chatter detection method and illustrate how process damping stabilizes the system. In (a), variation of the cutting forces is bounded and feed marks on the machined surface are clearly visible. Yet, in the sound spectrum there is a minor spike located at around the first mode which refers to the chatter frequency. This means that system tends to become unstable however the energy brought by regenerative component of the vibration is absorbed by ploughing forces and therefore amplitude of the vibration is prevented from growing up. On the contrary, (b) shows that energy dissipation due to process damping is not sufficient anymore to suppress the chatter. There are sharp variations in cutting forces, feed marks on the workpiece surface are difficult to identify and sound spectrum is fully dominated by the chatter frequency and its second harmonic.

Fig. 6. (a) Stable case at \( b = 4 \text{ mm}, \Omega = 1250 \text{ RPM} \). (b) Chatter at \( b = 5 \text{ mm}, \Omega = 1500 \text{ RPM} \).

Furthermore, it is interesting to see the chatter frequency being at around the first mode as the second mode seems to be much more flexible. This would confirm the discussions of Tlusty and Ismael [4] who claimed that high-frequency modes (tool modes) become inactive at low speeds. This phenomenon is illustrated schematically in Fig. 7.

Fig. 7. Variation of process damping included absolute stability in relation to different vibration modes.
Neglecting the effects of process damping, the absolute stability limit should be determined by the most flexible mode. However when the speed decreases, tool-workpiece interference causes an escalation in the stability limits. For low-frequency modes, this escalation happens at higher spindle speeds than it does for low-frequency modes. Therefore, it could be suggested for slender end-mills that the dynamics of process damping is governed by the spindle modes up to a certain speed.

Fig. 8 illustrate two stability diagrams computed for three amplitudes of 0.01 mm, 0.02 mm and 0.03 mm at two different wear land widths of 0.04 and 0.16 mm. On the basis of experimental results, only the first mode was taken into account. In both cases, frequency-domain model was also verified for $A_c=0.02 \text{ mm} \,$ (as shown in red lines) through plotted tool displacements. Modal parameters in $x$ and $y$ are $\omega_x=662 \text{ Hz}$, $\zeta_x=0.0506$, $\omega_y=662 \text{ Hz}$, $\zeta_y=0.0545$. For Ti6Al4V, tangential and radial cutting force coefficients were taken as $K_{tc}=1743 \text{ N/mm}^2$ and $K_{rc}=400 \text{ N/mm}^2$. Indentation constant $K_i$ was 30000 $\text{N/mm}^3$ as reported in [12]. Clearance angle was averaged to 9 degrees.

It can be seen from Fig. 8 (a) and (b) that the nonlinear relationship between vibration amplitude and process damping was correctly modeled. Improvement in the stability limit should be determined by the most flexible mode. However when the speed decreases, tool-workpiece interference causes an escalation in the stability limits. For low-frequency modes, this escalation happens at higher spindle speeds than it does for low-frequency modes. Therefore, it could be suggested for slender end-mills that the dynamics of process damping is governed by the spindle modes up to a certain speed.

5. Conclusion

In this study, a process damping model developed for flat-end milling has been presented. Effects of the tool vibration and tool geometry were not embedded in a constant but rather expressed as a function of vibration wavelength that varies with respect to changing cutting speed and chatter frequency in the stability solution. In computing stability lobes, vibration amplitude should be selected wisely to avoid under or over predictions. Moreover, further refinement in damping model is required to represent the effect of tool wear more accurately. Finally, even though the model is able to simulate the behavior of process damping in milling, more experimental effort is needed to test its reliability.

References