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Snarks without Small Cycles

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Snarks are nontrivial cubic graphs whose edges cannot be colored with three colors. Jaeger and Swart conjectured that any snark has girth (the length of the shortest cycle) at most 6. This problem is also known as the *girth conjecture* of snarks. The aim of this paper is to give a negative solution of this conjecture [illegible]
cyclically 5-edge-connected snarks of order n and with girth at least $([illegible] \pm o(1)) \log_2 n$.

1. INTRODUCTION

The study of cubic graphs whose edges cannot be colored with three colors began in 1880, when Tait [27] proved that the four-color theorem is equivalent with the statement that every cubic map is 3-edge-colorable. Since graphs of this kind are very difficult to find, Gardner [11] called them snarks borrowing this name from the Lewis Carroll's' "Hunting of the Snarks." Interest about these graphs has intensified since it is known that several significant conjectures about graphs would have snarks as minimal counterexamples.

In order to avoid trivial cases *snarks* are defined to be cyclically 4-edge-connected cubic graphs with girth at least 5 and chromatic index 4. Note that a graph is called *cyclically* k*-edge-connected* if deleting fewer than k edges does not disconnect the graph into components each containing a cycle. More detailed discussion about the history, motivation, and constructions of snarks can be found in the survey articles of Watkins and Wilson [33] and Chetwynd and Wilson [7].

There are two well-known conjectures about snarks. In the first one, due to Tutte [29], it is conjectured that every snark contains a graph

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homeomorphic to the Petersen graph. The second conjecture is known as the *girth conjecture* of snarks and was given by Jaeger and Swart [17] (see also [15, 6, 7, 33]). According to this conjecture every snark should have girth at most 6.

Note that, by Goddyn [12], any smallest counterexample to the *cycle double cover conjecture* must be a snark with girth at least 8. Furthermore Celmins [5] proved that any smallest counterexample to the 5-*flow conjecture* of Tutte [28] is a cyclically 5-edge-connected snark with girth at least 7 (see also [16, Theorem 9.3]). Thus, if the girth conjecture would hold, then it would imply the above-mentioned celebrated conjectures about graphs.

All snarks known until now have satisfied the conjectures of Tutte [29] and Jaeger and Swart [17]. See, e.g., the survey papers [33, 7]. We give a negative solution of the girth conjecture, constructing cyclically 4- and 5-edge-connected snarks with arbitrarily large girths in Sections 4 and 5.

In the third section we introduce a new method for constructing snarks called superposition. It is based on the idea that if we take a snark G and replace its edges by snarks and its vertices by arbitrary cubic graphs, we get a new snark. This method is very flexible and generalizes almost all constructions of snarks known until now. We have announced superposition in [20], where it was used by constructing a new family of cyclically 6-edge-connected snarks. More detailed discussion about applications of superposition and its ties with other constructions can be found in [21, 22].

2. Cubic Graphs with Large Girths

In the fourth section we present a constructive proof of the following theorem.

THEOREM 1. *Let G be a cyclically 5-edge-connected cubic graph of order n and with girth $c \geqslant 5$. Then there exists a cyclically 5-edge-connected snark $S(G)$ of order $22n+14$ and with girth c.*

In order to use Theorem 1 we need cubic graphs with large girths. Graphs with this property have been studied intensively by several authors. For instance, Weiss [34] extended an explicit construction of the sextet graphs of Biggs and Hoare [2] and, for any odd prime p, gave a bipartite connected cubic graph $\tilde{S}(p)$ whose order n and girth c satisfy

$$c \geqslant (\alpha \pm o(1)) \log_2 n, \tag{1}$$

where $\alpha = \frac{4}{3}$. The order of $\tilde{S}(p)$ is $q(q^2-1)/24$, where $q = p$ if $p \equiv \pm 1 \pmod 8$ and $q = p^2$ if $p \equiv \pm 3 \pmod 8$. Furthermore, any $\tilde{S}(p)$ is equal to a sextet graph or to the double cover (see, e.g., Biggs [1, p. 131]) of a sextet graph. Since the sextet graphs are 5-arc transitive, so are the graphs $\tilde{S}(p)$ (see [1, 2] for more details). Thus $\tilde{S}(p)$ is edge-transitive for any odd prime p.

Nedela and Škoviera [25, Theorems 7 and 8] have proved that the cyclic edge-connectivity of a connected edge- or vertex-transitive cubic graph is equal to its girth.

Therefore, from the results of [34, 2, 25] and Theorem 1 we have the following.

THEOREM 2. *There exists an infinite family of cyclically 5-edge-connected snarks such that if a snark of this family has order n then its girth is at least* $(\frac{4}{3} \pm o(1)) \log_2 n$.

Jendrol', Nedela, and Škoviera [19] constructed an infinite family of edge-transitive cubic graphs with girth c for any $c \geqslant 3$. Thus, by Theorem 1, we can conclude the following.

THEOREM 3. *For any given* $c \geqslant 5$, *there exists an infinite family of cyclically 5-edge-connected snarks with girth c.*

The families from Theorems 2 and 3 can be constructed explicitly, because our methods and the methods from [34, 2, 19] are constructive.

Note that other explicit constructions of cubic graphs satisfying the inequality (1) have been presented by Margulis [23] (for $\alpha = \frac{4}{3}$) and Imrich [13] (for $\alpha = 0.9602...$). Erdős and Sachs [9] used probabilistic methods and constructed cubic graphs satisfying the inequality (1), where $\alpha = 1$. Similar results have been obtained by Walther [31, 32]. Furthermore, Erdős and Sachs [9] found an upper bound of $c \leqslant (2 \pm o(1)) \log_2 n$ for the girth c of a cubic graph of order n. An infinite family of cyclically 5-edge-connected cubic graphs with girth $c \geqslant 5$ can be also constructed using the method from Sachs [26, Chap. 6].

3. SUPERPOSITION

Following the notation of Fiol [10] we consider graphs with semiedges. Such graphs are called multipoles. More formally, a *multipole* $M = (V, E, S)$ consists of a set of vertices $V = V(M)$, a set of edges $E = E(M)$, and a set of *semiedges* $S = S(M)$. Each semiedge is incident either with one vertex or with another semiedge, in which case the two mutually incident semiedges form a so-called *isolated edge*. Figure 6 shows a multipole with seven semiedges and two isolated edges. If $|S(M)| = k$, then M is also called a *k-pole*. Obviously, if $S(M) = \varnothing$, then M is a graph and we write $M = (V, E)$ in this case. All multipoles and graphs considered here are cubic; i.e., any vertex is incident with just three edges or semiedges.

By a *$(k_1, k_2, ..., k_n)$-pole* $M = (V, E, S_1, S_2, ..., S_n)$ we mean a k-pole, where $k = k_1 + \cdots + k_n$, V and E denote the vertex and edge sets, respectively, and the set $S(M)$ is partitioned into n pairwise disjoint nonempty sets $S_1, S_2, ..., S_n$ such that $|S_i| = k_i$, $i = 1, 2, ..., n$. The sets S_i are called *connectors* of the $(k_1, k_2, ..., k_n)$-pole M.

An edge incident with vertices u and v is denoted by (u, v) and a semiedge incident with a vertex v is denoted by (v). The length of the shortest path joining two vertices u and v is called the *distance* of u, v and denoted by $d(u, v)$. Otherwise we use the standard graph-theoretic terms that can be found in [4].

By a 3-*edge-coloring* of a multipole M we mean a mapping $\varphi\colon E(M) \cup S(M) \to \mathbb{Z}_2 \times \mathbb{Z}_2 - 0$ (the set of the nonzero elements of the group $\mathbb{Z}_2 \times \mathbb{Z}_2$) satisfying the following conditions:

— $\varphi(e_1) \neq \varphi(e_2)$ for any two (semi)edges $e_1, e_2 \in E(M) \cup S(M)$ having a vertex in common;

— $\varphi(s_1) = \varphi(s_2)$ for any two incident semiedges making up an isolated edge.

Using the addition in the group $\mathbb{Z}_2 \times \mathbb{Z}_2$ we can define, for any $X \subseteq S(M)$,

$$\bar{\varphi}(X) = \sum_{e \in X} \varphi(e). \tag{2}$$

Now recall the well-known *parity lemma*. According to Isaacs [14] it is due to Blanuša [3] and Descartes [8] independently (see also [7, 33]). We introduce it in the form from Fiol [10].

LEMMA 1. *Let M be a multipole having m semiedges and let it be* 3-*edge-colored by the nonzero elements of $\mathbb{Z}_2 \times \mathbb{Z}_2$. If m_i denotes the number of semiedges with color i, then $m_i \equiv m \pmod 2$ for any color i.*

Clearly, Lemma 1 is equivalent with the following statement.

LEMMA 2. *If $M = (V, E, S)$ is a multipole and φ is a* 3-*edge-coloring of M, then*

$$\bar{\varphi}(S(M)) = 0.$$

By a *superedge* we mean any multipole with two connectors and by a *supervertex* we mean any multipole with three connectors. From Lemma 2 we immediately have the following.

COROLLARY 1. *If* $M=(V, E, S_1, S_2)$ *is a superedge and* φ *is a 3-edge-coloring of* M, *then* $\bar{\varphi}(S_1)=\bar{\varphi}(S_2)$.

COROLLARY 2. *Let* $M=(V, E, S_1, S_2, S_3)$ *be a supervertex and let* φ *be a 3-edge-coloring of* M *such that* $\bar{\varphi}(S_i)\neq 0$ *for each* $i=1, 2, 3$. *Then* $\bar{\varphi}(S_1)$, $\bar{\varphi}(S_2)$, *and* $\bar{\varphi}(S_3)$ *are pairwise distinct.*

We shall call a multipole $M=(V, E, S_1, ..., S_n)$ *proper* if $\bar{\varphi}(S_i)\neq 0$ for every 3-edge-coloring φ of M and every $i=1, ..., n$. Clearly, any $(1, 1)$-pole is a proper superedge and any $(1, 1, 1)$-pole is a proper supervertex. Less trivial proper superedges can be constructed by the following.

Let G be a cubic graph and let u_1, u_2 be two nonadjacent vertices of G. Then by G_{u_1, u_2} we denote the $(3, 3)$-pole (V, E, S_1, S_2) arising after deleting u_1 and u_2 and replacing the edges incident with u_1 (resp. u_2) by semiedges of S_1 (resp. S_2).

LEMMA 3. *Let* G *be a snark and let* u_1, u_2 *be two nonadjacent vertices of* G. *Then the* $(3, 3)$-*pole* G_{u_1, u_2} *is a proper superedge.*

Proof. This follows from Corollary 1 and the fact that if $\bar{\varphi}(S_1)=\bar{\varphi}(S_2)=0$ then φ gives rise to a 3-edge-coloring of the snark G. ∎

Let $G=(V, E)$ be a cubic graph. Replace each edge $e\in E$ by a superedge $\mathscr{E}(e)$ and each vertex $v\in V$ by a supervertex $\mathscr{V}(v)$. Assume that if $v\in V$ is incident with $e\in E$, then one connector of $\mathscr{V}(v)$ is accompanied with one connector of $\mathscr{E}(e)$ and that these two connectors have the same cardinality. Join the semiedges of the accompanied pairs of connectors. The resulting cubic graph we shall call a *superposition* of G with $\mathscr{V}$ and $\mathscr{E}$ (or, simply, a superposition of G) and denote by $G(\mathscr{V}, \mathscr{E})$.

Furthermore, if $\mathscr{E}(e)$ is proper for every $e\in E$, then $G(\mathscr{V}, \mathscr{E})$ is called a *proper superposition* of G. The following theorem is the fundamental tool in our construction.

THEOREM 4. *If* G *is a snark and* $G(\mathscr{V}, \mathscr{E})$ *is a proper superposition of* G, *then* $G(\mathscr{V}, \mathscr{E})$ *is a snark.*

Proof. Suppose φ is a 3-edge-coloring of $G(\mathscr{V}, \mathscr{E})$. For any $e\in E$, let $\psi(e)$ denote the value $\bar{\varphi}(S_1)=\bar{\varphi}(S_2)$ (see Corollary 1), where S_1, S_2 are the connectors of the superedge $\mathscr{E}(e)$. For any $e\in E$, $\psi(e)\neq 0$ because $\mathscr{E}(e)$ is proper. Then, by Corollary 2, ψ is a 3-edge-coloring of G, which is impossible because G is a snark. ∎

We require that our superedges be proper, but we place no such restriction on supervertices. This will be used in our construction, replacing vertices by supervertices without small cycles and edges by proper superedges.

4. The Main Construction

Let W_1, W_2 and W_3 be the three subsets of the vertex set of the Petersen graph depicted in Figs. 1, 2, and 3, respectively.

Lemma 4. *Any cycle in the Petersen graph contains at least one vertex from each of W_1, W_2, and W_3.*

Proof. This follows from the fact that if the Petersen graph is denoted by P, then $P - W_1$, $P - W_2$, and $P - W_3$ are trees. ▮

Now we can prove Theorem 1.

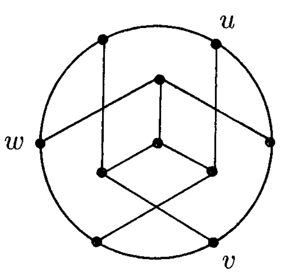
Proof of Theorem 1. The proof is composed from two steps.

Step 1. Let G be a cyclically 5-edge-connected cubic graph of order n and with girth c. Let $M^{(3)} = (V, E, S_1, S_2, S_3)$ be a $(1, 1, 3)$-pole arising as follows: choose a 2-path $u_1 u_2 u_3$ in G. Let $\{v_1, ..., v_5\}$ be the neighbor set of $\{u_1, u_2, u_3\}$ in G. Delete $\{u_1, u_2, u_3\}$ from G and add new semiedges $(v_1), ..., (v_5)$. Set $S_1 = \{(v_1)\}$, $S_2 = \{(v_2)\}$, and $S_3 = \{(v_3), (v_4), (v_5)\}$. Clearly,

$$d(v_i, v_j) \geqslant c - 4 \tag{3}$$

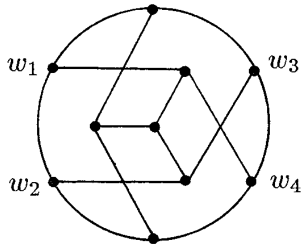
for any $i, j \in \{1, ..., 5\}$, $i \neq j$.

Let $M^{(2)}$ be the $(3, 3)$-pole $P_{u,v}$, where P is the copy of the Petersen graph from Fig. 1. By Lemma 3, $M^{(2)}$ is a proper superedge.



$W_1 = \{u, v, w\}$

Figure 1



$W_2 = \{w_1, w_2, w_3, w_4\}$

FIGURE 2

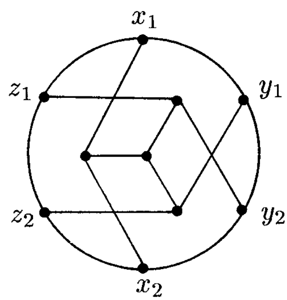
Take the Petersen graph P depicted in Fig. 2. Replace the vertices from W_2 by four copies of the $(1, 1, 3)$-pole $M^{(3)}$ and the edges (w_1, w_2), (w_3, w_4) by two copies of the $(3, 3)$-pole $M^{(2)}$, leaving the rest of P unchanged (more precisely, all other edges are replaced by copies of the $(1, 1)$-pole consisting of an isolated edge and the other vertices are replaced by copies of the $(1, 1, 1)$-pole consisting of a vertex incident with three semiedges). The two copies of w we shall denote by w and w'. Join the semiedges as indicated in Fig. 4 and denote the resulting graph by G'. By Lemma 3, G' is a proper superposition of P. Thus, from Theorem 4, Lemma 4, and (3) it follows:

(a) G' is a cyclically 4-edge-connected snark of order $4n+10$ and with girth 5;

(b) any cycle of G' not containing w or w' has length at least c.

Step 2. Denote by $H^{(2)}$ the $(3, 3)$-pole $G'_{w, w'}$ (see Fig. 4). It has $4n+8$ vertices and, by Lemma 3, it is a proper superedge.

Suppose $c \geqslant 7$. Let $H^{(3)} = (V, E, S_1, S_2, S_3)$ be a $(1, 3, 3)$-pole arising as follows: take a copy of G and a 4-path $r_1, ..., r_5$ in G. Let $\{t_1, ..., t_7\}$ be the neighbor set of $\{r_1, ..., r_5\}$ in G. Delete $\{r_1, ..., r_5\}$ from G and add



$W_3 = \{x_1, x_2, y_1, y_2, z_1, z_2\}$

FIGURE 3

new semiedges $(t_1), \ldots, (t_7)$. Set $S_1 = \{(t_1)\}$, $S_2 = \{(t_2), (t_3), (t_4)\}$, and $S_3 = \{(t_5), (t_6), (t_7)\}$. Suppose the notation is chosen so that

$$d(t_i, t_j) \geqslant c - 3 \tag{4}$$

holds in $H^{(3)}$ for any $t_i, t_j \in S_2(S_3)$, $i \neq j$. Clearly,

$$d(t_i, t_j) \geqslant c - 6 \tag{5}$$

for any $i, j \in \{1, \ldots, 7\}$, $i \neq j$.

Let P be the Petersen graph depicted in Fig. 3. Replace the vertices x_1 and x_2 by copies of $H^{(3)}$, the vertices y_1, y_2, z_1, and z_2 by copies of $M^{(3)}$ and the edges (x_1, y_1), (x_1, z_1), (x_2, y_2), and (x_2, z_2) by copies of $H^{(2)}$

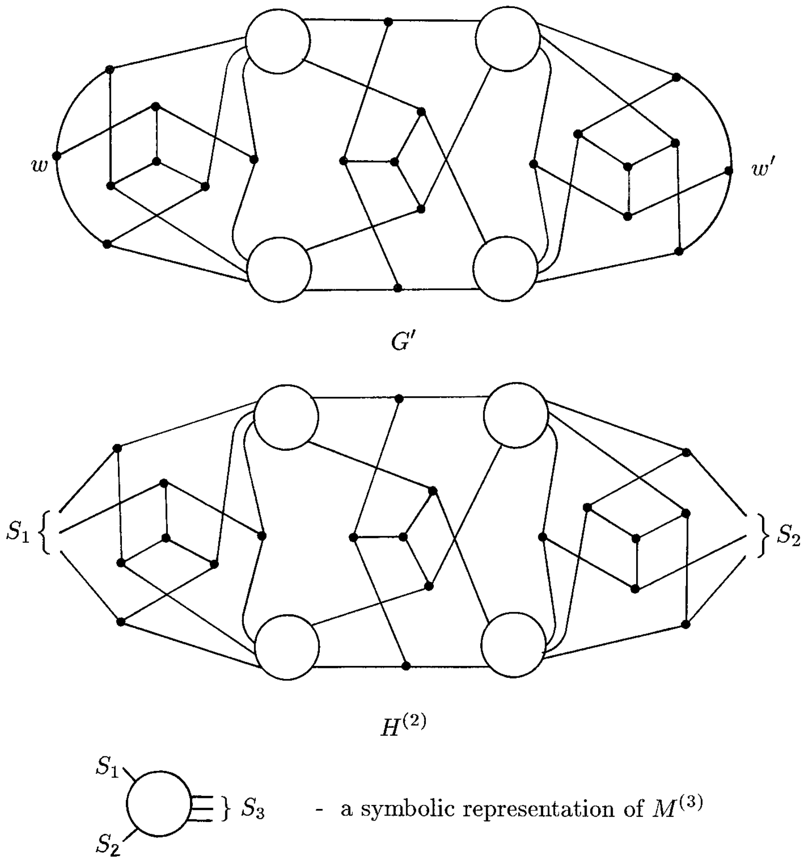


FIGURE 4

leaving the rest of P unchanged. Join the corresponding semiedges of the copies of $M^{(3)}$, $H^{(3)}$ and $H^{(2)}$ as indicated in Fig. 5 to obtain $S(G)$. The latter graph is cubic of girth at least c, has $22n+14$ vertices, and is a proper superposition of P. Thus, by Theorem 4, $S(G)$ is a snark. We can check that any cyclic 4-edge-cut of $H^{(2)}$ is no cut in $S(G)$, and, therefore, $S(G)$ has cyclic edge-connectivity 5.

If $c=6$, then Step 2 can be done analogously but we must allow $t_i=t_j$ for $i\neq j$ and similarly, if $c=5$ and r_1, r_5 are nonadjacent vertices. If (r_1, r_5)

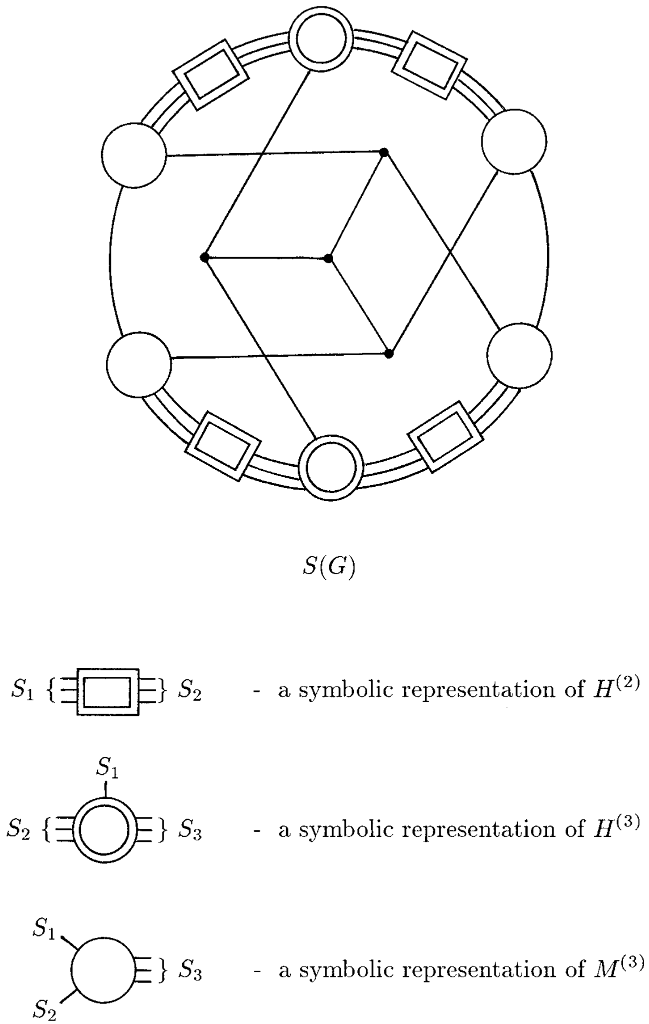


FIGURE 5

is an edge of G then let t_i denote the neighbor of r_i $(i = 1, ..., 5)$ and after deleting $\{r_1, ..., r_5\}$ we add semiedges $(t_1), ..., (t_5)$ and two adjacent semiedges s_1, s_2, forming an isolated edge. Then set $S_1 = \{(t_3)\}$, $S_2 = \{s_1, (t_1), (t_2)\}$, $S_3 = \{s_2, (t_4), (t_5)\}$ and continue similarly as for $c \geqslant 7$.

Finally note, that since $c \geqslant 5$ we can additionally suppose that G has a cycle C with length c such that C does not contain the vertices u_1, u_2, and u_3. Then $M^{(3)}$, G', and $S(G)$ contain a copy of C, and, therefore, $S(G)$ has girth equal to c. ∎

5. Two Additional Constructions

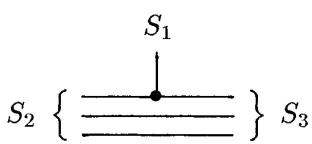
Now we shall present two variants of Theorem 1. Both of them we get after changing Step 2 in the above proof. The first improvement is possible for smaller girths.

Theorem 5. *Let G be a cyclically 5-edge-connected cubic graph of order n and with girth $9 \geqslant c \geqslant 5$. Then there exists a cyclically 5-edge-connected snark $R(G)$ of order $20n + 26$ and with girth c.*

Proof. $R(G)$ is constructed similarly as $S(G)$ with the exception that $H^{(3)}$ will be replaced by the $(1, 3, 3)$-pole H from Fig. 6. We can check that Fig. 5 can be completed such that the resulting graph $R(G)$ has girth c. Special attention must be given if $c = 9$. Figure 7 shows how H must be put into the graph $R(G)$ in order to guarantee that the girth stays as high as 9. The rest of the proof is similar as above. ∎

The second improvement can be obtained if we decrease the cyclic edge-connectivity.

Theorem 6. *Let G be a cyclically 4-edge-connected cubic graph of order n and with girth $c \geqslant 4$. Then there exists a cyclically 4-edge-connected snark $T(G)$ of order $12n + 10$ and with girth c.*



H

Figure 6

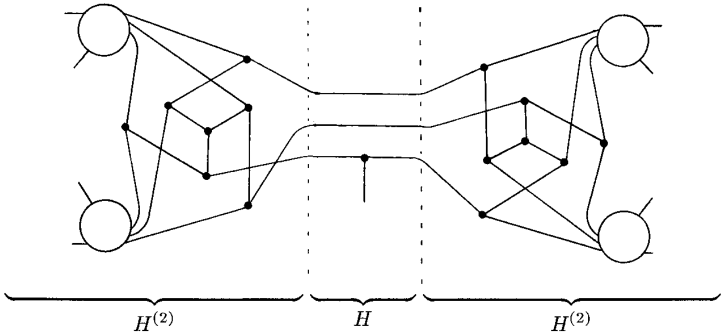


FIGURE 7

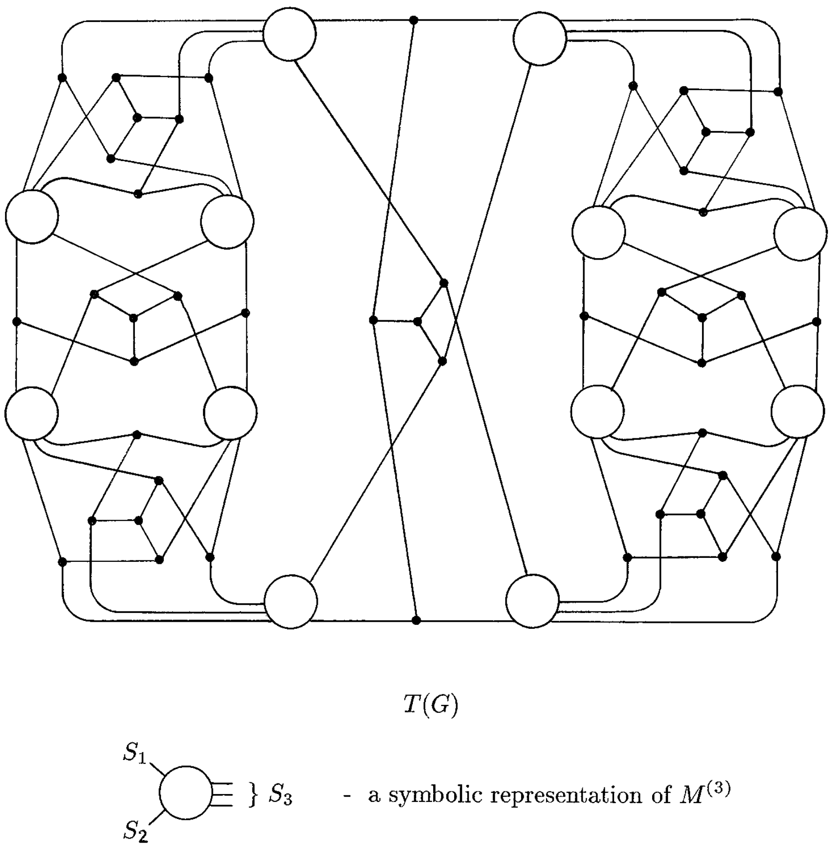
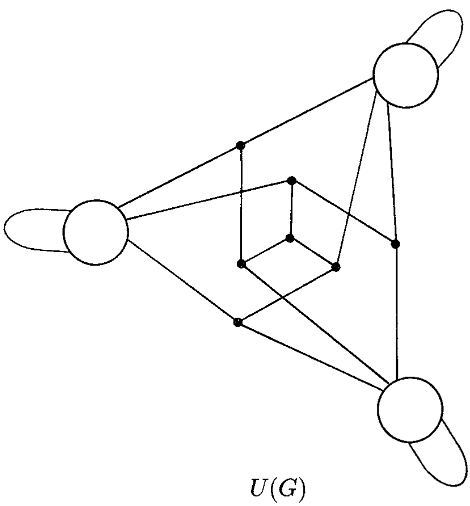


FIGURE 8

Proof. Construct G' and $H^{(2)}$ similarly as in the proof of Theorem 1. Then $T(G)$ is constructed similarly as G' with the exception that $H^{(2)}$ is used instead of $M^{(2)}$. The resulting graph $T(G)$ is depicted in Fig. 8. The rest of the proof is similar as above. ▮

The smallest cubic graph with girth 7 has 24 vertices (see McGee [24] or [4, p. 237; 26, p. 107]). Then using Theorems 6 and 5 we can construct cyclically 4- and 5-edge-connected snarks with girth 7 and of orders 298 and 506, respectively. They are the smallest counterexamples to the girth conjecture that can be obtained by our methods.

Applying the well-known trivial construction (see, e.g., [33] or [7]) we can construct a cyclically 3-edge-connected cubic graph $U(G)$ (depicted in Fig. 9) from a copy of the Petersen graph from Fig. 1 and three copies of $M^{(3)}$. $U(G)$ has order $3n-2$ and girth at most $c-3$. Clearly, from Lemma 1 follows that any 3-edge-coloring of $U(G)$ would provide a 3-edge-coloring of the Petersen graph. Thus $U(G)$ cannot be 3-edge-colored. Notice that $U(G)$ is not cyclically 4-edge-connected and, therefore, it is no snark. But we can easily check, that $T(G)$ can be obtained as dot product (see Isaacs [14] or [7, 33]) of four copies of $U(G)$ and three copies of the Petersen graph. This is no surprise because, as pointed out in [21, 22], the



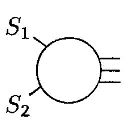


FIGURE 9

dot product is a special case of superposition. Thus counterexamples to the girth conjecture can be constructed with methods known before the superposition. On the other hand, the fact that the construction has not been obtained until now gives rise to the conviction that superposition is the method which is substantial for the solution of the girth conjecture. Moreover, applying superposition we obtain counterexamples with cyclic edge-connectivity 5, which cannot be done if we use the dot product.

We have shown that there exist cyclically 5-edge-connected snarks with large girths. In [14, 20, 21, 22] are constructed infinite families of cyclically 6-edge-connected snarks. Unfortunately we cannot increase this parameter supporting another conjecture of Jaeger and Swart [18] that any snark has cyclic edge-connectivity at most 6. Note that we have a suspicion that there could exist cyclically 6-edge-connected snarks with arbitrary large girths, but until now we do not have any idea how to construct such snarks.

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