Calculation of plates of variable rigidity on elastic foundation with variable coefficient of subgrade reaction

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Abstract

In the present paper is given the calculation of plates on an elastic basis with variable and constant coefficient of subgrade reaction. The calculation of plates bending is carried out by the finite element method. The calculation results are compared for different models of plate and an elastic basis. For a two-layer plate on an elastic basis, which has a heterogeneity in the plan, the results of calculation taking into account the increase of the height of the upper structure.

Keywords: Finite element method; Multilayer structure; Plate on an elastic basis; System «structure-foundation-base»; Models of basis.

1. Introduction

In the paper presents the results of calculations of the system «structure-foundation-base» using the two-layer and the single-layer slabs models on an elastic basis with variable and constant coefficients of subgrade reaction. The method of calculating two-layer slab with variable flexural rigidity on an elastic foundation was described in the author’s previous articles [1, 2], while in the present paper variable coefficients of subgrade reaction are taken into account. To solve this problem were used the finite element method.

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2. Statement of the problem

The problem refers to the calculation of the foundation slabs, loaded with the weight of the structure and contacting with the basis [1-5]. In the traditional statement of the problem is considered a single-layer slab, loaded with regular distributed load $q$, at the same time the weight of the structure and the weight of the foundation slabs are reduced to the top side (Fig. 1, b, d). In the present paper is given the calculation of a two-layer slab, the lower layer of which simulates the foundation, and the upper one - the structure, at the same time is considered the weight of each layer (Fig. 1, a, c).

Let us consider the bending of the two-layer slab freely supported on an elastic basis of Winkler’s type (Fig. 1, a, c), with the following characteristics: $a_1 = b_1 = 30 \text{ m}$, $\nu = 0.2$; $h_1 = 1 \text{ m}$, $E_1 = 10^7 \text{ kPa}$, $\gamma_1 = 25 \text{ kN/m}^3$; in the center of the plate: $a_2 = b_2 = 20 \text{ m}$, $h_2 = 0, 3, 6, 9, 12 \text{ m}$, $E_2 = 10^6 \text{ kPa}$, $\gamma_2 = 2.5 \text{ kN/m}^3$; in other part of the plate: $h_2 = 0$, $E_2 = 0$, $\gamma_2 = 0$ (Fig. 1, a). The coefficient of subgrade reaction at each point of the grid is different, where $C_2=0.3 \cdot C_1$; $C_3=0.4 \cdot C_1$; $C_4=0.6 \cdot C_1$; $C_5=0.7 \cdot C_1$; $C_6=0.8 \cdot C_1$; $C_7=0.8 \cdot C_1$; $C_8=0.9 \cdot C_1$; $C_9=0.9 \cdot C_1$; $C_{10}=0.9 \cdot C_1$; $C_1 = 10000 \text{ kN/m}^3$ (Fig. 2). For comparison, considered the variant, when over the entire area of the plate the coefficient of subgrade reaction $C_0 = C_1=\text{const}$ (Fig. 1, c).

![Figure 1](image-url)

Figure 1. The two-layer (a, c) and single-layer (b, d) plates models on an elastic basis with variable (a, b) and constant (c, d) coefficients of subgrade reaction.

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**Nomenclature**

- $a, b$: in-plane dimensions of plate
- $h_i$: thickness of plate
- $\gamma$: dead load (weight of material)
- $E$: modulus of elasticity
- $\nu$: Poisson's ratio
- $q$: regular distributed load
- $C$: coefficient of subgrade reaction
- $\sigma_x, \sigma_y$: normal stresses
- $\tau_{xy}$: shear stresses
Also, the problem considered in the traditional statement. We will calculate single-layer plate on an elastic foundation of Winkler’s with the characteristics which are specified higher for the lower layer of the plate. The bending of this plate is considered under the action of the variable surface load (Fig. 1, b). Also considered the variant, when the coefficient of subgrade reaction $C_0 = C_1 = \text{const}$ (Fig. 1, d).

3. Solution of the problem

The problem is solved using the finite element method [6, 7]. Ideas of the displacement method by using the finite element method were very popular, however in some cases application of ideas of the mixed method is very productive. In this case, the deflection and two distributed bending moment acting in two mutually perpendicular directions in each point of grid are taken unknown.

In the calculations the two-layer and single-layer plates are divided into 36 parts (6x6). The model is symmetric about the axes $Ox$ and $Oy$, is therefore considered only a quarter of the plate (Fig. 2).

Fig. 2. The partitioning scheme of quarter plate.

In order to compare the results of the calculations in Table 1 are given the values of the maximum bending moments, shear forces and the maximum vertical displacements, which are obtained by calculating the two-layer and single-layer plates models on an elastic basis with variable and constant modulus of subgrade reaction. The table shows the results for the height of the structure equal to 9 m.

<table>
<thead>
<tr>
<th>The calculation model</th>
<th>Coefficient of subgrade reaction</th>
<th>The maximum value of vertical displacements, (mm)</th>
<th>The maximum value of bending moments, (kNm / m)</th>
<th>The maximum value of shear forces, (kNm / m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The single-layer plate model</td>
<td>Constant</td>
<td>7,52</td>
<td>57,42</td>
<td>9,16</td>
</tr>
<tr>
<td></td>
<td>Variable</td>
<td>10,17</td>
<td>145,25</td>
<td>20,23</td>
</tr>
<tr>
<td>The two-layer plate model</td>
<td>Constant</td>
<td>5,71</td>
<td>1135,88</td>
<td>192,13</td>
</tr>
<tr>
<td></td>
<td>Variable</td>
<td>5,51</td>
<td>2196,25</td>
<td>350,23</td>
</tr>
</tbody>
</table>
The stress can be determined using the maximum values of the internal forces, according to the equations given in [1, 2]. The values of the normal stresses are presented in the form of diagrams $\sigma_x (\sigma_y)$ and $\sigma_z$ (Fig. 3, a, b) and shear stresses - in the form of diagrams $\tau_{xy}$ (Fig. 3, c), which are built on the basis of the internal forces from the calculation of the single-layer and the two-layer model plates using the method of finite elements. The height of the structure $h_2$ (or the thickness of the upper layer) is equal to 9 m.

Diagrams of stress $\sigma_x, \sigma_y$ are constructed for the cross section $x=5, y=0; \sigma_z$ - the cross section $x=0, y=0; \tau_{xy}$ - cross section $x=5, y=5$. For Figure 3, the following designations: 1 – a single-layer model of plate on elastic foundation with variable coefficient of subgrade reaction; 2 – a single-layer model of plate on elastic foundation with constant coefficient of subgrade reaction; 3 - a two-layer model of plate on elastic foundation with variable coefficient of subgrade reaction; 4 - a two-layer model of plate on an elastic foundation with constant coefficient of subgrade reaction.

The Figure 4 shows the diagrams of normal $\sigma_x, \sigma_z$ and shear $\tau_{xy}$ stresses for a two-layer plate of variable rigidity on elastic foundation Winkler’s type with variable coefficient of subgrade reaction. The results of calculation received with the increasing height structure. The numbers of the graphs correspond to values $h_2$: 1 - $h_2 = 0, 2 - h_2 = 3 \text{ m}, 3 - h_2 = 6 \text{ m}, 4 - h_2 = 9 \text{ m}, 5 - h_2 = 12 \text{ m}.$

Note the following features of the normal and shear stress diagrams in two-layer plates (see Fig. 3, 4):

- The diagrams of normal stresses $\sigma_x$ and shear stresses $\tau_{xy}$ on the interface of the two layers have a discontinuous jump, because the elastic modulus of foundation and structure materials are different.
• Diagrams of normal stresses $\sigma_z$ have the inflexion point on the border of the two layers.

Considering the overall differences between the statements of the problem, shown in Fig. 1, we note the following differences in the results. With consideration of the two-layer and the single-layer beam models with the same characteristics of the base, the values of internal forces, generated in two-layer beams, are obtained much more.

![Graphs showing normal and shear stresses](image)

Fig. 4. The diagrams of normal $\sigma_x$ (a), $\sigma_y$ (b) and shear $\tau_{xy}$ (c) stresses.

4. Conclusions

On the basis of the calculations we can make the following conclusion: for a more reliable determining the stress-strain state of the system «structure-foundation» on an elastic basis appropriate to carry out calculations by use of the contact model in the form of a two-layer plate on an elastic basis of Winkler’s type with variable coefficient of subgrade reaction, which is allows to take into account such factors as the changes of the rigidity base and the rigidity of the upper structure and formulas for the stresses given in [1, 2], can serve as for determining the stress-strain state of the system «structure-foundation».

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References


