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A fuzzy soft set theoretic approach to decision making problems

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Abstract

The problem of decision making in an imprecise environment has found paramount importance in recent years. A novel method of object recognition from an imprecise multiobserver data has been presented here. The method involves construction of a Comparison Table from a fuzzy soft set in a parametric sense for decision making.

Keywords: Fuzzy-soft-set; Resultant-fuzzy-soft-set; Comparison table; Object recognition

1. Introduction

A number of real life problems in engineering, social and medical sciences, economics etc. involve imprecise data and their solution involves the use of mathematical principles based on uncertainty and imprecision. Some of these problems are essentially humanistic and thus subjective in nature (e.g. human understanding and vision systems), while others are objective, yet they are firmly embedded in an imprecise environment. In recent years a number of theories have been proposed for dealing with such systems in an effective way. Some of these are theory of probability, fuzzy set theory [11–13], intuitionistic fuzzy sets [1,2], vague sets [3], theory of interval mathematics [2,4], rough set theory [8] etc. and these may be utilised as efficient tools for dealing with diverse types of uncertainties and imprecision embedded in a system. All these theories, however, are associated with an inherent limitation, which is the inadequacy of the parametrization tool associated with these theories.

Molodtsov [7] initiated a novel concept of soft theory as a new mathematical tool for dealing with uncertainties which is free from the above limitations. The soft set introduced by Molodtsov [7], Pawlak [10], etc. is a set associated with a set of parameters and has been applied in several directions.

In the present paper we present some results on an application of fuzzy-soft-sets in decision making problem. The problem of object recognition has received paramount importance in recent times. The recognition problem may be viewed as a multiobserver decision making problem, where the final identification of the object is based on the set of inputs from different observers who provide the overall object characterisation in terms of diverse sets of parameters.

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In this paper, we have presented a fuzzy-soft-set theoretic approach towards solution of the above decision making problem.

In the Section 2 we have presented a brief note on the preliminaries related to soft sets definitions centered around our problem. Section 3 deals with again the basics of Fuzzy soft sets and some relevant definitions. A decision making problem has been discussed and solved in the Section 4. We have some conclusions in the concluding Section 5.

2. Decision making in an imprecise environment

Most of our real life problems are imprecise in nature. The classical crisp mathematical tools are not capable of dealing with such problems. Fuzzy set theory has been used quite extensively to deal with such imprecisions [12,13].

2.1. Preliminaries

In this subsection, we present the basic definitions and results of soft set theory and fuzzy set theory which would be useful for subsequent discussions. Most of the definitions and results presented in this section may be found in [5–7].

Definition 1. Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subset E$.

A pair (F, A) is called a soft set over U, where F is a mapping given by

$$F : A \to P(U).$$

In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A).

Definition 2. For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

(i) $A \subset B$, and
(ii) $\forall \varepsilon \in A$, $F(\varepsilon)$ and $G(\varepsilon)$ are identical approximations.

We write $(F, A)\tilde{\subset}(G, B)$.

(F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\tilde{\supset}(G, B)$.

Definition 3. If (F, A) and (G, B) be two soft sets then "(F, A) AND (G, B)" denoted by $(F, A) \wedge (G, B)$ is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \bigcap G(\beta),\ \forall(\alpha, \beta) \in A \times B$.

Definition 4. If (F, A) and (G, B) be two soft sets then "(F, A) OR (G, B)" denoted by $(F, A) \vee (G, B)$ is defined by $(F, A) \vee (G, B) = (O, A \times B)$, where, $O(\alpha, \beta) = F(\alpha) \bigcup G(\beta),\ \forall(\alpha, \beta) \in A \times B$.

3. Fuzzy soft sets in decision making

In this section we present fuzzy-soft-set and some results of it. Most of them may be found in [5].

Let $U = \{o_1, o_2, \ldots, o_k\}$ be a set of k objects, which may be characterised by a set of parameters $\{A_1, A_2, \ldots, A_i\}$. The parameter space E may be written as $E \supseteq \{A_1 \cup A_2 \cup \cdots \cup A_i\}$. Let each parameter set A_i represent the ith class of parameters and the elements of A_i represents a specific property set. Here we assume that these property sets may be viewed as fuzzy sets.

In view of above we may now define a fuzzy soft set (F_i, A_i) which characterises a set of objects having the parameter set A_i.

Definition 5. Let $\mathscr{P}(U)$ denotes the set of all fuzzy sets of U. Let $A_i \subset E$.

A pair (F_i, A_i) is called a fuzzy-soft-set over U, where F_i is a mapping given by

$$F_i : A_i \to \mathscr{P}(U).$$

Definition 6. For two fuzzy-soft-sets (F, A) and (G, B) over a common universe U, (F, A) is a fuzzy-soft-subset of (G, B) if

(i) $A \subset B$, and
(ii) $\forall \varepsilon \in A$, $F(\varepsilon)$ is a fuzzy subset of $G(\varepsilon)$.

We write $(F, A)\tilde{\subset}(G, B)$.

(F, A) is said to be a fuzzy soft super set of (G, B), if (G, B) is a fuzzy-soft-subset of (F, A). We denote it by $(F, A)\tilde{\supset}(G, B)$.

In view of the above discussions, we now present an example below.

Example. Consider two fuzzy-soft-sets (F, A) and (G, B) over the same universal set $U = \{h_1, h_2, h_3, h_4, h_5\}$. Here U represents the set of houses, $A = \{\text{blackish, reddish, green}\}$ and $B = \{\text{blackish, reddish, green, large}\}$, and

$$F\ (\text{blackish}) = \{h_1/.4, h_2/.6, h_3/.5, h_4/.8, h_5/1\},$$

$$F\ (\text{reddish}) = \{h_1/1, h_2/.5, h_3/.5, h_4/1, h_5/.7\},$$

$$F\ (\text{green}) = \{h_1/.5, h_2/.6, h_3/.8, h_4/.8, h_5/.7\},$$

$$G\ (\text{blackish}) = \{h_1/.4, h_2/.7, h_3/.6, h_4/.9, h_5/1\},$$

$$G\ (\text{reddish}) = \{h_1/1, h_2/.6, h_3/.5, h_4/1, h_5/1\},$$

$$G\ (\text{green}) = \{h_1/.6, h_2/.6, h_3/.9, h_4/.8, h_5/1\},$$

$$G\ (\text{large}) = \{h_1/.4, h_2/.6, h_3/.5, h_4/.8, h_5/1\}.$$

Clearly, $(F, A)\tilde{\subset}(G, B)$.

Definition 7. If (F, A) and (G, B) be two fuzzy-soft-sets then "(F, A) AND (G, B)" is a fuzzy-soft-set denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha)\tilde{\cap}G(\beta),\ \forall \alpha \in A$ and $\forall \beta \in B$, where $\tilde{\cap}$ is the operation 'fuzzy intersection' of two fuzzy sets.

Comparison table is a square table in which the number of rows and number of columns are equal, rows and columns both are labelled by the object names $o_1, o_2, o_3, \ldots, o_n$ of the universe, and the entries are $c_{ij},\ i, j = 1, 2, \ldots, n$, given by $c_{ij} =$ the number of parameters for which the membership value of o_i exceeds or equal to the membership value of o_j.

Clearly, $0 \leqslant c_{ij} \leqslant k$, and $c_{ii} = k,\ \forall i, j$ where, k is the number of parameters present in a fuzzy soft set.

Thus, c_{ij} indicates a numerical measure, which is an integer number and o_i dominates o_j in c_{ij} number of parameters out of k parameters.

The row sum of an object o_i is denoted by r_i and is calculated by using the formula,

$$r_i = \sum_{j=1}^{n} c_{ij}.$$

Clearly, r_i indicates the total number of parameters in which o_i dominates all the members of U.

Likewise, the column sum of an object o_j is denoted by t_j and may be computed as

$$t_j = \sum_{i=1}^{n} c_{ij}.$$

The integer t_j indicates the total number of parameters in which o_j is dominated by all the members of U.

The score of an object o_i is S_i may be given as

$$S_i = r_i - t_i.$$

The problem here is to choose an object from the set of given objects with respect to a set of choice parameters P. We now present an algorithm for identification of an object, based on multiobservers input data characterised by colour, size and surface texture features.

3.1. Algorithm

1. Input the fuzzy-soft-sets (F, A), (G, B) and (H, C).
2. Input the parameter set P as observed by the observer.
3. Compute the corresponding resultant-fuzzy-soft-set (S, P) from the fuzzy soft sets (F, A), (G, B), (H, C) and place it in tabular form.
4. Construct the Comparison-table of the fuzzy-soft-set (S, P) and compute r_i and t_i for o_i, $\forall i$.
5. Compute the score of o_i, $\forall i$.
6. The decision is S_k if, $S_k = \max_i S_i$.
7. If k has more than one value then any one of o_k may be chosen.

4. Application in a decision making problem

Let $U = \{o_1, o_2, o_3, o_4, o_5, o_6\}$, be the set of objects having different colours, sizes and surface texture features. The parameter set, E={blackish, dark brown, yellowish, reddish, large, small, very small, average, very large, course, moderately course, fine, extra fine}. Let A, B and C denote three subsets of the set of parameters E. Also let A represent the colour space and B represents the size of the object. A = {blackish, dark brown, yellowish, reddish}, B = {large, very large, small, very small, average}. The subset C represents the surface texture granularity i.e. C = {course, moderately course, fine, extra fine}.

Assuming that the fuzzy-soft-set (F, A) describe the 'objects having colour space', the fuzzy-soft-set (G, B) describes the 'objects having size' and the fuzzy-soft-set (H, C) describes the 'texture feature of the object surface'. The problem is to identify an unknown object from the multiobservers fuzzy data, specified by different observers, in terms of fuzzy soft sets (F, A), (G, B) and (H, C), as specified earlier. These fuzzy soft sets may be computed as below.

The fuzzy-soft-set (F, A) is defined as (F, A)={objects having blackish colour={$o_1/.3, o_2/.3, o_3/.4, o_4/.8, o_5/.7, o_6/.9$}, objects having dark brown colour ={$o_1/.4, o_2/.9, o_3/.5, o_4/.2, o_5/.3, o_6/.2$}, objects having yellowish colour ={$o_1/.6, o_2/.3, o_3/.8, o_4/.4, o_5/.6, o_6/.4$}, objects having reddish colour ={$o_1/.9, o_2/.5, o_3/.7, o_4/.8, o_5/.5, o_6/.3$}}.

The fuzzy-soft-set (G, B) is defined as (G, B) = {objects having large size = {$o_1/.4, o_2/.8, o_3/.6, o_4/.9, o_5/.2, o_6/.3$}, objects having very large size ={$o_1/.2, o_2/.6, o_3/.4, o_4/.8, o_5/.1, o_6/.2$}, objects having small size ={$o_1/.8, o_2/.3, o_3/.4, o_4/.2, o_5/.9, o_6/.8$}, objects having very small size ={$o_1/.6, o_2/.1, o_3/.1, o_4/.1, o_5/.8, o_6/.6$}, objects having average size ={$o_1/.5, o_2/.7, o_3/.7, o_4/.6, o_5/.7, o_6/.5$}}

The fuzzy-soft-set (H, C) is defined as (H, C) = {objects having course texture = {$o_1/.3, o_2/.6, o_3/.5, o_4/.7, o_5/.6, o_6/.8$}, objects having moderately course texture ={$o_1/.4, o_2/.5, o_3/.6, o_4/.6, o_5/.6, o_6/.7$}, objects having fine texture ={$o_1/.1, o_2/.4, o_3/.3, o_4/.6, o_5/.5, o_6/.7$}, objects having extra fine texture ={$o_1/.9, o_2/.5, o_3/.6, o_4/.3, o_5/.4, o_6/.9$}} The tabular representation of the fuzzy-soft-sets (F, A), (G, B) and (H, C) are shown in Figs. 1(a)–(c), respectively.

Let (F, A) and (G, B) be any two fuzzy-soft-sets over the common universe U. After performing some operations (like AND, OR etc.) on the fuzzy-soft-sets for some particular parameters of A and B, we obtain another fuzzy-soft-set. The newly obtained fuzzy-soft-set is termed as resultant-fuzzy-soft-set of (F, A) and (G, B).

(a)

U	‘blackish = a_1’	‘dark brown = a_2’	‘yellowish = a_3’	‘reddish = a_4’
o_1	0.3	0.4	0.6	0.9
o_2	0.3	0.9	0.3	0.5
o_3	0.4	0.5	0.8	0.7
o_4	0.8	0.2	0.4	0.8
o_5	0.7	0.3	0.6	0.5
o_6	0.9	0.2	0.4	0.3

(b)

U	‘large = b_1’	‘very large = b_2’	‘small = b_3’	‘very small = b_4’	‘average = b_5’
o_1	0.4	0.2	0.8	0.6	0.5
o_2	0.8	0.6	0.3	0.1	0.7
o_3	0.6	0.4	0.4	0.1	0.7
o_4	0.9	0.8	0.2	0.1	0.4
o_5	0.2	0.1	0.9	0.8	0.7
o_6	0.3	0.2	0.8	0.6	0.5

(c)

U	‘course = c_1’	‘moderately course = c_2’	‘fine = c_3’	‘extra fine = c_4’
o_1	0.3	0.4	0.1	0.9
o_2	0.6	0.5	0.4	0.5
o_3	0.5	0.6	0.3	0.6
o_4	0.7	0.6	0.6	0.3
o_5	0.6	0.6	0.5	0.4
o_6	0.8	0.7	0.7	0.9

Fig. 1.

Considering the above two fuzzy-soft-sets (F, A) and (G, B) if we perform “(F, A) AND (G, B)” then we will have $4 \times 5 = 20$ parameters of the form e_{ij}, where $e_{ij} = a_i \wedge b_j$, for all $i = 1, 2, 3, 4.$ and $j = 1, 2, 3, 4, 5$. If we require the fuzzy-soft-set for the parameters $R = \{e_{11}, e_{15}, e_{21}, e_{24}, e_{33}, e_{44}, e_{45}\}$, then the resultant-fuzzy-soft-set for the fuzzy-soft-sets (F, A) and (G, B) will be (K, R), say.

So, after performing the “(F, A) AND (G, B)” for some parameters the tabular representation of the resultant-fuzzy-soft-set, say, will take the form as

U	‘e_{11}’	‘e_{15}’	‘e_{21}’	‘e_{24}’	‘e_{33}’	‘e_{44}’	‘e_{45}’
o_1	0.3	0.3	0.4	0.4	0.6	0.6	0.5
o_2	0.3	0.3	0.8	0.1	0.3	0.1	0.5
o_3	0.4	0.4	0.5	0.1	0.4	0.1	0.7
o_4	0.8	0.4	0.2	0.1	0.2	0.1	0.4
o_5	0.2	0.7	0.2	0.3	0.6	0.5	0.5
o_6	0.3	0.5	0.2	0.2	0.4	0.3	0.3

Let us now see how the algorithm may be used to solve our original problem. Consider the fuzzy-soft-sets $(F, A), (G, B)$ and (H, C) as defined above. Suppose that $P = \{e_{11} \wedge c_1, e_{15} \wedge c_3, e_{21} \wedge c_2, e_{24} \wedge c_4, e_{33} \wedge c_3, e_{44} \wedge c_3, e_{45} \wedge c_4\}$, be the set of choice parameters of an observer. On the basis of this parameter we have to take the decision from the availability set U. The tabular representation of resultant-fuzzy-soft-set (S, P) will be as

U	'$e_{11} \wedge c_1$'	'$e_{15} \wedge c_3$'	'$e_{21} \wedge c_2$'	'$e_{24} \wedge c_4$'	'$e_{33} \wedge c_3$'	'$e_{44} \wedge c_3$'	'$e_{45} \wedge c_4$'
o_1	0.3	0.1	0.4	0.4	0.1	0.1	0.5
o_2	0.3	0.3	0.5	0.1	0.3	0.1	0.5
o_3	0.4	0.3	0.5	0.1	0.3	0.1	0.6
o_4	0.7	0.4	0.2	0.1	0.2	0.1	0.3
o_5	0.2	0.5	0.2	0.3	0.5	0.5	0.4
o_6	0.3	0.5	0.2	0.2	0.4	0.3	0.3

The Comparison-table of the above resultant-fuzzy-soft-set is as below.

Table 1

	o_1	o_2	o_3	o_4	o_5	o_6
o_1	7	4	2	4	4	4
o_2	6	7	5	5	3	3
o_3	6	7	7	5	3	3
o_4	4	4	4	7	2	3
o_5	3	4	4	6	7	6
o_6	4	5	4	6	3	7

Next we compute the row-sum, column-sum, and the score for each o_i as shown below:

Table 2

	row-sum (r_i)	column-sum (t_i)	Score (S_i)
o_1	25	30	−5
o_2	29	31	−2
o_3	31	26	5
o_4	24	33	−9
o_5	30	22	8
o_6	29	26	3

From the above score table, it is clear that the maximum score is 8, scored by o_5 and the decision is in favour of selecting o_5.

5. Conclusion

In his pioneer work Molodtsov [7] originated the soft set theory as a general mathematical tool for dealing with uncertain, fuzzy, or vague objects. In the present paper we give an application of fuzzy soft set theory in object recognition problem. The recognition strategy is based on multiobserver input parameter data set. The algorithm involves Construction of Comparison table from the resultant fuzzy soft set and the final decision is taken based on the maximum score computed from the Comparison Table (Tables 1 and 2).

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Further reading

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