



ELSEVIER

SCIENCE @ DIRECT®

PHYSICS LETTERS B

Physics Letters B 599 (2004) 62–74

www.elsevier.com/locate/physletb

Top flavour violating decays in general supersymmetric models

D. Delépine^a, S. Khalil^{b,c}

^a Instituto de Física, Universidad de Guanajuato, Loma del Bosque, 103 Col. Loma del Campestre, CP 37150 Leon, Gto, Mexico

^b IPPP, Physics Department, Durham University DH1 3LE, Durham, UK

^c Ain Shams University, Faculty of Science, Cairo 11566, Egypt

Received 7 July 2004; received in revised form 9 August 2004; accepted 13 August 2004

Available online 25 August 2004

Editor: P.V. Landshoff

Abstract

We analyse the top flavour violating decays in general supersymmetric model using the mass insertion approximation. In particular, we discuss the impact of a light right-handed top-squark and large mixing between the first or second and third generation of up-squarks on processes as $t \rightarrow q\gamma, g$. We also take into account the relevant experimental constraints from B -physics and the requirements for a successful electroweak baryogenesis on squark mixings. We show that for general large mixings in squarks mass matrix, the branching ratio of the $t \rightarrow q\gamma, g$ ($q = u, c$) can be as large as 10^{-6} .

© 2004 Published by Elsevier B.V. Open access under [CC BY license](https://creativecommons.org/licenses/by/4.0/).

Keywords: Baryogenesis; Rare decays; Supersymmetry phenomenology

1. Introduction

The standard model of electroweak and strong interactions (SM) has had an impressive success when confronted with experiment. However, it has been established that the strength of CP violation in the standard model is not sufficient to account for the cosmological baryon asymmetry of the Universe (BAU) [1]. One of the most attractive mechanisms to generate the observed BAU is that of electroweak baryogenesis in supersymmetric (SUSY) extensions of the SM. It was shown that supersymmetric extensions of the SM have all the necessary requirements to generate enough BAU. In particular, SUSY models offer new sources of CP violation, and in the presence of a light stop the phase transition becomes much stronger [2]. However, the bound of the neutron electric dipole moment (EDM) imposes severe constraints on the flavour diagonal phases [3] and may possibly rule out scenarios of SUSY electroweak baryogenesis based on CP-violating chargino currents. A possible way to overcome this problem, and to generate enough BAU while satisfying the EDM constraints, is to assume that SUSY CP violation has a flavour

E-mail address: shaaban.khalil@durham.ac.uk (S. Khalil).

character as in the SM [4–6]. These models share the common features of requiring the presence of a light top-squark and predicting a large mixing between the third and first or second generations of up squarks. Since both the latter requirements play an important role in top-quark physics, one is likely to expect an enhancement in flavour changing top decays.

In SM, processes like $t \rightarrow u, c\gamma, g$ are absent at the tree level, and are highly suppressed by the GIM mechanism at the one-loop level. Within the SM, the prediction for the branching ratio (Br) of these decays is of order 10^{-13} [7]. Therefore, the observation of $t \rightarrow u, c\gamma$ decays, either at the LHC or at a future linear e^+e^- collider, will constitute a sign of new physics. In supersymmetric models, new channels (mainly through chargino and gluino exchange) emerge to compete with those of the SM.

In this Letter, we study flavour changing top decays as $t \rightarrow u, c\gamma, g$ in the minimal supersymmetric standard model (MSSM) with a light right-handed top-squark. In order to obtain a model-independent analysis of the low-energy MSSM, we will use the generalised mass insertion approximation (MIA). In this framework, a basis for fermions and sfermions is adopted in such a way that the couplings of these particles to neutral gauginos are flavour diagonal, while flavour-violating effects are encoded in the non-diagonality of the sfermion propagators. In addition, it is assumed that one of the eigenvalues of the up-squark mass matrix is much lighter than the other (degenerate) eigenvalues. We take into account the constraints that the relevant mass insertions in the up sector must fulfill in order to generate a successful baryogenesis at the electroweak scale and we consider the bounds on the squark mass insertions derived from experimental measurements of B decays. In view of the above, we investigate the possibility of observing flavour violating top decays at the LHC or at a forthcoming linear collider.

These flavour violating top decays have been previously studied in the literature [8–15]. In these papers, the top decay has been computed within the physical basis for all supersymmetric particles. Our results are fully compatible with Refs. [10–14]. In Ref. [11], the gluino contributions have been considered and in Ref. [10] they took into account only the chargino and neutralino SUSY contributions in both case for the flavor universal and non-universal soft SUSY breaking terms framework. All previous works [8,9] to these papers have been working within the framework of minimal supersymmetric standard model with flavor-universal soft SUSY breaking terms. Although, it is possible to make within the MSSM the full computation of FCNC processes as it has been done in all previous work on FCNC in top decays. However, given the variety of extensions of the MSSM, it is important to have a way to extract readily from all FCNC processes a set of upper limits on quantities easily computed in any SUSY frame [16]. One parametrization of FCNC which answers to this question is the so-called generalised mass insertion. In this framework, one works in a basis for the fermions and sfermions where all the couplings of these particles to neutral gauginos are flavour diagonal, while the flavour changing appears in the non-diagonality of the sfermions propagators. Denoting by Δ_{AB}^q , with $A, B = L, R$ and with $q = u, d$ for, respectively, the up (down) sfermions, the off-diagonal terms of the sfermions mass matrices, the sfermion propagators are expanded in series in terms of $\delta_{AB}^{u,d} = \Delta_{AB}^{u,d}/\tilde{m}^2$ where \tilde{m}^2 is the average sfermion mass. This approach is well defined as long as Δ_{AB}^q is significantly smaller than \tilde{m}^2 .

In this Letter, in order to be able to apply easily our approach to any supersymmetric models, we shall use this generalised mass insertion approximation. This approach presents two main advantages compared to previous work. First, it does not require a lot of computation to apply our results to any SUSY models. It is enough to get the predictions for the δ 's to get a prediction for $\text{Br}(t \rightarrow c\gamma, g)$.¹ Secondly, it is straightforward to use the limits on the δ 's coming from other FCNC processes in B or K physics.

The Letter is organised as follows. In Section 2 we provide analytical results of the SUSY contributions the amplitudes of $t \rightarrow q\gamma, g$ decays, using the generalised MIA. Section 3 is devoted to the presentation of the numerical results, analysing the constraints from BAU and FCNC processes and how they affect the branching ratio of these

¹ It is a great advantages compared to the full SUSY computation as done in the previous work where for each SUSY models, one has to diagonalize all sfermion and gaugino mass matrices to get the mixing and then to compute the branching ratio.

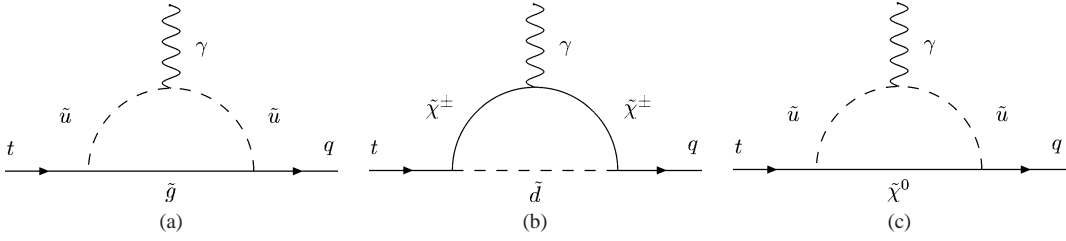


Fig. 1. Feynman diagrams for the decay $t \rightarrow q\gamma$: (a) gluino mediated, (b) chargino mediated and (c) neutralino exchange. On diagram (b) the photon line can be also coupled to the internal down-squark line.

decays. We also comment on the prospects of observing the $t \rightarrow q\gamma, g$ process in the upcoming experiments. Our conclusions are summarised in Section 4.

2. $t \rightarrow q\gamma$ in the MSSM with a light stop

The total amplitude for the $t \rightarrow q\gamma$ decay can be written as

$$\mathcal{A}_{\text{total}}(t \rightarrow q\gamma, g) = \sum_i (\mathcal{A}_{iR}^{\gamma,g} \mathcal{O}_{LR}^{\gamma,g} + \mathcal{A}_{iL}^{\gamma,g} \mathcal{O}_{RL}^{\gamma,g}), \quad (1)$$

where i denotes the mediator in the loop, and

$$\mathcal{O}_{LR}^{\gamma,g} = \epsilon_{\mu\bar{q}} i \sigma^{\mu\nu} p_\nu P_R t, \quad \mathcal{O}_{RL}^{\gamma,g} = \epsilon_{\mu\bar{q}} i \sigma^{\mu\nu} p_\nu P_L t, \quad (2)$$

with $\sigma^{\mu\nu} \equiv \frac{i}{2} [\gamma^\mu, \gamma^\nu]$, and p_ν the momentum of the outgoing photon (gluon).

In the SM, the $t \rightarrow q\gamma$ decay is mediated by charged W bosons, so that $\mathcal{A}_{\text{SM}}^\gamma = \mathcal{A}_W^\gamma$. In the framework of the MSSM, one finds four new sets of diagrams inducing the effective \mathcal{O}^γ operators, namely via the exchange of charged Higgs bosons, gluinos, charginos and neutralinos, as illustrated in Fig. 1. Thus the amplitude for the $t \rightarrow q\gamma$ decay can be parametrised as $\mathcal{A}_i^\gamma = \{\mathcal{A}_{\text{SM}}^\gamma, \mathcal{A}_{H^\pm}^\gamma, \mathcal{A}_g^\gamma, \mathcal{A}_{\tilde{\chi}^\pm}^\gamma, \mathcal{A}_{\tilde{\chi}^0}^\gamma\}$.

Both the SM and charged Higgs contributions rely on the Cabibbo–Kobayashi–Maskawa matrix (V_{CKM}) as the sole source of flavour violation. In what follows, we will study each of the above contributions. In particular, we will compute the sfermion mediated decays (gluino, chargino and neutralino) using the mass insertion approximation.

2.1. W and H^\pm contributions

For completeness we include here the SM contribution as well as the one associated with charged Higgs exchange. These contributions are given by [17]

$$\mathcal{A}_{W,R}^\gamma = \frac{\alpha_w \sqrt{\alpha}}{4\sqrt{\pi}} \frac{3m_t}{m_W^2} (V_{\text{CKM}})_{qb} (V_{\text{CKM}}^*)_{tb} x_{bW} [e_D F_1(x_{bW}) + F_2(x_{bW})], \quad (3)$$

$$\begin{aligned} \mathcal{A}_{H^\pm,R}^\gamma &= \frac{\alpha_w \sqrt{\alpha}}{4\sqrt{\pi}} \frac{m_t}{m_W^2} (V_{\text{CKM}})_{qb} (V_{\text{CKM}}^*)_{tb} x_{bh} \\ &\times \{ \tan^2 \beta [e_D F_1(x_{bh}) + F_2(x_{bh})] + [e_D F_3(x_{bh}) + F_4(x_{bh})] \}. \end{aligned} \quad (4)$$

In the above e_D is the charge of the down-type quarks running in the loop ($e_D = -1/3$), and $F_{1,2,3,4}$ the associated loop functions, given in Appendix A, with $x_{bW,h}$ defined as the mass ratios $x_{bW,h} = m_b^2/m_{W,H^\pm}^2$, respectively. α is the fine-structure constant and $\alpha_w = g_w^2/(4\pi)$ where g_w is the weak coupling constant. Due to the smallness of

the associated Yukawa couplings, the contribution of d and s quarks are negligible, and hence we consider only the dominant bottom-quark terms. We also neglect the contribution of the partial amplitudes $\mathcal{A}_{W(H^\pm),L}^\gamma$ which are suppressed by a factor of m_q/m_t when compared with $\mathcal{A}_{W(H^\pm),R}^\gamma$. Since $m_b \ll m_H, m_W$, one can easily obtain an estimate of the charged Higgs and W boson contributions to the $t \rightarrow q\gamma$ decays. One finds

$$\Gamma(t \rightarrow q\gamma) = \frac{m_t^3}{16\pi} |A_{W,R}^\gamma + A_{H,R}^\gamma|^2. \quad (5)$$

Regarding the associated branching ratio, $\text{Br}(t \rightarrow q\gamma)\Gamma(t \rightarrow q\gamma)/\Gamma_{\text{total}}$, let us recall that the total decay width of the top-quark is dominated by the $t \rightarrow bW$ channel, which is given by

$$\Gamma_t \approx \Gamma(t \rightarrow bW) = \frac{G_F}{8\sqrt{2}\pi} |V_{tb}|^2 m_t^3 \left(1 - \frac{m_W^2}{m_t^2}\right) \left(1 + \frac{m_W^2}{m_t^2} - 2\frac{m_W^4}{m_t^4}\right). \quad (6)$$

Thus, the W and charged Higgs contributions to the $\text{Br}(t \rightarrow q\gamma)$ are given, to a very approximation, by

$$\text{Br}(t \rightarrow q\gamma) \approx \frac{m_t^3}{16\pi} |A_{W,R}^\gamma + A_{H,R}^\gamma|^2 \frac{1}{\Gamma(t \rightarrow bW)}. \quad (7)$$

Numerically ($m_t = 174$ GeV, $m_b = 5$ GeV, $\tan\beta = 10$, $m_{H^\pm} \simeq 100$ GeV), one has

$$\begin{aligned} W: \quad & \text{Br}(t \rightarrow u\gamma) = 7.5 \times 10^{-15}, & \text{Br}(t \rightarrow c\gamma) = 6.3 \times 10^{-13}, \\ H^\pm: & \text{Br}(t \rightarrow u\gamma) = 4.6 \times 10^{-12}, & \text{Br}(t \rightarrow c\gamma) = 3.8 \times 10^{-10}. \end{aligned} \quad (8)$$

Mostly due to CKM suppression, both W and charged Higgs contributions are indeed very small, and we shall neglect them in our numerical analysis.

2.2. Gluino contribution

In the super-CKM basis, the quark–squark–gluino interaction is given by

$$\mathcal{L}_{u\tilde{u}\tilde{g}} = \sqrt{2}g_s T_{bc}^a (\bar{u}^b P_L \tilde{g}^a \tilde{u}_R^c - \bar{u}^b P_R \tilde{g}^a \tilde{u}_L^c + \text{H.c.}), \quad (9)$$

where T^a are the $SU(3)_c$ generators, and b, c are colour indices. As aforementioned, we will use the MIA to express the gluino contribution to the $t \rightarrow c\gamma$ amplitude. We begin by considering the *generalised* MIA scenario, in which one of the scalars in the loop (typically the right-handed top-squark), is considerably lighter than the other squarks, $m_{\tilde{t}_R}^2 \ll m_{\tilde{q}_{L,R}}^2$. In this case, the amplitude for the gluino mediated $t \rightarrow q\gamma$ decays reads:

$$\begin{aligned} \mathcal{A}_{\tilde{g},R}^\gamma &= -\frac{4}{3} \frac{\alpha_s \sqrt{\alpha}}{\sqrt{\pi}} e_U \frac{\langle \tilde{m}^2 \rangle}{m_{\tilde{g}}^2} \left\{ \frac{m_t}{m_{\tilde{g}}^2} (\delta_{LL}^u)_{q3} \left(\frac{F_2(z_t, z_{\tilde{q}_L}) - F_2(z_t, z_{\tilde{t}_L})}{z_{\tilde{q}_L} - z_{\tilde{t}_L}} \right) \right. \\ &\quad \left. - \frac{1}{m_{\tilde{g}}} (\delta_{LR}^u)_{q3} \left(\frac{F_4(z_t, z_{\tilde{q}_L}) - F_4(z_t, z_{\tilde{t}_R})}{z_{\tilde{q}_L} - z_{\tilde{t}_R}} \right) \right\}, \end{aligned} \quad (10)$$

$$\begin{aligned} \mathcal{A}_{\tilde{g},L}^\gamma &= -\frac{4}{3} \frac{\alpha_s \sqrt{\alpha}}{\sqrt{\pi}} e_U \frac{\langle \tilde{m}^2 \rangle}{m_{\tilde{g}}^2} \left\{ \frac{m_t}{m_{\tilde{g}}^2} (\delta_{RR}^u)_{q3} \left(\frac{F_2(z_t, z_{\tilde{q}_R}) - F_2(z_t, z_{\tilde{t}_R})}{z_{\tilde{q}_R} - z_{\tilde{t}_R}} \right) \right. \\ &\quad \left. - \frac{1}{m_{\tilde{g}}} (\delta_{RL}^u)_{q3} \left(\frac{F_4(z_t, z_{\tilde{q}_R}) - F_4(z_t, z_{\tilde{t}_L})}{z_{\tilde{q}_R} - z_{\tilde{t}_L}} \right) \right\}. \end{aligned} \quad (11)$$

In the above, e_U is the charge of the up-type quarks ($e_U = 2/3$), $z_{t,\tilde{q}}$ are, respectively, defined as the mass ratio $(m_{t,\tilde{q}}/m_{\tilde{g}})^2 \equiv 1/x_{t,\tilde{q}}$, and the loop functions $F_{2,4}(x, y)$ can be found in [Appendix A](#). $\langle \tilde{m}^2 \rangle$ is the mean value of the squark mass matrix. We have also neglected in $\mathcal{A}_{\tilde{g},R}^\gamma$ ($\mathcal{A}_{\tilde{g},L}^\gamma$) terms associated to δ_{RR}^u (δ_{LL}^u), since these would be suppressed by $m_{u,c}$.

In a scenario where one can approximate $m_{\tilde{t}_R}^2 \approx m_{\tilde{t}_L}^2 \approx m_{\tilde{q}_L}^2$, $m_{\tilde{g}} \gg m_t$, the finite differences would tend to the usual MIA derivatives.

Regarding the $t \rightarrow cg$ amplitude, one has

$$\begin{aligned} \mathcal{A}_{\tilde{g},R}^g = & -\frac{\alpha_s \sqrt{\alpha} \langle \tilde{m}^2 \rangle}{\sqrt{\pi} m_{\tilde{g}}^2} \left\{ \frac{m_t}{m_{\tilde{g}}^2} (\delta_{LL}^u)_{q3} \left(\frac{\tilde{F}_2(z_t, z_{\tilde{q}_L}) - \tilde{F}_2(z_t, z_{\tilde{t}_L})}{z_{\tilde{q}_L} - z_{\tilde{t}_L}} \right) \right. \\ & \left. - \frac{1}{m_{\tilde{g}}} (\delta_{LR}^u)_{q3} \left(\frac{\tilde{F}_4(z_t, z_{\tilde{q}_L}) - \tilde{F}_4(z_t, z_{\tilde{t}_R})}{z_{\tilde{q}_L} - z_{\tilde{t}_R}} \right) \right\}, \end{aligned} \quad (12)$$

$$\begin{aligned} \mathcal{A}_{\tilde{g},L}^g = & -\frac{4}{3} \frac{\alpha_s \sqrt{\alpha}}{\sqrt{\pi}} eU \frac{\langle \tilde{m}^2 \rangle}{m_{\tilde{g}}^2} \left\{ \frac{m_t}{m_{\tilde{g}}^2} (\delta_{RR}^u)_{q3} \left(\frac{\tilde{F}_2(z_t, z_{\tilde{q}_R}) - \tilde{F}_2(z_t, z_{\tilde{t}_R})}{z_{\tilde{q}_R} - z_{\tilde{t}_R}} \right) \right. \\ & \left. - \frac{1}{m_{\tilde{g}}} (\delta_{RL}^u)_{q3} \left(\frac{\tilde{F}_4(z_t, z_{\tilde{q}_R}) - \tilde{F}_4(z_t, z_{\tilde{t}_L})}{z_{\tilde{q}_R} - z_{\tilde{t}_L}} \right) \right\}, \end{aligned} \quad (13)$$

where the functions $\tilde{F}_{2,4}$ are defined as

$$\tilde{F}_{2,4}(x, y) = \left[\frac{4}{3} - \frac{C(G)}{2} \right] F_{2,4}(x, y) - \frac{C(G)}{2} F_{1,3}(x/y, 1/y)$$

with $C(G)$ the quadratic Casimir operator of the adjoint representation of $SU(3)_C$.

2.3. Chargino contributions

The relevant Lagrangian terms for the chargino–quark–squark interaction are given by

$$\begin{aligned} \mathcal{L}_{u\tilde{d}\tilde{\chi}^+} = & \sum_{A=1}^2 \sum_{i,j=1}^3 \left\{ \tilde{u}_R^i [V_{A2}^* (Y_u^{\text{diag}} V_{\text{CKM}})_{ij}] \tilde{\chi}_A^+ \tilde{d}_L^j - \tilde{u}_L^i [g U_{A1} (V_{\text{CKM}})_{ij}] \tilde{\chi}_A^+ \tilde{d}_L^j \right. \\ & \left. + \tilde{u}_L^i [U_{A2} (V_{\text{CKM}} Y_d^{\text{diag}})_{ij}] \tilde{\chi}_A^+ \tilde{d}_R^j \right\} + \text{H.c.}, \end{aligned} \quad (14)$$

where the indices i, j label fermion and sfermion flavour eigenstates while A refers to chargino mass eigenstates. $Y_{u,d}^{\text{diag}}$ are the diagonal up- and down-quark Yukawa couplings, and V, U are the usual chargino rotation matrices defined by $U^* M_{\tilde{\chi}} + V^{-1} = \text{diag}(m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+})$. Keeping the terms whose flavour violation stems from the V_{CKM} , and neglecting those proportional to m_q/m_t , the chargino contribution now reads

$$\begin{aligned} \mathcal{A}_{\tilde{\chi}^\pm,R} = & -\frac{\alpha_w \sqrt{\alpha}}{2\sqrt{\pi}} m_t \sum_{A=1}^2 \frac{1}{m_{\tilde{\chi}_A}^2} \left\{ g^2 |U_{A1}|^2 [(V_{\text{CKM}})_{qi} (V_{\text{CKM}}^\dagger)_{j3} \delta_{ij} [F_1(x_t, x_A) + e_D F_2(z_t, z_A)] \right. \\ & + (V_{\text{CKM}})_{qi} (\delta_{LL}^d)_{ij} (V_{\text{CKM}}^\dagger)_{j3} [G_1(x_t, x_A) + e_D G_2(z_t, x_A)] \\ & + |U_{A2}|^2 [(V_{\text{CKM}})_{qi} (Y_d^{\text{diag}})_{ii}^2 (V_{\text{CKM}}^\dagger)_{i3} [F_1(x_t, x_A) + e_D F_2(z_t, z_A)] \\ & + (V_{\text{CKM}})_{qi} (Y_d^{\text{diag}})_{ii} (\delta_{RR}^d)_{ij} (Y_d^{\text{diag}})_{jj} (V_{\text{CKM}}^\dagger)_{j3} [G_1(x_t, x_A) + e_D G_2(z_t, x_A)] \\ & - g U_{A1} U_{A2}^* [(V_{\text{CKM}})_{qi} (Y_d^{\text{diag}})_{ii} (\delta_{LR}^d)_{ij} (Y_d^{\text{diag}})_{jj} (V_{\text{CKM}}^\dagger)_{j3} [G_1(x_t, x_A) + e_D G_2(z_t, x_A)]] \\ & - g U_{A1}^* U_{A2} [(V_{\text{CKM}})_{qi} (Y_d^{\text{diag}})_{ii} (\delta_{RL}^d)_{ij} (V_{\text{CKM}}^\dagger)_{j3} [G_1(x_t, x_A) + e_D G_2(z_t, x_A)]] \\ & \left. - \left(\frac{m_{\tilde{\chi}_A}}{m_t} \right) \left\{ -g U_{A1} V_{A2} [(V_{\text{CKM}})_{qi} \delta_{ij} (V_{\text{CKM}}^\dagger)_{j3} (Y_u^{\text{diag}})_{33} [F_3(x_t, x_A) + e_D F_4(z_t, z_A)] \right. \right. \end{aligned}$$

$$\begin{aligned}
& + (V_{\text{CKM}})_{qi} (\delta_{LL}^d)_{ij} (V_{\text{CKM}}^\dagger)_{j3} (Y_u^{\text{diag}})_{33} [G_3(x_t, x_A) + e_D G_4(z_t, x_A)] \\
& + U_{A2} V_{A2} [(V_{\text{CKM}})_{qi} (Y_d^{\text{diag}})_{ii} (\delta_{RL}^d)_{ij} (V_{\text{CKM}}^\dagger)_{j3} (Y_u^{\text{diag}})_{33} [G_3(x_t, x_A) + e_D G_4(z_t, x_A)]] \Big\}, \quad (15)
\end{aligned}$$

where $x_{A,t} = (m_{\tilde{\chi}_{A,t}^\pm} / m_{\tilde{d}})^2 \simeq m_{\tilde{\chi}_{A,t}^\pm}^2 / m_{\tilde{q}_L}^2$, $z_t = (m_t / m_{\tilde{\chi}_A^\pm})^2$ and the additional loop functions G_i can be also found in [Appendix A](#). For the chargino contributions, one has $\mathcal{A}_{\tilde{\chi}^\pm, L} = \mathcal{O}(m_q)$ ($q \neq t$).

To get the chargino contribution to $t \rightarrow q\gamma$, one should only keep the terms proportional to the functions $F_{2,4}(x, y)$ and $G_{2,4}(x, y)$ and clearly changing $\sqrt{\alpha} e_D$ by $\sqrt{\alpha} s$.

2.4. Neutralino contributions

In this case, the relevant Lagrangian terms are

$$\begin{aligned}
\mathcal{L}_{u\tilde{u}\tilde{\chi}^0} = & \sum_{a=1}^4 \sum_{i=1}^3 \left\{ \tilde{u}_R^i N_{a1}^* \frac{4}{3} \frac{g}{\sqrt{2}} \tan\theta_W \tilde{\chi}_A^0 \tilde{u}_R^i - \tilde{u}_R^i N_{a4}^* Y_u^{\text{diag}} \tilde{\chi}_A^0 \tilde{u}_L^i \right. \\
& \left. - \tilde{u}_L^i \frac{g}{\sqrt{2}} \left(N_{a2} + \frac{1}{3} N_{a1} \tan\theta_W \right) \tilde{\chi}_A^0 \tilde{u}_L^i - \tilde{u}_L^i N_{a4} Y_u^{\text{diag}} \tilde{\chi}_A^0 \tilde{u}_R^i \right\}, \quad (16)
\end{aligned}$$

where N is the 4×4 rotation matrix which diagonalises the neutralino mass matrix M_N , $N^* M_N N^{-1} = \text{diag}(m_{\tilde{\chi}_a^0})$. Using $\mathcal{L}_{u\tilde{u}\tilde{\chi}^0}$ one derives the neutralino contributions to the flavour changing top decay $t \rightarrow q\gamma$, which are given by

$$\begin{aligned}
A_{\tilde{\chi}^0, R}^\gamma = & \frac{\alpha_w \sqrt{\alpha} \langle \tilde{m}^2 \rangle}{2\sqrt{\pi} m_{\tilde{\chi}_a^0}^2} \sum_{a=1}^4 \left\{ \left[\frac{m_t}{2m_W \sin\beta} N_{a4} \left(N_{a2} + \frac{1}{3} \tan\theta_W N_{a1} \right) \frac{1}{m_{\tilde{\chi}_a^0}} \left(\frac{\tilde{F}_4(z_t, z_{\tilde{q}_L}) - \tilde{F}_4(z_t, z_{\tilde{l}_L})}{z_{\tilde{q}_L} - z_{\tilde{l}_L}} \right) \right. \right. \\
& + \left. \frac{(N_{a2} + \frac{1}{3} \tan\theta_W N_{a1})^2}{2} \frac{m_t}{m_{\tilde{\chi}_a^0}^2} \left(\frac{F_2(z_t, z_{\tilde{q}_L}) - F_2(z_t, z_{\tilde{l}_L})}{z_{\tilde{q}_L} - z_{\tilde{l}_L}} \right) \right] (\delta_{LL}^u)_{q3} \\
& + \left[\frac{2}{3} \tan\theta_W N_{a1} \left(N_{a2} + \frac{1}{3} \tan\theta_W N_{a1} \right) \frac{1}{m_{\tilde{\chi}_a^0}} \left(\frac{\tilde{F}_4(z_t, z_{\tilde{q}_L}) - \tilde{F}_4(z_t, z_{\tilde{i}_R})}{z_{\tilde{q}_L} - z_{\tilde{i}_R}} \right) \right. \\
& \left. \left. - \frac{m_t}{2m_W \sin\beta} N_{a4}^* \left(N_{a2} + \frac{1}{3} \tan\theta_W N_{a1} \right) \frac{m_t}{m_{\tilde{\chi}_a^0}^2} \left(\frac{F_2(z_t, z_{\tilde{q}_L}) - F_2(z_t, z_{\tilde{i}_R})}{z_{\tilde{q}_L} - z_{\tilde{i}_R}} \right) \right] (\delta_{LR}^u)_{q3} \right\}, \quad (17)
\end{aligned}$$

$$\begin{aligned}
A_{\tilde{\chi}^0, L}^\gamma = & \frac{\alpha_w \sqrt{\alpha} \langle \tilde{m}^2 \rangle}{2\sqrt{\pi} m_{\tilde{\chi}_a^0}^2} \sum_{a=1}^4 \left\{ \left[-\frac{m_t}{m_W \sin\beta} \frac{2}{3} \tan\theta_W N_{a1}^* N_{a4} \frac{1}{m_{\tilde{\chi}_a^0}} \left(\frac{\tilde{F}_4(z_t, z_{\tilde{q}_R}) - \tilde{F}_4(z_t, z_{\tilde{i}_R})}{z_{\tilde{q}_R} - z_{\tilde{i}_R}} \right) \right. \right. \\
& + \left. \frac{8}{9} \tan^2\theta_W |N_{a1}|^2 \frac{m_t}{m_{\tilde{\chi}_a^0}^2} \left(\frac{F_2(z_t, z_{\tilde{q}_R}) - F_2(z_t, z_{\tilde{i}_R})}{z_{\tilde{q}_R} - z_{\tilde{i}_R}} \right) \right] (\delta_{RR}^u)_{q3} \\
& + \left[\frac{2}{3} \tan\theta_W N_{a1}^* \left(N_{a2}^* + \frac{1}{3} \tan\theta_W N_{a1}^* \right) \frac{1}{m_{\tilde{\chi}_a^0}} \left(\frac{\tilde{F}_4(z_t, z_{\tilde{q}_R}) - \tilde{F}_4(z_t, z_{\tilde{l}_L})}{z_{\tilde{q}_R} - z_{\tilde{l}_L}} \right) \right. \\
& \left. \left. - \frac{2}{3} \frac{m_t}{m_W \sin\beta} N_{a4} N_{a1}^* \tan\theta_W \frac{m_t}{m_{\tilde{\chi}_a^0}^2} \left(\frac{F_2(z_t, z_{\tilde{q}_R}) - F_2(z_t, z_{\tilde{l}_L})}{z_{\tilde{q}_R} - z_{\tilde{l}_L}} \right) \right] (\delta_{RL}^u)_{q3} \right\}. \quad (18)
\end{aligned}$$

The fourth contributions for each amplitudes are usually neglected for processes involving light quarks or leptons as $b \rightarrow s\gamma$ or $\mu \rightarrow e\gamma$. But clearly, for the top-quark, they cannot be neglected. As for the chargino case, to get their contribution to $t \rightarrow c\gamma$, one should just replace α by α_s .

3. Numerical results

In this section we will explore the parameter space of the general MSSM in order to find the maximum allowed values of $\text{Br}(t \rightarrow c\gamma)$ in the presence of a light stop, while observing the constraints on squark mixing imposed by B -physics and by generating the correct BAU. In our analysis we take into account experimental bounds on the masses of the SUSY particles [18] and all available constraints from FCNC and rare decays [21–28].

As shown in Ref. [5], the observed ratio of the baryon number to photon number in the Universe [23,24]

$$\eta \equiv \frac{n_B}{n_\gamma} = (6.3 \pm 0.3) \times 10^{-10} \quad (19)$$

can be accommodated in the framework of flavour-dependent supersymmetric electroweak baryogenesis. In this scenario, complying with the value of η requires two key ingredients: a very light stop, $105 \text{ GeV} \lesssim m_{\tilde{t}_R} \lesssim 165 \text{ GeV}$, and a sizable mixing in the LR up-squark sector.² In fact, in this framework, the BAU can be written as

$$\eta \sim 10^{-9} I_{RR} \mu Y_t^2 \frac{\langle m_{\tilde{q}}^2 \rangle}{m_t} \text{Im}(\delta_{LR}^u)_{3i}^*, \quad (20)$$

where I_{RR} is given in [5], and $Y_t \equiv (Y_u^{\text{diag}})_{33}$. The requisite of LR up-squark mixing depends on the other parameters involved in the computation of n_B/n_γ , namely on $m_{\tilde{q}}$ and on the value of the bilinear μ -term. In particular, for $m_{\tilde{q}} \simeq 1 \text{ TeV}$ and $\mu \sim 700 \text{ GeV}$, complying with the observed BAU imposes

$$\text{Im}(\delta_{LR}^u)_{3i}^* \gtrsim 0.15, \quad (21)$$

which in turn implies

$$|(\delta_{LR}^u)_{3i}| \gtrsim 0.15. \quad (22)$$

It is important to notice that since $(\delta_{LR})_{ij} = (\delta_{RL})_{ji}^*$, thus the BAU constraint can be written as

$$|(\delta_{RL}^u)_{i3}| \gtrsim 0.15. \quad (23)$$

Regarding $m_{\tilde{t}_R}$, in agreement with collider bounds on the mass of the lightest top-squark [19,20], and unless otherwise stated, throughout the analysis we will always consider $m_{\tilde{t}_R} = 110 \text{ GeV}$.

After these considerations, we turn again our attention to the SUSY contributions to the inclusive width of the $t \rightarrow q\gamma$ decay. These are given by

$$\Gamma(t \rightarrow q\gamma) = \frac{m_t^3}{16\pi} \left\{ \left| \sum_i \mathcal{A}_{i,R}(t \rightarrow q\gamma) \right|^2 + \left| \sum_i \mathcal{A}_{i,L}(t \rightarrow q\gamma) \right|^2 \right\}, \quad (24)$$

$$\Gamma(t \rightarrow qg) = C(R) \frac{m_t^3}{16\pi} \left\{ \left| \sum_i \mathcal{A}_{i,R}(t \rightarrow qg) \right|^2 + \left| \sum_i \mathcal{A}_{i,L}(t \rightarrow qg) \right|^2 \right\}, \quad (25)$$

where $C(R) = 4/3$.

Where $i = \tilde{g}, \tilde{\chi}^\pm, \tilde{\chi}^0$. As we usual in the framework of the MIA, we analyse each contribution separately, so that the branching ratio associated with each of the above terms is defined as

$$\text{Br}(t \xrightarrow{i} q\gamma, g) = \frac{\Gamma_i(t \rightarrow q\gamma, g)}{\Gamma(t \rightarrow bW)}, \quad (26)$$

² We note here that in these scenarios, the strength of the EWPT is typically too small. Nevertheless, this problem can be overcome by the introduction of new degrees of freedom, as is the case of extensions of the MSSM with additional Higgs scalars [25–27].

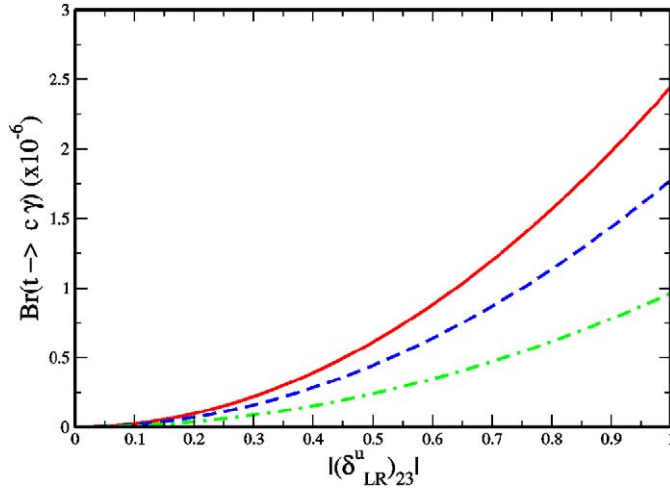


Fig. 2. Gluino contributions to $\text{Br}(t \rightarrow c\gamma)$ as a function of $|(\delta_{LR}^u)_{23}|$ for different pairs of $(m_{\tilde{g}}, m_{\tilde{q}})$: (300 GeV, 1 TeV), (300 GeV, 500 GeV) and (500 GeV, 1 TeV), corresponding to solid, dashed and dot-dashed lines, respectively.

with $\Gamma(t \rightarrow bW) \simeq 1.52$ GeV and

$$\Gamma_i(t \rightarrow q\gamma) = \frac{m_t^3}{16\pi} \{ |\mathcal{A}_{i,R}(t \rightarrow q\gamma)|^2 + |\mathcal{A}_{i,L}(t \rightarrow q\gamma)|^2 \},$$

$$\Gamma_i(t \rightarrow qg) = C(R) \frac{m_t^3}{16\pi} \{ |\mathcal{A}_{i,R}(t \rightarrow qg)|^2 + |\mathcal{A}_{i,L}(t \rightarrow qg)|^2 \},$$

with $i = \tilde{g}, \tilde{\chi}^\pm, \tilde{\chi}^0$.

We start our analysis by considering gluino mediated top decays. As it can be seen from Eqs. (10), (11), the gluino contribution to $\text{Br}(t \rightarrow q\gamma)$ essentially depends on three parameters: the gluino mass $m_{\tilde{g}}$, the average squark mass $m_{\tilde{q}}$ and the mass of the light top-squark $m_{\tilde{t}_R}$. Regarding the flavour structure, the gluino mediated $t \rightarrow c\gamma$ decay is a function of the $(\delta_{LL}^u)_{23}$ and $(\delta_{LR}^u)_{23}$ mass insertions. An illustrative example of the dependence of the $\text{Br}(t \rightarrow c\gamma)$ on the relevant mass insertions can be drawn by considering a representative point in the parameter space, which complies with the BAU requirements. For $m_{\tilde{g}} = 300$ GeV and $m_{\tilde{q}} \sim 1$ TeV, the branching ratio reads:

$$\begin{aligned} \text{Br}(t \xrightarrow{\tilde{g}} c\gamma) = & 2.6 \times 10^{-10} (\delta_{LL}^u)_{23}^2 - 5 \times 10^{-8} (\delta_{LL}^u)_{23} (\delta_{LR}^u)_{23} + 2.4 \times 10^{-6} (\delta_{LR}^u)_{23}^2 + 1.6 \times 10^{-8} (\delta_{RL}^u)_{23}^2 \\ & - 7.1 \times 10^{-8} (\delta_{RL}^u)_{23} (\delta_{RR}^u)_{23} + 7.7 \times 10^{-8} (\delta_{RR}^u)_{23}^2. \end{aligned} \quad (27)$$

The leading gluino contributions to the Br always stems from the terms proportional to $(\delta_{LR}^u)_{23}^2$, with an associated coefficient of order $\mathcal{O}(10^{-6})$. Therefore, the bound from Eq. (22) will not affect the dominant $(\delta_{LR}^u)_{23}^2$ term in the branching ratio contrary to naive expectations that the large mixing between first/second and the third up-squark generation required for a successful electroweak baryogenesis implies enhancement in top flavour violating decays branching ratio. It is worth mentioning that in the class of SUSY models with Hermitian or symmetric trilinear couplings, the magnitude of $|(\delta_{LR}^u)_{23}|$ is of the same order $|(\delta_{LR}^u)_{32}|$, hence the BAU leads to a lower bound on the branching ratio $\text{Br}(t \rightarrow c\gamma)$ of order 10^{-7} as it can be seen from Fig. 2.

In Fig. 2 we plot the $\text{Br}(t \xrightarrow{\tilde{g}} c\gamma)$ as a function of $|(\delta_{LR}^u)_{23}|$, for several values of $(m_{\tilde{g}}, m_{\tilde{q}})$, fixing all the other mass insertions to be zero. As can be seen from this figure, larger values of the average squark mass strongly enhance the gluino contributions to the branching ratio. This can be easily understood by inspection of Eq. (10) as in such a case, the $x_{\tilde{q}_L} \approx x_{\tilde{t}_L} \rightarrow 0$ and the dominant terms only comes from the light right-handed top-squark

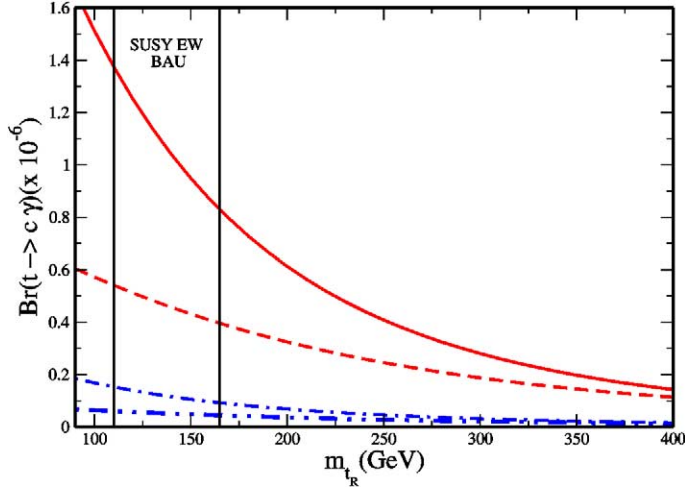


Fig. 3. Gluino contributions to $\text{Br}(t \rightarrow c\gamma)$ as a function of $m_{\tilde{t}_R}$ for different pairs of $(m_{\tilde{g}}, (\delta_{LR}^u)_{23})$: (300 GeV, 0.75), (500 GeV, 0.75) and (300 GeV, 0.25), (500 GeV, 0.25) corresponding, respectively, to solid, dashed, dot-dashed and double-dot-dashed lines. $(\delta_{LL}^u)_{23}$ is fixed as 0.1 and $m_{\tilde{q}} = 1$ TeV.

contributions. In fact, it can be verified that the $\text{BR}(t \rightarrow c\gamma)$ monotonically increases with $m_{\tilde{q}}$, saturating at $\text{BR} \sim 10^{-5}$ for $m_{\tilde{q}} \sim \mathcal{O}(4 \text{ TeV})$.

In Fig. 3 we present a plot for the branching ratio $\text{Br}(t \rightarrow c\gamma)$ as a function of $m_{\tilde{t}_R}$ which is a crucial parameter for enhancing the BAU and also the branching ratio of top decay. As can be seen from this figure, in case of large mixing between the third and the first or the second generation of quarks ($(\delta_{LR})_{i3} \approx 0.1$ or bigger), imposing the right-handed stop masses to be within the range needed for electroweak baryogenesis imposes to the $\text{Br}(t \rightarrow q\gamma)$ to be bigger than 10^{-7} .

For completeness, let us get the value of the gluino contributions in case of a no-BAU inspired models. In that case, we shall use $m_{\tilde{g}} = 300$ GeV, $m_{\tilde{t}_R} = 100$ GeV and $m_{\tilde{q}} = 500$ GeV. One gets

$$\begin{aligned} \text{Br}(t \xrightarrow{\tilde{g}} c\gamma) = & 1.9 \times 10^{-9} (\delta_{LL}^u)_{23}^2 - 1.2 \times 10^{-7} (\delta_{LL}^u)_{23} (\delta_{LR}^u)_{23} + 1.97 \times 10^{-6} (\delta_{LR}^u)_{23}^2 \\ & + 8.4 \times 10^{-8} (\delta_{RL}^u)_{23}^2 - 1.5 \times 10^{-7} (\delta_{RL}^u)_{23} (\delta_{RR}^u)_{23} + 6.98 \times 10^{-8} (\delta_{RR}^u)_{23}^2. \end{aligned} \quad (28)$$

Finally, we address the additional phenomenological constraints that should be applied to the computation of the $\text{Br}(t \rightarrow c\gamma)$. First, we point out that the main constraint on the mass insertions $(\delta_{AB}^u)_{23}$ is associated with having the latter involved in the chargino contribution to the $b \rightarrow s\gamma$ decay. From the analysis conducted in Ref. [21], one finds that the current measurements of the $\text{Br}(b \rightarrow s\gamma) = (3.21 \pm 0.43 \pm 0.27) \times 10^{-4}$ [22], can only constrain the $(\delta_{LL}^u)_{23}$ at large $\tan\beta$, while the relevant mass insertions to the gluino mediated top decay, $(\delta_{LR,RL}^u)_{23}$, remain unconstrained.

Regarding the chargino contributions, their contribution is always very suppressed compared to gluino contributions but it is important to emphasize to the fact that their contributions are proportional to δ_{AB}^d . Let us recall that $B_d^0 - \bar{B}_d^0$ mixing constrains $(\delta_{LL}^d)_{13}$ to be of $\mathcal{O}(0.1)$ [28]. Nevertheless, the mass insertion $(\delta_{LL}^d)_{23}$ is essentially unconstrained since nor $b \rightarrow s\gamma$ limits nor $B_s^0 - \bar{B}_s^0$ mixing impose any bound on this parameter [29], so that $(\delta_{LL}^d)_{23}$ could be of order one. Even so, chargino contributions to $\text{Br}(t \rightarrow c\gamma)$ can be at most of order 10^{-9} , and play a secondary role when compared to those of the gluino.

Respect the neutralino contributions, it can be seen from Eq. (18) that as in the case of the gluinos, these contributions depend on $(\delta_{AB}^u)_{23}$. However, their associated coefficients are comparatively more suppressed. For

instance, the coefficient of the dominant $(\delta_{LR}^u)_{23}$ term is suppressed by a factor $\frac{\alpha_W}{\alpha_S} \frac{1}{C(R)} \frac{m_{\tilde{\chi}_a^0}}{m_{\tilde{g}}} \tan \theta_W N_{a1} (N_{a2} + 1/3 \tan \theta_W N_{a1})$ which is of order 10^{-3} – 10^{-4} , implying that neutralino contributions will be clearly subdominant when compared to those of the gluino and chargino.

To conclude our analysis, we briefly comment on the experimental prospects for the observation of the SUSY mediated $t \rightarrow q\gamma, g$ decays here discussed. First, let us notice that the present CDF limit on these processes is very weak [18,30]

$$\text{Br}(t \rightarrow \gamma q) \leq 0.032. \quad (29)$$

However, significant progresses are likely to occur in the near future, with new data from Tevatron Run II, which should be able to improve these limits by a factor 10 [31]. At longer terms, the next generation of colliders as LHC or a linear collider like TESLA, is expected to ameliorate the current bound Eq. (29) by a few orders of magnitude [32]. In particular, after one year of operation, it should be possible to reach the following limits at LHC and TESLA, respectively, [33,34]:

$$\text{Br}(t \rightarrow c\gamma) \leq 7.7 \times 10^{-6} \quad (\text{LHC}), \quad (30)$$

$$\text{Br}(t \rightarrow c\gamma) \leq 3.7 \times 10^{-6} \quad (\text{TESLA}), \quad (31)$$

$$\text{Br}(t \rightarrow cg) \leq 1.0 \times 10^{-5} \quad (\text{LHC}). \quad (32)$$

From the comparison of these values to the results of the analysis conducted in this section, one can conclude that in the presence of a light top-squark and provided large mixing between the first/second and third up squark generations, the observation of processes as $t \rightarrow c\gamma$ will soon be within experimental reach.

4. Conclusions

In this Letter we have studied in a completely model independent way top flavour violating decays in general supersymmetric models using the generalised MIA. We have computed in a model-independent way the gluino, chargino and neutralino contributions to the branching ratio of $t \rightarrow q\gamma$. We have shown that in a light \tilde{t}_R scenario, gluino mediated decays provide the leading contribution to the branching ratio of $t \rightarrow q\gamma, g$ for a gluino mass between 300 and 500 GeV.

We have verified that the present experimental constraints on B physics did not prevent us to get $\text{Br}(t \rightarrow q\gamma) \gtrsim 10^{-6}$ and $\text{Br}(t \rightarrow qg) \gtrsim 10^{-5}$. These results are particularly interesting, since such a sensitivity could be reached by the LHC or by a linear collider like TESLA. In particular, after a few years of operation, one should be able to observe the $t \rightarrow q\gamma, g$ decays.

As a corollary of our approach, we have shown that contrary to the naive expectations, the large mixing between up-squarks needed to generate the BAU at electroweak scale does not affect top flavour violating decay, except in particular SUSY models where $(\delta_{LR}^u)_{23}$ is related to $(\delta_{LR}^u)_{32}$ (see for instance SUSY models with Hermitian texture for the trilinear terms).

Note added

In Ref. [15] working in the physical basis for all supersymmetric particles, they claimed to be able to reach for $\text{Br}(t \rightarrow c\gamma, g)$, respectively, 10^{-6} , 10^{-4} . We want to emphasize that these results are consistent with the one we have for $\text{Br}(t \rightarrow c\gamma)$. As we said in this Letter, it is possible to get $\text{Br}(t \rightarrow c\gamma)$ close to 10^{-5} (which correspond to $\text{Br}(t \rightarrow cg) \approx 10^{-4}$ if $m_{\tilde{q}} \approx$ a few TeV and $m_{\tilde{t}_R}$ around 110 GeV. But we should stress that a such spectrum is quite improbable and difficult to justify.

Acknowledgements

We are indebted to A. Teixeira for her exhaustive and fruitfull discussions on this topic. We are grateful to D. Emmanuel-Costa for a critical reading of the manuscript. This work was partially supported by the Nato Collaborative Linkage Grants. The work of D.D. was supported by project PROMEP/103.5/04/1335 (Mexico) and by CONACYT, Mexico under project 37234-E. The work of S.K. was supported by PPARC.

Appendix A. Loop functions

$$F_1(z, y) = \frac{-y}{2} \int_0^1 dx \frac{(1-x)}{z} \ln\left(\frac{x+y(1-x)-z(1-x)x}{x+y(1-x)}\right), \quad (\text{A.1})$$

$$F_2(z, y) = -\frac{1}{2} \int_0^1 dx \frac{(1-x)}{z} \ln\left(\frac{x+y(1-x)-z(1-x)x}{x+y(1-x)}\right), \quad (\text{A.2})$$

$$F_3(z, y) = -y \int_0^1 dx \frac{(1-x)}{xz} \ln\left(\frac{x+y(1-x)-z(1-x)x}{x+y(1-x)}\right), \quad (\text{A.3})$$

$$F_4(z, y) = -\int_0^1 dx \frac{1}{z} \ln\left(\frac{x+y(1-x)-z(1-x)x}{x+y(1-x)}\right). \quad (\text{A.4})$$

In the limit $z \rightarrow 0$, one recovers the usual loop functions:

$$F_1(0, x) = x \left(\frac{x^3 - 6x^2 + 3x + 2 + 6x \log(x)}{12(x-1)^4} \right) = x F_1(x), \quad (\text{A.5})$$

$$F_2(0, 1/x) = x \left(\frac{2x^3 + 3x^2 - 6x + 1 - 6x^2 \log(x)}{12(x-1)^4} \right) = x F_2(x), \quad (\text{A.6})$$

$$F_3(0, x) = x \left(\frac{x^2 - 4x + 3 + 2 \log(x)}{2(x-1)^3} \right) = x F_3(x), \quad (\text{A.7})$$

$$F_4(0, 1/x) = x \left(\frac{x^2 - 1 - 2x \log(x)}{2(x-1)^3} \right) = x F_4(x), \quad (\text{A.8})$$

where the $F_{1,2,3,4}(x)$ functions are defined in Ref. [17].

$$G_1(x, y) = -y \frac{\partial F_1(x, y)}{\partial y} + x \frac{\partial F_1(x, y)}{\partial x}, \quad (\text{A.9})$$

$$G_3(x, y) = -y \frac{\partial F_3(x, y)}{\partial y} + x \frac{\partial F_3(x, y)}{\partial x}, \quad (\text{A.10})$$

$$G_2(x, y) = \frac{1}{y} \frac{\partial F_2(x, z)}{\partial z} \Big|_{z=1/y}, \quad (\text{A.11})$$

$$G_4(x, y) = \frac{1}{y} \frac{\partial F_4(x, z)}{\partial z} \Big|_{z=1/y}. \quad (\text{A.12})$$

It is easy to check that in the limit $x \rightarrow 0$, one recovers the usual MIA loop functions:

$$G_1(0, x) = -x \frac{-1 - 9x + 9x^2 + x^3 - 6x(1+x)\log(x)}{6(x-1)^5}, \quad (\text{A.13})$$

$$G_2(0, x) = -x \frac{-1 + 9x + 9x^2 - 17x^3 + 6x^2(3+x)\log(x)}{12(x-1)^5}, \quad (\text{A.14})$$

$$G_3(0, x) = -x \frac{-5 + 4x + x^2 - 2(1+2x)\log(x)}{2(x-1)^4}, \quad (\text{A.15})$$

$$G_4(0, x) = -x \frac{1 + 4x - 5x^2 + 2x(2+x)\log(x)}{2(x-1)^4}. \quad (\text{A.16})$$

$$f_2(x) \equiv -\frac{\partial(x F_2(x))}{\partial x} \quad (\text{A.17})$$

$$= \frac{1 - 9x - 9x^2 + 17x^3 - 18x^2 \ln x - 6x^3 \ln x}{12(x-1)^5}, \quad (\text{A.18})$$

$$f_4(x) \equiv -\frac{\partial(x F_4(x))}{\partial x} \quad (\text{A.19})$$

$$= \frac{-1 - 4x + 5x^2 - 4x \ln x - 2x^2 \ln x}{2(x-1)^4}. \quad (\text{A.20})$$

References

- [1] G.R. Farrar, M.E. Shaposhnikov, Phys. Rev. Lett. 70 (1993) 2833;
G.R. Farrar, M.E. Shaposhnikov, Phys. Rev. Lett. 71 (1993) 210, Erratum;
M.B. Gavela, P. Hernandez, J. Orloff, O. Pene, C. Quimbay, Nucl. Phys. B 430 (1994) 382;
M.B. Gavela, M. Lozano, J. Orloff, O. Pene, Nucl. Phys. B 430 (1994) 345;
P. Huet, E. Sather, Phys. Rev. D 51 (1995) 379.
- [2] D. Delepine, J.M. Gerard, R. Gonzalez Felipe, J. Weyers, Phys. Lett. B 386 (1996) 183, hep-ph/9604440;
D. Delepine, hep-ph/9609346;
M. Carena, M. Quiros, C.E. Wagner, Phys. Lett. B 380 (1996) 81;
M. Carena, M. Quiros, C.E. Wagner, Nucl. Phys. B 524 (1998) 3;
J.R. Espinosa, Nucl. Phys. B 475 (1996) 273;
J.M. Cline, K. Kainulainen, Nucl. Phys. B 482 (1996) 73;
J.M. Cline, M. Joyce, K. Kainulainen, Phys. Lett. B 417 (1998) 79;
J.M. Cline, M. Joyce, K. Kainulainen, Phys. Lett. B 448 (1999) 321, Erratum.
- [3] S. Pokorski, J. Rosiek, C.A. Savoy, Nucl. Phys. B 570 (2000) 81;
S. Abel, S. Khalil, O. Lebedev, Nucl. Phys. B 606 (2001) 151, and references therein;
D. Chang, W.Y. Keung, A. Pilaftsis, Phys. Rev. Lett. 82 (1999) 900;
D. Chang, W.Y. Keung, A. Pilaftsis, Phys. Rev. Lett. 83 (1999) 3972, Erratum;
D. Chang, W.F. Chang, W.Y. Keung, Phys. Lett. B 478 (2000) 239;
D. Chang, W.F. Chang, W.Y. Keung, hep-ph/0205084;
A. Pilaftsis, hep-ph/0207277.
- [4] S. Abel, D. Bailin, S. Khalil, O. Lebedev, Phys. Lett. B 504 (2001) 241, hep-ph/0012145.
- [5] D. Delépine, R. González Felipe, S. Khalil, A.M. Teixeira, Phys. Rev. D 66 (2002) 115011, hep-ph/0208236.
- [6] S. Khalil, V. Sanz, Phys. Lett. B 576 (2003) 107, hep-ph/0306171.
- [7] G. Eilam, J.L. Hewett, A. Soni, Phys. Rev. D 44 (1991) 1473;
G. Eilam, J.L. Hewett, A. Soni, Phys. Rev. D 59 (1999) 039901, Erratum.
- [8] C.S. Li, R.J. Oakes, J.M. Yang, Phys. Rev. D 49 (1994) 293;
C.S. Li, R.J. Oakes, J.M. Yang, Phys. Rev. D 56 (1997) 3156, Erratum.
- [9] G. Couture, C. Hamzaoui, H. Konig, Phys. Rev. D 52 (1995) 1713, hep-ph/9410230.

- [10] J.L. Lopez, D.V. Nanopoulos, R. Rangarajan, Phys. Rev. D 56 (1997) 3100, hep-ph/9702350.
- [11] G.M. de Divitiis, R. Petronzio, L. Silvestrini, Nucl. Phys. B 504 (1997) 45, hep-ph/9704244.
- [12] J.J. Cao, Z.H. Xiong, J.M. Yang, Nucl. Phys. B 651 (2003) 87, hep-ph/0208035.
- [13] J. Guasch, J. Solá, Nucl. Phys. B 562 (1999) 3.
- [14] S. Bejar, J. Guasch, J. Solá, hep-ph/0101294.
- [15] J.S. Liu, C.S. Li, L.L. Yang, L.G. Jin, hep-ph/0406155.
- [16] F. Gabbiani, et al., Nucl. Phys. B 477 (1996) 321.
- [17] S. Bertolini, F. Borzumati, A. Masiero, G. Ridolfi, Nucl. Phys. B 353 (1991) 591.
- [18] K. Hagiwara, et al., Particle Data Group Collaboration, Phys. Rev. D 66 (2002) 010001.
- [19] V.M. Abazov, et al., D0 Collaboration, Phys. Rev. Lett. 88 (2002) 171802, hep-ex/0108018.
- [20] D. Acosta, et al., CDF Collaboration, Phys. Rev. Lett. 90 (2003) 251801, hep-ex/0302009.
- [21] E. Gabrielli, S. Khalil, E. Torrente-Lujan, Nucl. Phys. B 594 (2001) 3, hep-ph/0005303.
- [22] S. Chen, et al., CLEO Collaboration, Phys. Rev. Lett. 87 (2001) 251807, hep-ex/0108032.
- [23] D.N. Spergel, et al., Astrophys. J. Suppl. 148 (2003) 175, astro-ph/0302209.
- [24] M. Tegmark, et al., SDSS Collaboration, astro-ph/0310723.
- [25] M. Pietroni, Nucl. Phys. B 402 (1993) 27, hep-ph/9207227.
- [26] A.T. Davies, C.D. Froggatt, R.G. Moorhouse, Phys. Lett. B 372 (1996) 88, hep-ph/9603388.
- [27] S.J. Huber, M.G. Schmidt, Nucl. Phys. B 606 (2001) 183, hep-ph/0003122.
- [28] E. Gabrielli, S. Khalil, Phys. Rev. D 67 (2003) 015008, hep-ph/0207288.
- [29] P. Ball, S. Khalil, E. Kou, hep-ph/0311361.
- [30] F. Abe, et al., CDF Collaboration, Phys. Rev. Lett. 80 (1998) 2525.
- [31] D. Chakraborty, J. Konigsberg, D. Rainwater, hep-ph/0303092.
- [32] R. Frey, et al., hep-ph/9704243.
- [33] J.A. Aguilar-Saavedra, Phys. Lett. B 502 (2001) 115, hep-ph/0012305.
- [34] M. Beneke, et al., hep-ph/0003033.