Experimental investigation and constitutive modeling for the hardening behavior of 5754O aluminum alloy sheet under two-stage loading

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The two-stage loading tests of 5754O aluminum alloy sheet were carried out. In the first loading stage, the uniaxial tensile tests of big sheets were carried out. Then small specimens were cut off from the pre-strained big sheets along different directions. In the second loading stage, the uniaxial tensile tests of the small specimens were performed. From the experimental results, it is found that the initial yield stress of each specimen in the second loading stage decreases when the strain path changes. In addition, when the strain path changes the transient effect appeared and no obvious permanent softening was observed. In this study, in order to describe the hardening behavior of 5754O aluminum alloy sheet under two-stage loading, the Chaboche type combined isotropic–kinematic hardening models were adopted with Yld2000-2d and Hill48 as yield functions. It is proven that no permanent softening can be described with Chaboche type model in two-stage loading. Three methods for determining the parameters of the hardening models were developed in order to establish accurate isotropic–kinematic hardening model describing the hardening behavior of 5754O aluminum alloy sheet under two-stage loading. The established constitutive models were implemented into the commercial FEM code ABAQUS as a user material subroutine (UMAT) for numerical simulations. By comparing the experimental and simulated results of the two-stage loading tests, the isotropic–kinematic hardening models describing the hardening behavior of 5754O aluminum alloy sheet under two-stage loading were accurately determined. Also, the influences of the characterization method of Hill48 yield function on the accuracy of the resulting hardening models were discussed. It is also found that the established isotropic–kinematic hardening model describing the hardening behavior under two-stage loading can describe reasonably the springback profile of the three point bending of the pre-strained specimen.

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1. Introduction

Sheet metal forming is usually a complicated deformation process, during which the strain path of the material points may change. The deformation behavior of materials under complex loading condition is different from that under monotonic loading condition since the loading path change will affect the hardening behavior, flow rule and forming ability, etc. When the strain path changes, some phenomena such as transient effect, increase or decrease of the yield stress and the hardening rate of the stress–strain curves will appear.

Among the strain path change loading conditions, reverse loading conditions have been widely investigated for a long time. During reverse loading, Bauschinger effect, transient effect and permanent softening are commonly observed. In order to investigate the effect of the reverse loading on the hardening behavior of sheet metals, many cyclic tension–compression loading tests have been performed in which some methods for preventing buckling during compression loading have been developed. Kuwabara et al. (1995, 2009) carried out tension–compression tests without buckling of the specimen using a specially designed equipment mounted with comb-type dies. In their tension–compression tests, a decrease in the flow stresses due to the Bauschinger effect was clearly observed. Chen et al. (1999) developed reverse torsion tests to investigate the Bauschinger effect and multiaxial yield behavior of mild steel. Yoshida et al. (2002) carried out in-plane cyclic tension–compression tests at large strain by employing adhesively bonded sheet laminate specimens as well as a special device for preventing the buckling of specimens. They found that the cyclic hardening was strongly influenced by cyclic strain range and mean strain. The transient softening and work hardening stagnation during stress reversals were also observed. Boger et al. (2005) performed continuous strain reversal tests with a special specimen geometry and solid flat plates as buckling constraints. The Bauschinger effect, room-temperature creep, and anelasticity after strain reversals in commercial sheet alloys...
were investigated. Lee et al. (2005) carried out in-plane uniaxial cyclic tension–compression tests where solid plates made of the hardened steel were placed along the side of the sheet specimen in order to prevent buckling of the thin sheet specimen. With their experimental results, the material parameters of the combined isotropic–kinematic hardening law (Chung et al., 2005) were determined. Cao et al. (2009) developed a fixture with a regular tension–compression machine, where the normal support was provided to the entire length of the specimen during the tension–compression cycle so that the potential buckling of sheet specimen can be prevented. The cyclic tension–compression behavior during reverse loading and the non-symmetric behavior during reloading were observed and described.

Since sheet metal forming is a complicated process, the strain path change of sheet metals during forming process is various. Besides in-plane uniaxial cyclic tension–compression tests, some two/multi-stage loading tests such as tension–tension, tension–shearing and two-stage biaxial loading have also been performed in order to investigate the hardening behavior of sheet metals under complicated loading paths. Wagoner and Laukonis (1983) carried out strain path–change tests (plane strain tension followed by uniaxial tensile tests) to evaluate the residual work-hardening behavior. They found that the subsequent hardening curves depended primarily on the relative direction between major strain axes in the two deformation stages and very little on the specific pre-strain procedure. Doucet and Wagoner (1989) performed tensile tests along the transverse direction on specimens which are pre-strained along the rolling direction under plane strain condition. Khan and Wang (1990) developed a non-proportional biaxial compression experiment in which a rectangular block first undergoes uniaxial compression and then, after finite deformation, was subjected to biaxial compression. Hu et al. (1992), Thuiller and Rauch (1994) performed two-stage experiments (including shear and tension tests) to investigate the deformation behavior of metals. Hiwatashi et al. (1997, 1998) performed non-proportional biaxial loading experiments to evaluate the crystal plasticity. Kuwabara et al. (2000) carried out strain path–change experiments and found the important differences between the yield surface shapes found by the strain path change procedure and the shapes found by probing the yield points from the elastic region. Barlat et al. (2003b) obtained non-linear deformation paths using uniaxial tension followed by simple shear tests. The flow stress was represented as a function of the plastic work in order to eliminate the influence of the initial plastic anisotropy and to compare the results in a consistent manner. Based on experimental and simulation results, the relative contributions of the crystallographic texture and dislocation microstructure evolution to the anisotropic hardening behavior of the material were discussed. Bouvier et al. (2005) carried out two-stage tests (shear–shear and tension–shear) to investigate the deformation behavior. Bouvier et al. (2006) investigated the isotropic, kinematic and distortional hardening of rolled metal sheets at moderate and finite strains using two-stage non-proportional loadings involving sequences of simple shear and uniaxial tensile deformations. Kim and Yin (1997) performed three-stage tests, where in the first loading stage, the sheets were stretched in the rolling direction, in the second loading stage the sheets were stretched at certain angles to the rolling direction, and in the third loading tensile test were performed along different directions. From tensile tests, effects of the two-stage pre-strains on the subsequent yielding were investigated. They observed that the orientations of the orthotropic axes change drastically over a few percent of tensile strain. Kuwabara et al. (2002) carried out the same test on an IF steel sheet. From the three-stage tests, Haahm and Kim (2008) found that the r-value distribution is hardly affected by the pre-strain. Wu et al. (2005) carried out uniaxial tensile tests along different angles from the rolling direction for both as-received and pre-strained sheet to investigate the effect of pre-strain on material anisotropy. They concluded that the conventional methodology for determining material anisotropy overestimated the effect of the pre-strain. Tari-gopula et al. (2008) investigated the elastic-plastic behavior of sheet metals under two-stage loading and observed Bauschinger effect, transient effect and permanent softening phenomena. Manninen et al. (2009) carried out two-stage tests and investigated the effect of pre-strain on material anisotropy. Considerable Bauschinger effect, transient effect and permanent softening were observed. Khan et al. (2009, 2010a,b) investigated the initial and subsequent yield surfaces of aluminum alloy. The subsequent yield surfaces were determined during various loading paths. It is found that the initial yield surface is very close to the von-Mises yield surface and the subsequent yield surfaces undergo translation and distortion. On subsequent yield surface a “nose” in the loading direction and flattened shape in the reverse loading direction were observed. They found that the yield surfaces after unloading depict strong anisotropy, positive cross-effect and exhibits different proportion of distortion in each loading condition. Verma et al. (2011) performed two-stage uniaxial tests along with uniaxial cyclic tests and biaxial tests to evaluate the effect of more general strain path changes on the deformation behaviors of sheet metals. They proposed the combined isotropic–kinematic hardening model which can reasonably describe various experimental phenomena under more general strain path changes such as the hardening stagnation, cross-effect and asymmetry in tension and compression, Bauschinger effect and the transient behaviors.

In order to describe the hardening behavior of sheet metals under strain path change loading, many hardening models including microstructural and phenomenological models have been proposed. Teodosiu and Hu (1995) proposed a microstructural model which can reasonably describe the macroscopic hardening behavior under strain path change loading condition (Haddadi et al., 2006; Oliveira et al., 2007). Bouvier et al. (2005) proposed a physically–based phenomenological model using four internal state tensor variables based on Teodosiu and Hu's model. The accuracy and the efficiency of the model were evaluated with the springback simulations of the stamping of a curved rail. However, compared to the phenomenological models, the microstructural models are computationally expensive and the parameters identification is very complex so that they have not been widely used.

Two kinds of phenomenological models have been used in order to describe the in-plane uniaxial cyclic tension–compression hardening behavior: one is based on kinematic hardening (shifting of a single-yield surface), and the other one involves multiple yield surfaces (Khan and Huang, 1995). For the first kind of models, Chaboche type combined isotropic–kinematic hardening model (Chaboche, 1986) generalized from the Armstrong–Frederick type model (Armstrong and Frederick, 1966) has been used widely for a long time. In order to more accurately describe the hardening behavior during strain path change loading, mainly for reverse loading, some new hardening models and some new methods for determining the kinematic hardening parameters have been developed based on Chaboche type model (Chaboche, 1991; Chun et al., 2002; Geng et al., 2002; Geng and Wagoner, 2000, 2002; Khan and Huang, 1995; Ohno and Wang, 1993a,b; Yoshida et al., 2002). Based on the generalized plastic work equivalence principle, Chung et al. (2005) proposed one modified Chaboche type combined isotropic–kinematic hardening law, which can accurately describe the transient behavior by considering the slope of the loading and unloading curves. Since no permanent softening can be predicted by this model (Kim et al., 2006), Ahn et al. (2009), Chung et al. (2010) and Lee et al. (2006) improved it by introducing softening parameter. Some two-surface models (Dafalias and Popov, 1976; Krieg, 1975; Lee et al., 2007; Yoshida and Uemori, 2002) have also
been proposed in order to describe the hardening behavior during reverse loading accurately.

As to two-stage loading for some materials, hardening behaviors such as Bauschinger effect and transient behavior have been observed (Gardey et al., 2005; Hahm and Kim, 2008; Manninen et al., 2009; Tarigopula et al., 2008, 2009; Yoshida et al., 2011). Some attempts have been made to describe the hardening behavior when strain path changes. Hahm and Kim (2008) made an attempt to approximate the observed yield and flow behavior under two/multi-stage tests based on a modified isotropic–kinematic hardening model with Hill48 yield function. The yield stress distributions were predicted to some degree. Tarigopula et al. (2008) used Chaboche type combined isotropic–kinematic hardening model with a high-exponent yield function (Hershey, 1954). The parameters of the hardening model were deduced by numerical tests and correlation with certain experimental stress-plastic strain curve. This model can describe the remarkable yield stress decrease and the transient effect when strain path changes with reasonable accuracy. The general trends of the experimental results were described reasonably. Then Tarigopula et al. (2009) evaluated the performance of the established Chaboche type hardening model for other deformation modes of dual-phase steel subjected to non-proportional loading. It is found that the Chaboche type model predicted the general trends of the experimental results such as transitory hardening and overall work hardening. However, the transient hardening behavior subsequent to strain path changes cannot be described accurately. Manninen et al. (2009) analyzed the evolution of kinematic and isotropic hardening components in two-stage tests using Chaboche type combined isotropic–kinematic hardening model with Mises yield function, while no predicted results of experimental curves were presented.

In this study, the hardening behaviors of 5754O aluminum alloy sheet under a two-stage loading (tension–tension) were investigated. The primary objective is to establish an isotropic–kinematic hardening model that can describe the hardening behavior of 5754O aluminum alloy sheet under two-stage loading accurately. In order to describe the hardening behavior under two-stage loading, the Chaboche type combined isotropic–kinematic hardening model with Yld2000-2d yield function (Barlat et al., 2003a) was adopted, and three methods for determining the parameters of the hardening model were established. The Chaboche type combined isotropic–kinematic hardening model with Hill48 yield function was also used to describe the hardening behavior. The established constitutive models were implemented intoABAQUS software and were verified by comparing the theoretical and experimental results. Considering more general strain path changes, three point bending tests of pre-strained specimens were performed to verify further the established hardening model further.

2. Material characterization

Simple tension tests of the test material 5754O aluminium alloy sheet along the rolling, 45° off and transverse directions were carried out. The measured hardening curves along the three directions are shown in Fig. 1. In this study, the simple tension along the rolling direction is adopted as the reference curve, which will be shown in Section 6.1 (Figs. 8 and 13). The r-value was calculated with Eq. (1) (Chung et al., 2011):

$$r = \frac{d\varepsilon_{yy}/d\sigma_{zz} - d\varepsilon_{yy}/(d\sigma_{xx} + d\varepsilon_{yy})}{(1 + d\varepsilon_{yy}/d\sigma_{xx})}$$

where $\varepsilon_{xx}$, $\varepsilon_{yy}$ and $\varepsilon_{zz}$ are the true strains along the longitudinal, width and thickness directions in simple tension test, respectively. The value of $d\varepsilon_{yy}/d\sigma_{xx}$ was determined by linearly interpolating the measured strains along the width and the longitudinal directions, respectively ($\varepsilon_{yy}$ and $\varepsilon_{xx}$) and the measured $d\varepsilon_{yy}/d\sigma_{xx}$ of simple tension tests along various directions are shown in Fig. 3.
The measured normalized stresses and the \( r \)-values of 5754O aluminum alloy are listed in Table 1.

### 3. Experimental

#### 3.1. Experimental procedure

The two-stage loading tests were made up of two steps. First, the uniaxial tensile tests were performed in one group of big sheets. After some pre-strains, small specimens were cut off along different directions from the center of the pre-strained specimens and then uniaxial tensile tests of the cut off specimens were performed. The dimensions and directions of the specimens in the first and the second loading stages are shown in Fig. 4.

Three groups of two-stage loading tests with strain path changes were performed:

**Group (1):** As shown in Fig. 4(a), the first and the second loading stages are along the rolling and the transverse directions, respectively. The plastic pre-strains in the first loading stage were 0.013, 0.02, 0.028, 0.047, 0.077 and 0.107, respectively.

**Group (2):** As shown in Fig. 4(a), the first and the second loading stages are along the transverse and the rolling directions, respectively. The plastic pre-strains in the first loading stage were 0.047 and 0.077, respectively.

**Group (3):** As shown in Fig. 4(b), the first and the second loading stages are along the rolling and the 45° directions, respectively. The plastic pre-strains in the first loading stage were 0.018 and 0.057, respectively.

In addition, the tensile tests without strain path change were performed and the dimensions of the specimens are shown in Fig. 4(c).

#### 3.2. Experimental results

The experimental stress–strain curves in the second loading stage are shown in Fig. 5. As shown in Fig. 5(a)–(c), Bauschinger effect and transient effect appear due to strain path changes, i.e. the initial yield stress decreases and the slopes of the stress–strain curves change rapidly. After some deformation, the stress–strain curves in the second loading stage converge to that in monotonic loading, i.e., there is no permanent softening.

As shown in Fig. 5(d), when the first and the second loading stages are along the same direction (i.e., no strain path change), the stress–strain curves coincide with those in monotonic loading. In other words, no above phenomena such as Bauschinger effect and transient effect appear when the strain path does not change.

For the two-stage loading where the first and the second loading stages are along different directions, the experimental stress–strain curves can only be analyzed qualitatively in the same coordinate system due to anisotropy.

In order to obtain a reasonable equivalence for the stress–strain curves in different directions, the stress should be represented as a function of the plastic work per unit volume (Khan and Huang, 1995), and then a direct comparison between the assumption of isotropic hardening and the real hardening exhibited by the material can be carried out (Barlat et al., 2003b).

The stress-plastic work per unit volume curves in the second loading stage are plotted in Fig. 6. As shown in Fig. 6(a)–(c), when strain path changes, Bauschinger effect and transient effect were observed by comparing the stress-plastic work curves in the second loading stage and those in monotonic loading. After some plastic work, the stress-plastic work curves in the second loading stage converge to that in the monotonic loading, i.e., no permanent softening appears. As shown in Fig. 6(d), when strain path does not change, no Bauschinger effect or transient effect appears, and during plastic deformation process the stress-plastic work curves coincide with that in monotonic loading.

Therefore, isotropic hardening will not be sufficient to describe the observed Bauschinger effect and transient effect under two-stage loading, and kinematic hardening should be considered to model the observed hardening behavior under two-stage loading.

### 4. Isotropic–kinematic hardening model

#### 4.1. Yield function

In this study, Yld2000-2d yield function proposed by Barlat et al. (2003a) was adopted to establish the isotropic–kinematic
Fig. 5. Stress–strain curves in the second loading stage with different pre-strains. (a) Curves along the transverse direction with pre-strains along the rolling direction. (b) Curves along the 45° direction with pre-strains along the rolling direction. (c) Curves along the rolling direction with pre-strains along the transverse direction. (d) Curves along the rolling direction with pre-strains along the rolling direction.

Fig. 6. Stress-plastic work per unit volume curves in the second loading stage. (a) Curves along the transverse direction with pre-strains along the rolling direction. (b) Curves along the 45° direction with pre-strains along the rolling direction. (c) Curves along the rolling direction with pre-strains along the transverse direction. (d) Curves along the rolling direction with pre-strains along the rolling direction.
hardening model that can describe the hardening behavior under two-stage loading accurately.

Yld2000-2d yield function is given by

$$\phi = \phi' + \phi'' = 2\bar{\sigma}^m$$

(2)

where exponent m is a material coefficient and

$$\phi' = |X'_1 - X'_2|^m$$
$$\phi'' = |2X'_1 + X'_2|^m$$

(3)

Here, $\phi$ is the sum of two isotropic functions, which are symmetric with respect to $X'_1$ and $X'_2$ as well as $X''_1$ and $X''_2$. The principal values of the matrices, $X'$ and $X''$ are

$$X'_1 = \frac{1}{2}(X_{11} + X_{22} + \sqrt{(X_{11} - X_{22})^2 + 4X_{12}^2})$$
$$X'_2 = \frac{1}{2}(X_{11} + X_{22} - \sqrt{(X_{11} - X_{22})^2 + 4X_{12}^2})$$

(4a)

and

$$X''_1 = \frac{1}{2}(X''_{11} + X''_{22} + \sqrt{(X''_{11} - X''_{22})^2 + 4X''_{12}^2})$$
$$X''_2 = \frac{1}{2}(X''_{11} + X''_{22} - \sqrt{(X''_{11} - X''_{22})^2 + 4X''_{12}^2})$$

(4b)

Components of $X$ and $X''$ are obtained from the following linear transformation of the Cauchy stress:

$$X' = L\sigma, \quad X'' = L'\sigma$$

(5)

where

$$L_{11} = \frac{2}{3}, \quad L_{12} = -\frac{1}{3}, \quad L_{21} = 0, \quad L_{22} = 0, \quad L_{66} = 0$$

$$L'_{11} = -\frac{2}{3}, \quad L'_{12} = \frac{2}{3}, \quad L'_{21} = 0, \quad L'_{22} = 0, \quad L''_{11} = \frac{1}{5}, \quad L''_{12} = \frac{1}{5}, \quad L''_{21} = \frac{1}{5}, \quad L''_{22} = \frac{1}{5}$$

(6)

In Eqs. (5)-(7), $\sigma$ is Cauchy stress and $\beta_1$ - $\beta_6$ are eight anisotropy coefficients. The procedure for solving $\beta_1$ - $\beta_6$ numerically was developed according to the method proposed by Barlat et al. (2003a). In this study $m = 8$ since 5754O aluminum alloy is an FCC material.

Hill48 yield function is also adopted to establish the isotropic-kinematic hardening model and under a plane stress condition it is given by

$$(G + H)\sigma^2 + 2H\sigma_1\sigma_2 + (F + H)\sigma_3^2 + 2N_2\sigma_3^2 = \sigma^2$$

(8)

where $F$, $G$, $H$ and $N$ are material coefficients which can be calibrated with normalized stresses $(\sigma_1/\sigma_0$, $\sigma_4/\sigma_0$, $\sigma_6/\sigma_0)$, and $\sigma_0/\sigma_0$ or the r-values along different directions ($f_{0}, f_{45}$ and $f_{90}$) (Park and Chung, 2012).

In this study, for convenience, the Hill48 yield function characterized with r-values is called Hill48 (1) while that characterized with normalized stresses is called Hill48 (2). Then the established isotropic-kinematic hardening models with different yield functions (Yld2000-2d, Hill48 (1) and Hill48 (2)) will be compared with experimental results, and the influences of characterization method of Hill48 on the accuracy of the resulting hardening model will be analyzed.

### 4.2. Hardening model

#### 4.2.1. Chaboche type model

The kinematic hardening law in Chaboche type model, i.e., Armstrong and Frederick evolution rule (Armstrong and Fredrick, 1966), can be expressed as

$$d\bar{\sigma} = \frac{c}{\bar{\sigma}}d\bar{\varepsilon}^p - \gamma d\bar{\varepsilon}^p$$

(9)

where $\bar{\sigma}$ is Cauchy stress tensor, $\bar{\varepsilon}$ is backstress tensor, $c$ and $\gamma$ are material parameters, $d\bar{\varepsilon}^p$ is the equivalent plastic strain and $\bar{\varepsilon}$ is the equivalent stress (the isotropic hardening). Here only one term of the backstress in Chaboche type model was adopted.

Chaboche type model as well as its modified model has been widely used for describing the hardening behavior of sheet metals under cyclic tension-compression loading. The kinematic hardening parameters can be assumed to be constant or be represented as the functions of the plastic deformation (Ahn et al., 2009; Cao et al., 2009; Chung et al., 2005, 2010; Geng et al., 2002; Geng and Wagoner, 2002; Khan and Huang, 1995; Kim et al., 2006; Lee et al., 2005) according to actual needs.

Chaboche type model can describe the Bauschinger effect and the transient effect during reverse loading reasonably whereas it cannot describe the permanent softening in the uniaxial cyclic tension-compression loading (Chun et al., 2002; Chung et al., 2005; Yoshida et al., 2002), and Kim et al. (2006) presented the detailed theoretical explanation.

By replacing the stress tensor $\sigma$ in Yld2000-2d yield function with $\sigma - \alpha$, the Chaboche type combined isotropic-kinematic hardening model with Yld2000-2d yield function can be expressed as

$$\phi(\sigma - \alpha) = 2\bar{\sigma}^m$$

(10)

For this two-stage loading, there are only two principle stresses, which are along the rolling and the transverse directions, respectively. So we have

$$\phi(\sigma_1 - \alpha_1, \sigma_2 - \alpha_2) = 2\bar{\sigma}^m$$

(11)

where $\sigma_1$ and $\alpha_1$ are the stress and backstress variables along the rolling direction and $\sigma_2$ and $\alpha_2$ are those along the transverse direction. In this study, the simple tension along the rolling direction is adopted as the reference state.

Then the performance of Chaboche hardening model for the permanent softening in two-stage loading is analyzed in Appendix. It is proven that no permanent softening can be described with Chaboche type model in two-stage loading. Coincidentally, no obvious permanent softening was observed in the experimental results in this study. Therefore, we make an attempt to describe the hardening behavior in the two-stage loading using Chaboche type model.

#### 4.2.2. Calculation of the backstress and the isotropic hardening

The method for determining the parameters of the hardening models in uniaxial cyclic tension-compression loading cannot be directly used for that in two-stage loading since the loading paths are different. Also take the two-stage loading where the first and the second loading stages are along the rolling and the transverse directions respectively as an example. According to the Chaboche type combined isotropic-kinematic hardening model, besides the isotropic expansion, the yield surface will also shift along the loading direction in the first loading stage.

At the end of the first loading stage (uniaxial tension in the rolling direction), $\sigma_2 = 0$ and $\alpha_2 = 0$. Then from the isotropic-kinematic hardening model we have

$$\sigma_1 - \alpha_1 = \bar{\sigma}$$

(12)
where $\sigma_1$ and $\sigma_2$ are the backstress and the measured stress along the rolling direction at the end of the first loading stage.

At the initial yield point of the second loading stage, $\sigma_1 = 0$ and $\sigma_2 = 0$. Then from Eq. (11) we have

$$\phi(-\sigma_1, \sigma_2) = 2\sigma^m$$ (13)

where $\sigma_2$ is the measured yield stress at the initial yield point of the second loading stage.

Then the isotropic hardening $\sigma$ and the backstress $\sigma_2$ can be obtained by solving Eqs. (12) and (13) numerically. The results based on the Chaboche type model with Hill48 yield function can also be obtained just by replacing Yld2000-2d with Hill48 yield function in Eqs. (12) and (13).

4.2.3. Determination of the parameters of the hardening model

With several two-stage loading tests with different pre-strains in the first loading stage, a group of $\sigma$ and $\sigma_1$ can be obtained from Eqs. (12) and (13). After the calculation of the backstress and the isotropic hardening, three methods for determining the parameters of the hardening model are presented below utilizing the experimental data of group (1) (the first and the second loading are along the rolling and the transverse directions, respectively).

Method (1)

The kinematic hardening parameters of Chaboche type model, $c$ and $\gamma$, are assumed to be constants. Then in the first loading stage of group (1), from Eq. (9) we have

$$\sigma_1 = \frac{c}{\gamma}(1 - \exp(-\gamma\bar{e}))$$ (14)

Voyc type hardening function was adopted to describe the isotropic hardening $\sigma$:

$$\sigma = \sigma_0 + q(1 - \exp(-b\bar{e}))$$ (15)

where $\sigma_0$, $q$ and $b$ are materials parameters.

c and $\gamma$ were obtained by fitting a group of data ($\sigma_1$, $\bar{e}$) while $\sigma_0$, $q$ and $b$ were obtained by fitting data $(\sigma, \bar{e})$.

Method (2)

The kinematic hardening parameters of Chaboche type model, $c$ and $\gamma$, were determined with Method (1).

As for the isotropic hardening $\sigma$, we use the expression below:

$$\sigma = \sigma_1 - \sigma_1 = \sigma_0 + q(1 - \exp(-b\bar{e})) - \frac{c}{\gamma}(1 - \exp(-\gamma\bar{e}))$$ (16)

where $\sigma_0$, $q$ and $b$ were obtained by fitting the uniaxial tensile test data, $(\sigma_1, \bar{e})$, in the rolling direction.

Method (3)

Voyc type hardening function was adopted for the isotropic hardening $\sigma$ and the parameters $\sigma_0$, $q$ and $b$ were determined with Method (1).

For the first loading stage of group (1), from Eqs. (9) and (12) we have

$$\frac{d\sigma_1}{d\bar{e}} = c - \gamma\bar{e}_1$$

$$\frac{d\sigma_1}{d\sigma} = \frac{d\sigma_1}{d\bar{e}} = \frac{d\sigma_1}{d\sigma}$$ (17)

where $d\sigma_1/d\bar{e}$ can be obtained from the experimental curve in the first loading stage of group (1), and $d\sigma_1/d\sigma$ can be obtained from Eq. (15) or from the resulting data $(\sigma, \bar{e})$ if they are sufficient.

At the initial yield point of the second loading stage of group (1), $\sigma_2 = 0$ and $\sigma_1 = 0$. Then according to Eq. (9)

$$\frac{d\sigma_1}{d\bar{e}} = c(\frac{\sigma_1 - \sigma_1}{\sigma}) - \gamma\bar{e}_1 = -\frac{c}{\sigma}(c + \gamma)$$ (18)

and

$$\frac{d\sigma_2}{d\bar{e}} = c(\frac{\sigma_2 - \sigma_1}{\sigma}) - \gamma\bar{e}_2 = -\frac{c}{\sigma}$$ (19)

For the first loading stage of group (1), according to Drucker’s postulate and plastic work equivalent theorem

$$\frac{d\sigma_1}{d\sigma} = \frac{d\sigma_1}{d\bar{e}}$$ (20)

where $d\sigma_1/d\sigma$ is the plastic strain increment along the transverse direction. Then we have

$$\frac{d\sigma_2}{d\sigma} = \frac{d\sigma_2}{d\sigma} = \frac{d\sigma_2}{d\sigma}$$ (21)

The differentiation of the Chaboche type combined isotropic-kinematic hardening model with Yld2000-2d yield function (Eq. (10)) leads to

$$\frac{d\sigma}{d\sigma_1} = \frac{d\sigma}{d\sigma_2} + \frac{d\sigma}{d\sigma_2} + \frac{d\sigma}{d\sigma_2} = \frac{d\sigma}{d\sigma_2}$$ (22)

For the second loading stage of group (1), according to Drucker’s postulate and plastic work equivalent theorem

$$\frac{d\sigma}{d\sigma_1} = \frac{d\sigma}{d\sigma_2}$$ (23)

Since at the initial yield point of the second loading stage of group (1), $\sigma_2 = \sigma_1 = 0$ we have

$$\frac{d\sigma}{d\sigma_1} = \frac{d\sigma}{d\sigma_2}$$ (24)

Then we have

$$\frac{d\sigma_2}{d\sigma_1} = \frac{d\sigma_2}{d\sigma_2}$$ (25)

From Eqs. (18), (19), (22), and (24) we have

$$-\frac{c}{\sigma}(c + \gamma) = \frac{d\sigma_1}{d\sigma_2} + \frac{d\sigma_2}{d\sigma_2} + \frac{d\sigma_2}{d\sigma_2} + \frac{c}{\sigma}\frac{d\sigma_2}{d\sigma_2}$$ (26)

Then from Eqs. (17) and (26) we have

$$c = P(1 - K - \frac{d\sigma_1}{d\sigma_1})$$ (27)

$$\gamma = \frac{P}{\gamma_1}(1 - K - \frac{d\sigma_1}{d\sigma_1})$$ (28)

where $I = \frac{d\sigma_1}{d\sigma_1}$, $K = \frac{d\sigma_1}{d\sigma_1} + \frac{d\sigma_2}{d\sigma_2} + \frac{d\sigma_2}{d\sigma_2}$ and $M = \frac{d\sigma_1}{d\sigma_1} - \frac{d\sigma_1}{d\sigma_1}$. $P = \frac{(1 - K - \frac{d\sigma_1}{d\sigma_1})}{\gamma_1}$. Here, $\gamma_1$ can be obtained from the experimental curve in the second loading stage.

With the test data of six two-stage loadings of group (1), a set of data $(c, \bar{e})$ and $(\gamma, \bar{e})$ can be obtained from Eqs. (27) and (28). Here, the kinematic hardening parameters, $c$ and $\gamma$, were represented as the exponentially decaying functions of the equivalent plastic strain $\bar{e}$:

$$c(\bar{e}) = c_1 + c_2 e^{-c_3 \bar{e}}$$ (29)

$$\gamma(\bar{e}) = \gamma_1 + \gamma_2 e^{-\gamma_3 \bar{e}}$$ (30)

By fitting the obtained data $(c, \bar{e})$ and $(\gamma, \bar{e})$, the parameters of Eqs. (29) and (30) $(c_1, c_2, c_3, \gamma_1, \gamma_2, \gamma_3)$ can be determined. The results based on the Chaboche type model with Hill48 yield function can also be obtained from the above equations just by replacing Yld2000-2d with Hill48 yield function.
5. Numerical simulations

The established Chaboche type combined isotropic–kinematic hardening models with Yld2000-2d and Hill48 yield functions were implemented into ABAQUS/Standard codes using the user-subroutines UMAT and the Backward Euler method. The basic equations of the numerical formulations for the Chaboche type isotropic–kinematic hardening model with Yld2000-2d yield function are as follows.

\[
\Delta \sigma_n = \sigma_n - \sigma_0 = C(\Delta \varepsilon^p_n) + D\Delta \sigma_n + C(\Delta \varepsilon^p_n)\Delta \sigma_n \quad (31a)
\]

\[
\Delta \varepsilon^p_n = \frac{\partial}{\partial \sigma} \left( \frac{\sigma}{\sigma_0} \right) \quad (31b)
\]

\[
\Delta \sigma_n = \sigma_n - \sigma_0 = C(\Delta \varepsilon^p_n)\Delta \sigma_n \quad (31c)
\]

\[
\Delta \varepsilon^p_n = \Delta \varepsilon^p_n - \frac{\partial}{\partial \sigma} \left( \frac{\sigma}{\sigma_0} \right) \quad (31d)
\]

And

\[
\rho = \rho_0(\Delta \sigma_n) = \rho_0(\Delta \varepsilon^p_n + \Delta \sigma_n) \quad (32a)
\]

\[
c = c_0(\Delta \sigma_n) = c_0(\Delta \varepsilon^p_n + \Delta \sigma_n) \quad (32b)
\]

\[
\gamma = \gamma_0(\Delta \sigma_n) = \gamma_0(\Delta \varepsilon^p_n + \Delta \sigma_n) \quad (32c)
\]

where \( \varepsilon^p \) is plastic strain, \( C \) is elastic matrix and \( \rho \) is the reference hardening curve. The subscript denotes the process time step.

The predictor-corrector scheme based on the Newton–Raphson method was used to solve \( \Delta \sigma_n \) in Eq. (31). To effectively solve the non-linear equation, the solution was obtained progressively by adding several yield surfaces between the trial elastic stress \( \sigma_n = \sigma_0 + C\Delta \varepsilon^p_n \) and the initial stress \( \sigma_0 \) (Yoon et al., 1999).

As for the isotropic–kinematic hardening model with Hill48 yield function, only Chaboche (3) was implemented into ABAQUS since Hill48 yield function and the Chaboche type model with constant parameters are already included in ABAQUS.

Then the finite element simulations of the above two-stage tests were performed where the S4R shell elements were adopted.

6. Results and discussions

6.1. Verifications of the hardening models

The anisotropic coefficients of Yld2000-2d are calculated with the seven measured material properties (in Table 1) shown in Table 2 assuming \( \beta_3 = \beta_6 \) (therefore, \( L_{12} = L_{23} \)) in Eq. (7) by solving a group of non-linear equations (Barlat et al., 2003a; Lee et al., 2005).

The anisotropic coefficients of Hill48 yield functions are listed in Table 3. In Table 3, the coefficients of Hill48 yield functions are calculated with two different relationships (Park and Chung, 2012) with \( r \)-values and normalized stresses presented in Table 1 respectively as shown in Section 4.1. The resulting parameters of Chaboche type model combined isotropic–kinematic model with Yld2000-2d and Hill48 are listed in Tables 4–9. The values in Tables 4 and 5 were obtained with Yld2000-2d, those in Tables 6 and 7 were obtained with Hill48 (1) and those in Tables 8 and 9 were obtained with Hill48 (2).

The predicted reference curve, isotropic hardening and backstress based on Chaboche (1), (2) and (3) with Yld2000-2d yield function are compared with the experimental results in Fig. 7. In Fig. 7, the experimental reference curve is the simple tension curve along the rolling direction. The experimental isotropic hardening and backstress are obtained by solving Eqs. (12) and (13), where \( \sigma_1 \) and \( \sigma_2 \) are measured from the two stage loading test of Group (1) as presented in Section 3.1, i.e. the first loading and the second loading are along the rolling and the transverse directions, respectively. For each two-stage loading test, one pair of stresses \( \sigma_1 \) and \( \sigma_2 \) are measured from the first loading and the second loading tests, respectively. Then one pair of values of the isotropic hardening and backstress can be obtained by solving Eqs. (12) and (13). Therefore six pairs of values of the isotropic hardening and backstress are obtained since there are six tests in the two-stage test of Group (1).

It can be seen that all the three models can describe the measured isotropic hardening and backstress reasonably. Chaboche (2) and (3) can describe the uniaxial tensile test curves in the rolling directions accurately whereas Chaboche (1) cannot. For Chaboche (1), there is a bifurcation point between the simulated and experimental curve in the rolling direction, after which the error of Chaboche (1) becomes larger with the increase of strain. This is because that only the limited experimental data including the backstress and the isotropic hardening were used for determining the parameters of Chaboche (1). When determining the parameters of the isotropic hardening of Chaboche (2), the whole experimental reference curve (uniaxial tensile curves in the rolling direction) was used so that the simulated and experimental reference curves in the rolling direction coincide with each other. Besides the experimental isotropic hardening and backstress, the slopes of the experimental curves were also used for determining the parameters of Chaboche (3) which may lead to the reasonable prediction of the experimental reference curve.

The uniaxial tensile test curves simulated based on Chaboche (1), (2) and (3) with Yld2000-2d yield function along the 45° and the transverse directions are shown in Fig. 8. In this study, the experimental uniaxial tensile curves along the rolling direction was adopted as the reference curve and the stress anisotropy changes during deformation process were considered in Yld2000-2d yield function. Therefore, as shown in Fig. 8, Chaboche (2) and (3) can describe the uniaxial curves along 45° and the transverse directions well since the reference curves based on them coincide with the experimental ones. Chaboche (1) can not describe the experimental curves along 45° and the transverse direction reasonably since it can not predict the reference curve reasonably.

The simulated and experimental stress–strain curves in the second loading stage of the two-stage loading, where the first and the second loading stages are along the rolling and the transverse
For all the three models, the initial yield stress of the second loading stage can be predicted accurately, because the experimental backstress and isotropic hardening were adopted to determine the parameters of the isotropic–kinematic hardening model.

As shown in Fig. 9, neither the initial yield stress of the second loading stage nor the transient effect can be described reasonably by the isotropic hardening model. It can also be seen that the error of the isotropic hardening model becomes larger with the increase of the pre-strain. However, for each curve, after some deformation the results based on the isotropic hardening model converge to those based on Chaboche (3) gradually. This is because the monotonic tension curve along the rolling direction based on Chaboche (3) is adopted as the reference and no permanent softening exists in Chaboche model (shown in Appendix). If the simple tensile stress–strain curve based on another Chaboche model (Chaboche (1) or Chaboche (2)) is adopted as the reference curve, the similar result based on the isotropic hardening will be observed which will not be shown here.

Figs. 10 and 11 shows the experimental and the predicted backstress, isotropic hardening, and the reference curve based on Chaboche (1), (2) and (3) with Hill48 (1) and Hill48 (2) respectively. The results are obtained with the same method as above (as those with Yld2000-2d yield function). As shown in Fig. 10, for the case with Hill48 (1) characterized with r-values, all the three models can describe the isotropic hardening reasonably. Chaboche (2) describes the reference curve better than Chaboche (1) and Chaboche (3). At the initial stage of deformation, Chaboche (3) underestimates the backstress and the reference curve. After some deformation, the error of the reference curve based on Chaboche (1) with Hill48 becomes larger with the increase of strain. As shown in Figs. 11 and 7, the predicted results based on the three Chaboche models with Hill48 (2) characterized with normalized stresses are similar to those with Yld2000-2d, respectively which will no longer be discussed.

The uniaxial tensile curves in 45° and the transverse directions simulated based on Chaboche (1), (2) and (3) with Hill48 yield function are shown in Figs. 12 and 13. For the case with Hill48 (1) with r-value stresses, as shown in Fig. 12, all the three models with Hill48 yield function cannot describe the curves in 45° and the transverse directions reasonably. Rather, as shown in Fig. 13, for the case with Hill48 (2) characterized with normalized stresses, all the three models describe the experimental curves reasonably as well as those with Yld2000-2d yield function.

Figs. 14 and 15 shows the results in the second loading stage of the two-stage loading of group (1) (the first and the second loading are along the rolling and transverse directions) based on Chaboche (1), (2) and (3) with Hill48 (1) and Hill48 (2). By comparing Figs. 9 and 15, the performances of the three Chaboche model and isotropic hardening model with Hill48 (2) characterized with normalized stresses are same as those with Yld2000-2d yield function. The results based on Chaboche (3) with Hill48 (2) are as good as those based Chaboche (3) with Yld2000-2d. As for the case with Hill48 (1) (characterized with r-values), as shown in Fig. 14, none of three Chaboche models can describe the experimental curves of the second loading stage reasonably though the initial yield stress can be predicted well.

As seen from the above results, Chaboche (3) with Yld2000-2d and Hill48 (2) (Characterized with normalized stress) have the highest accuracy in describing the experimental results under two-stage loadings.

In this study, the parameters of the isotropic–kinematic hardening models were determined with the experimental data under the two-stage loading where the first and second loading stages are along the rolling and the transverse directions, respectively. In order to further evaluate the accuracy of the established hardening model, the simulated and experimental results under directions, respectively, are compared with each other as shown in Fig. 9. The simulated results based on the isotropic hardening model are also presented where the monotonic tension curve based on Chaboche (3) along the rolling directions is adopted as the reference curve. It can be seen that the stress–strain curves simulated based on Chaboche (3) with Yld2000-2d yield function are in good agreement with the experimental results. Chaboche (3) can describe the transient effect of the entire experimental results well, because the slopes of the experimental curves in the second loading stage were considered. When the pre-strain is not large (0.013, 0.02 and 0.028), there is no significant difference between Chaboche (2) and Chaboche (3). And when the pre-strains are 0.013, 0.02 and 0.028, Chaboche (1) and Chaboche (2) describe the transient effect reasonably whereas they do not act well on other pre-strains.

Table 5
The parameters of Chaboche (3) with Yld2000-2d yield function.

<table>
<thead>
<tr>
<th>σ0</th>
<th>q</th>
<th>b</th>
<th>c1</th>
<th>c2</th>
<th>c3</th>
<th>γ1</th>
<th>γ2</th>
<th>γ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>92.19</td>
<td>140.49</td>
<td>18.25</td>
<td>4283.08</td>
<td>-4041.92</td>
<td>3.72</td>
<td>54.12</td>
<td>-64.77</td>
<td>42.13</td>
</tr>
</tbody>
</table>

Table 6
The parameters of Chaboche (1) and Chaboche (2) with Hill48 (1).

<table>
<thead>
<tr>
<th>σ0</th>
<th>q</th>
<th>b</th>
<th>c</th>
<th>γ1</th>
<th>γ2</th>
<th>γ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chaboche (1)</td>
<td>90.43</td>
<td>125.81</td>
<td>17.31</td>
<td>1075.09</td>
<td>23.80</td>
<td></td>
</tr>
<tr>
<td>Chaboche (2)</td>
<td>99.11</td>
<td>180.79</td>
<td>14.45</td>
<td>1075.09</td>
<td>23.80</td>
<td></td>
</tr>
</tbody>
</table>

Table 7
The parameters of Chaboche (1) and Chaboche (2) with Hill48 (2).

<table>
<thead>
<tr>
<th>σ0</th>
<th>q</th>
<th>b</th>
<th>c</th>
<th>γ1</th>
<th>γ2</th>
<th>γ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chaboche (1)</td>
<td>97.82</td>
<td>132.30</td>
<td>17.82</td>
<td>423.598</td>
<td>8.787</td>
<td></td>
</tr>
<tr>
<td>Chaboche (2)</td>
<td>99.11</td>
<td>180.79</td>
<td>14.45</td>
<td>423.598</td>
<td>8.787</td>
<td></td>
</tr>
</tbody>
</table>

Table 8
The parameters of Chaboche (3) with Hill48 (1).

<table>
<thead>
<tr>
<th>σ0</th>
<th>q</th>
<th>b</th>
<th>c1</th>
<th>c2</th>
<th>c3</th>
<th>γ1</th>
<th>γ2</th>
<th>γ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.43</td>
<td>125.81</td>
<td>17.31</td>
<td>3703.25</td>
<td>-3524.10</td>
<td>3.43</td>
<td>26.92</td>
<td>-65.88</td>
<td>30.49</td>
</tr>
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</table>

Table 9
The parameters of Chaboche (3) with Hill48 (2).

<table>
<thead>
<tr>
<th>σ0</th>
<th>q</th>
<th>b</th>
<th>c1</th>
<th>c2</th>
<th>c3</th>
<th>γ1</th>
<th>γ2</th>
<th>γ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>97.82</td>
<td>132.30</td>
<td>17.82</td>
<td>6481.50</td>
<td>-6088.40</td>
<td>2.23</td>
<td>52.05</td>
<td>-53.72</td>
<td>32.40</td>
</tr>
</tbody>
</table>

Fig. 7. Isotropic hardening, backstress and reference curve based on Chaboche type model with Yld2000-2d yield function.
Fig. 8. Uniaxial tensile stress–strain curves simulated based on Chaboche type model with Yld2000-2d yield function. (a) Curves along the 45° direction. (b) Curves along the transverse direction.

Fig. 9. Comparison of the experimental and simulated stress–strain curves based on Chaboche type model with Yld2000-2d yield function (for the second loading of the two-stage tests where the first loading is along the rolling direction and the second loading is along the transverse direction). (a) Pre-strain of 0.013, (b) Pre-strain of 0.02, (c) Pre-strain of 0.028, (d) Pre-strain of 0.047, (e) Pre-strain of 0.077, (f) Pre-strain of 0.107.
other two-stage loadings are compared with each other. Figs. 16 and 17 show the comparisons between the experimental and simulated results based on Chaboche (1), (2) and (3) with Yld2000-2d and Hill48 (2) (characterized with normalized stress). In Fig. 16(a), (b) and Fig. 17(a), (b) the first and the second loading stages are along the rolling and the 45° directions, respectively, while in Fig. 16(c), (d) and Figs. 17(c), (d) the first and the second loading stages are along the transverse and the rolling directions, respectively. As shown in Figs. 16 and 17, the performances of the three Chaboche models based on Hill48 (2) are the same as those based on Yld2000-2d. Chaboche (3) with Hill48 (2) and Yld2000-2d describe the entire experimental results reasonably. For the isotropic hardening model, neither the initial yield stress nor the transient effect of the second loading stage can be described reasonably whereas the predicted results coincide with the experimental results gradually with the increase of the deformation due to no permanent softening.

6.2. Performance of the yield functions on the resulting hardening model

For the hardening behavior under in-plane uniaxial cyclic tension–compression test, the predicted results do not depend on the choice of the yield function since the loading is always along the same axis (forward or reverse loading). However, for the two-stage loading, the loading paths are along two different directions, the anisotropy also plays an important role on the predicted results as well as the kinematic hardening. In this study, the objective is to characterize the stress–strain curves and the stress anisotropy is dominant. Therefore, when Hill48 is characterized with normalized stress the predicted results are very close to those with Yld2000-2d function and Chaboche (3) model with both of them describe the experimental results reasonably. Rather, when Hill48 is characterized with r-values, the results can not be described reasonably. The above results show that a proper characterization method is as important as the yield function itself. Therefore, it is also important to choose a proper characterization method of the yield function when establishing the combined isotropic–kinematic hardening model for two-stage loading besides a proper kinematic hardening law.

Fig. 18 shows the plastic work contours of Yld2000-2d, Hill48 (1) and Hill48 (2) at different equivalent plastic strain. As shown in Fig. 18, the plastic work contours of Hill48 (1) and Hill48 (2) with the two characterization methods are remarkably different. The difference of the predicted stress in the transverse direction will lead to the differences of the backstress and the isotropic hardening solved from Eqs. (12) and (13). The stresses in the transverse direction with Hill48 (2) always coincide with those with Yld2000-2d yield function which lead to the almost same results. In this study, the isotropic hardening and the backstress were obtained by solving Eqs. (12) and (13), from which it can be concluded that the end loading point of the first loading stage (A1) and the initial yield point of the second loading stage (A2) should be on the resulting subsequent yield surface as shown in Fig. 19, where the equivalent plastic strain is 0.107. OY, OH1 and OH2 denote the resulting centers of the subsequent yield surfaces of Yld2000-2d, Hill48 (1) and Hill48 (2), respectively, which demonstrates the difference of the backstresses based on Hill48 (1) and Hill48 (2).
Fig. 13. Uniaxial tensile stress-plastic strain curves simulated based on Chaboche type model with Hill48 (2). (a) Curves along the 45° direction. (b) Curves along the transverse direction.

Fig. 14. Comparison of the experimental and simulated stress–strain curves based on Chaboche type model with Hill48 (1) (for the second loading of the two-stage tests where first loading is along the rolling direction and the second loading is along the transverse direction). (a) Pre-strain of 0.013, (b) Pre-strain of 0.02, (c) Pre-strain of 0.028, (d) Pre-strain of 0.047, (e) Pre-strain of 0.077, (f) Pre-strain of 0.107.
Fig. 20 shows the resulting isotropic hardening and backstress vs. the equivalent plastic strain curves based on Chaboche (3) with Yld2000-2d, Hill48 (1) and Hill48 (2), from which the difference between Hill48 (1) and Hill48 (2) can be seen clearly. As shown in Fig. 20, the resulting curves based with Yld2000-2d and Hill48 (2) almost coincide with each other.

Though the resulting isotropic hardening and the backstress based on Yld2000-2d, Hill48 (1) and Hill48 (2) are different, they both satisfy Eqs. (12) and (13). Therefore, both resulting initial yield stress of the second loading stage based on them coincide with the experimental ones, as shown in Figs. 9, 14 and 15.

The established combined isotropic–kinematic hardening model Chaboche (3) with Yld2000-2d and Hill48 (2) can describe the Bauschinger effect and the transient effect under two-stage loading reasonably, but it can not describe permanent softening effect. In order to describe the hardening behavior with softening effect, a modification such as adding softening parameters is needed.

7. Three point bending tests of the pre-strained specimen

7.1. Experimental and numerical simulation

In order to further evaluate and verify the established hardening model considering more general loading path changes, the three point bending test of the pre-strained specimen was performed. As shown in Fig. 21, in the first step, a rectangle sheet with the length of 190 mm was stretched along the length direction and the second loading is along the transverse direction. As shown in Fig. 21, the resulting curves based with Yld2000-2d and Hill48 (2) almost coincide with each other.

Though the resulting isotropic hardening and the backstress based on Yld2000-2d, Hill48 (1) and Hill48 (2) are different, they both satisfy Eqs. (12) and (13). Therefore, both resulting initial yield stress of the second loading stage based on them coincide with the experimental ones, as shown in Figs. 9, 14 and 15.
Fig. 16. Comparisons of the experimental and the simulated stress–strain curves based on Chaboche type model with Yld2000-2d yield function. (a) Curves along the 45° direction with pre-strain of 0.018 along the rolling direction. (b) Curves along the 45° direction with pre-strain of 0.057 along the rolling direction. (c) Curves along the rolling direction with pre-strain of 0.047 along the transverse direction. (d) Curves along the rolling direction with pre-strain of 0.077 along the transverse direction.

Fig. 17. Comparisons of the experimental and the simulated stress–strain curves based on Chaboche type model with Hill48 (2). (a) Curves along the 45° direction with pre-strain of 0.018 along the rolling direction. (b) Curves along the 45° direction with pre-strain of 0.057 along the rolling direction. (c) Curves along the rolling direction with pre-strain of 0.047 along the transverse direction. (d) Curves along the rolling direction with pre-strain of 0.077 along the transverse direction.
The width of the rectangle sheet before the first step is 40 mm. Two specimens with the length along the rolling and the transverse directions respectively were used. The diameters of the punch and die of three point bending are 20 mm and the span of the die is 40 mm as shown in Fig. 21(b). The displacement of the punch in three point bending is 4.5 mm.

The three point bending test of no pre-strained specimen with the same dimensions as those of the pre-strained specimen was also performed.

In the simulation of three point bending, S4R shell element was adopted for the blank with 5 integration points through thickness direction. R3D4 discrete rigid element was adopted for the punch and die and the friction coefficient \( \mu = 0.2 \) was adopted.

For the three point-bending test, elasticity (and its anisotropy) is as important as plasticity (Chung et al., 2011). Therefore, orthotropic elasticity was used under the planes stress condition:

\[
\begin{bmatrix}
    \sigma_{xx} \\
    \sigma_{yy} \\
    \sigma_{xy}
\end{bmatrix} =
\begin{bmatrix}
    E_x & \nu_{xy} & 0 \\
    \nu_{yx} & E_y & 0 \\
    0 & 0 & G_{xy}
\end{bmatrix} \begin{bmatrix}
    \varepsilon_{xx} \\
    \varepsilon_{yy} \\
    \varepsilon_{xy}
\end{bmatrix} \tag{33}
\]

where \( \nu_{xy}E_x = \nu_{yx}E_y, G_{xy} = \frac{E}{2(1+\nu)} \).

In Eq. (33), \( x, y, z \) are the materially embedded principal anisotropic axes: \( x \) for the rolling, \( y \) and \( z \) for transverse and thickness directions, respectively. The elastic modulus along the rolling, 45°-off and transverse directions \( E_0, E_{45} \) and \( E_{90} \) are shown in Table 10.

Two models were used in the simulations:

- Isotropic hardening model with Yld2000-2d yield function.

#### 7.2. Results and discussions

From the simulated results of three point bending, it can be seen that the material points at the edges of the specimen (with or without pre-strain) are under uniaxial loading while those at the center of the specimen are nearly under plane strain loading. During bending process of the pre-strained specimens, the material points under compressive loading (i.e., the material points above the neutral layer of the specimen) will experience reverse loading since they undergo tensile loading in the first loading stage. Fig. 22 shows the comparisons between the experimental and the simulated springback profiles of the three point bending
test of the specimens without pre-strain, while Fig. 23 shows those with pre-strains.

As shown in Fig. 22, for the three point bending of the specimens without pre-strain, both the isotropic–kinematic hardening model Chaboche (3) with Yld2000-2d yield function and the isotropic hardening model with Yld2000-2d yield function describe the springback profile reasonably. As shown in Fig. 23, the isotropic–kinematic hardening model Chaboche (3) with Yld2000-2d yield function can describe springback profiles of the two pre-strained specimens reasonably whereas the isotropic hardening model with Yld2000-2d yield function can not.

The main objective of this study is to establish a proper isotropic–kinematic hardening model that can describe the hardening behavior under two-stage loading accurately. It is interesting that the isotropic–kinematic hardening model Chaboche (3) with Yld2000-2d yield function established with the experimental data from two-stage loading can reasonably describe the springback profile of the three point bending of the pre-strained specimen where some material points undergo reverse loading.

8. Conclusions

In the present work, the two-stage loading tests were performed for 5754O aluminum alloy and the Bauschinger effect and transient effect were observed whereas no permanent softening appeared. The Chaboche type isotropic–kinematic hardening models were adopted with Yld2000-2d and Hill48 yield functions. From theoretical analysis it is found that the Chaboche type model can not describe the permanent softening in two-stage loading. Three methods for determining the parameters of the isotropic–kinematic hardening model were presented. The two-stage loading tests were simulated in ABAQUS software based on the established models. The isotropic–kinematic hardening models describing the experimental results under two-stage loading were accurately determined. The influences of the characterization of Hill48 yield function on the accuracy of the resulting isotropic–kinematic hardening model were analyzed. It is concluded that a proper characterization method of the yield function is important when establishing accurate isotropic–kinematic hardening model for two-stage loading. It is interesting that the established isotropic–kinematic hardening model describing the deformation behavior under two-stage loading can also describe reasonably the springback profile of three point bending of the pre-strained specimen.

The experimental and theoretical investigation of the anisotropy evolution of sheet metals under two-stage loading will be carried out in the future.

Acknowledgements

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Appendix A. Performance of Chaboche type model for the permanent softening in two-stage loading

Take the two-stage loading where the first and the second loading stages are along the rolling and the transverse directions respectively, as an example.

For the monotonic loading along the direction of the second loading stage (the transverse direction), we have $\sigma_2 = 0$ according to Chaboche type model (Eq. (8)) since $\sigma_1 = 0$. Then according to the expression of Eq. (9) (Chaboche type isotropic–kinematic hardening model with Yld2000-2d yield function) we have

$$\sigma_2 - \sigma_2 = \eta((\vec{\varepsilon})_T) \sigma$$

where

$$\eta((\vec{\varepsilon})_T) = \frac{\sigma_2 - \sigma_2}{\sigma_{\eta}(\vec{\varepsilon})}$$

In Eq. (A.2), $\sigma_2$ and $\sigma_{\eta}(\vec{\varepsilon})$ are the present yield stresses along the rolling and the transverse directions during deformation. Then according to Eq. (8) we have

$$d\sigma_2 = c((\vec{\varepsilon})^p) \eta((\vec{\varepsilon})_T) d\vec{\varepsilon}^p - \gamma((\vec{\varepsilon})_T) d\vec{\varepsilon}^p$$

Here the kinematic hardening parameters, $c$ and $\gamma$, were assumed as the functions of the equivalent plastic strain $\vec{\varepsilon}^p$. In special cases $c$ and $\gamma$ are constant.

Then from Eq. (A.3),

$$\varepsilon_2(\vec{\varepsilon}) = \exp \left( - \int_{\vec{\varepsilon}^p}^{\vec{\varepsilon}} \gamma((\vec{\varepsilon})^p) d\vec{\varepsilon}^p \right) \left( \int_{\vec{\varepsilon}^p}^{\varepsilon(\vec{\varepsilon})} \gamma((\vec{\varepsilon})_T) c((\vec{\varepsilon})_T)^p d\vec{\varepsilon}^p + \varepsilon_2(\vec{\varepsilon})^p \right)$$

Then

$$\lim_{\vec{\varepsilon}^p \rightarrow \infty} \varepsilon_2(\vec{\varepsilon}) = \lim_{\vec{\varepsilon}^p \rightarrow \infty} \int_{\vec{\varepsilon}^p}^{\varepsilon(\vec{\varepsilon})} \gamma((\vec{\varepsilon})_T) c((\vec{\varepsilon})_T)^p d\vec{\varepsilon}^p + \varepsilon_2(\vec{\varepsilon})^p$$

$$= \lim_{\vec{\varepsilon}^p \rightarrow \infty} \left( \eta((\vec{\varepsilon})_T) c((\vec{\varepsilon})_T)^p \right)$$

Then according to Eqs. (A.1) and (A.6)

$$\lim_{\vec{\varepsilon}^p \rightarrow \infty} \sigma^{\text{monotonic}}_2 = \lim_{\vec{\varepsilon}^p \rightarrow \infty} \eta((\vec{\varepsilon})_T) \sigma_T(\vec{\varepsilon}) + \lim_{\vec{\varepsilon}^p \rightarrow \infty} \varepsilon_2(\vec{\varepsilon}) = \lim_{\vec{\varepsilon}^p \rightarrow \infty} \eta((\vec{\varepsilon})_T) c((\vec{\varepsilon})_T)^p \gamma((\vec{\varepsilon})_T)$$

In the first loading stage of this two-stage loading, $\sigma_2 = 0$. Then we have $\sigma_2 = 0$ according to Eq. (8). Then according to Eq. (8) we have

$$d\varepsilon_2 = c((\vec{\varepsilon})_T) \frac{\sigma_2 - \sigma_2}{\sigma} d\vec{\varepsilon}^p - \gamma((\vec{\varepsilon})_T) d\vec{\varepsilon}^p$$

After the first loading stage and then in the second loading stage $\sigma_1 = 0$. Then according to Eq. (8) we have

$$d\varepsilon_2 = -\varepsilon_2 \left( \frac{c((\vec{\varepsilon})_T)}{\sigma_{\eta}((\vec{\varepsilon})_T)} + \gamma((\vec{\varepsilon})_T) \right) d\vec{\varepsilon}^p$$

And from Eq. (8) we have

$$d\varepsilon_2 = c((\vec{\varepsilon})_T) \eta((\vec{\varepsilon})_T) d\vec{\varepsilon}^p - \gamma((\vec{\varepsilon})_T) d\vec{\varepsilon}^p$$

where

$$\eta((\vec{\varepsilon})_T) = \frac{\sigma_2 - \sigma_2}{\sigma}$$

Since $\sigma_1 \neq 0$, according to the expression of Eq. (10) we have $\eta((\vec{\varepsilon})_T) \neq \eta((\vec{\varepsilon})_T)$. From Eq. (A.9),

$$\varepsilon_1((\vec{\varepsilon})_T) = \exp \left( - \int_{\vec{\varepsilon}^p}^{\varepsilon((\vec{\varepsilon})_T)} \left( \frac{c((\vec{\varepsilon})_T)}{\sigma_{\eta}((\vec{\varepsilon})_T)} + \gamma((\vec{\varepsilon})_T) \right) d\vec{\varepsilon}^p \right) \varepsilon_1((\vec{\varepsilon})_T)$$

Then after some operations with the expressions of $c((\vec{\varepsilon})_T)$, $\gamma((\vec{\varepsilon})_T)$ and $\eta((\vec{\varepsilon})_T)$ presented below, we have

$$\lim_{\vec{\varepsilon}^p \rightarrow \infty} \varepsilon_1(\vec{\varepsilon}) = 0$$

With the similar calculation as Eq. (A.6) we have

$$\lim_{\vec{\varepsilon}^p \rightarrow \infty} \varepsilon_2 = \lim_{\vec{\varepsilon}^p \rightarrow \infty} \eta((\vec{\varepsilon})_T) \left( \frac{\sigma_2 - \sigma_2}{\sigma_{\eta}((\vec{\varepsilon})_T)} \right)$$

$$= \lim_{\vec{\varepsilon}^p \rightarrow \infty} 2 \left( \frac{c((\vec{\varepsilon})_T)^p}{\eta((\vec{\varepsilon})_T)} \right)$$

Then according to the expression of Eq. (10) we have

$$\lim_{\vec{\varepsilon}^p \rightarrow \infty} \left( \sigma_2 - \sigma_2 \right) = \lim_{\vec{\varepsilon}^p \rightarrow \infty} \frac{\sigma_2 - \sigma_2}{\sigma_{\eta}((\vec{\varepsilon})_T)}$$

Then

$$\lim_{\vec{\varepsilon}^p \rightarrow \infty} \lim_{\vec{\varepsilon}^p \rightarrow \infty} \left( \frac{\sigma_2 - \sigma_2}{\sigma_{\eta}((\vec{\varepsilon})_T)} \right) = \lim_{\vec{\varepsilon}^p \rightarrow \infty} \left( \frac{\sigma_2 - \sigma_2}{\sigma_{\eta}((\vec{\varepsilon})_T)} \right)$$

$$= \lim_{\vec{\varepsilon}^p \rightarrow \infty} \left( \frac{\sigma_2 - \sigma_2}{\eta((\vec{\varepsilon})_T)} \right)$$

$$= \lim_{\vec{\varepsilon}^p \rightarrow \infty} \left( \frac{\sigma_2 - \sigma_2}{\gamma((\vec{\varepsilon})_T)} \right)$$

For the two-stage loading where the first and the second loading stages are along other directions, the same result can be obtained. With the Chaboche type model with Hill48 yield function, the same result can also be obtained. Therefore, it can be seen from Eq. (A.19) that no permanent softening can be described with Chaboche type model in two-stage loading.

References