Peculiarities of expert estimation. Comparison methods

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Abstract
This paper is devoted to integrated assessment of the alternative courses of action (objects) in the decision-making process. Expert investigation is frequently used method of comparison practical activities aspects. In some cases this method is only possible, as awareness about object behaviour can be obtained only from hands-on experience. Although attempts to create an automated system for obtaining expert evaluations to exclude the human factor are proceeding. In the paper comparison method used in getting criteria weights are described.

Keywords: comparison method, the decision-making process, expertise, criteria.

1 Introduction

Expertise is the basis of credibility of a person who is perceived to be knowledgeable in an area or topic due to study, training, or experience in the subject matter. The difficulty of getting evidential expert estimation is that there is difference in experts opinion, while as estimation based on only one expert opinion can be one-side. Therefore it is necessary to choose an exact mathematic method allowed to get formalized expert opinion [1].

To allow comparison of several objects, each of which is characterized by a set of different criteria, there is a need for to form a single integrated assessment, which will be subsequently used to compare and select one or more objects of the set in question, and their priority in terms of predetermined goals. Each valued object or effect is specific, and therefore, their evaluation requires the development of specific criteria.

Nowadays expert estimation is frequently used as this is the only way to assay actions (objects) and to make forecasting. At the same time scientists are looking for ways to create automated expert system, allowing to minimize involvement of a person in evaluation process or eliminate it completely. Expert estimation often considered as functional aspects of thinking, which can be
described at a computational level and reproduced as function of the automated system [2]. But it is necessary to analyze peculiarities of the evaluation process more detailed.

2 Main peculiarities of the evaluation process

To make decisions about some action (objects) is necessary to get a correct integral evaluation. Integral evaluation verity is based on several steps [3]:

1. Criterion selection - a quality criterion must be chosen to compare the objects.
2. Comparison method – method of comparing action (objects) must be chosen.
3. Estimations collection - the expert estimations of the objects must be collected.
4. Definition of criteria weights – criteria must be compared against each other.
5. Analysis of expert evaluations

The primary focus of this paper is on comparison methods.

2.1. Criterion selection

The choice of criteria depends on the goal of investigation. It may be due to standards or legal documents, the specificity of line of investigation, the process organization etc. [4].

2.2. Comparison method

The method of simple ranging

The method of simple ranging is based on expert’s arranging objects as personal preference. 1 sets to the most important object. 2 – the following object and so on. Results of ranging can be tabulated.

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Table 1: Expert estimates

$a_{ij}$ – an order of preference of one object above the other;
$x_j$ – comparison objects, $j$ from 1 to $n$;
$m_{kj}$ – number of the experts;

Integrated expert opinion is calculated on the basis of table (1).

$$ S_j = \frac{\sum_{i=1}^{m} a_{ij}}{m_{kj}} $$

The received values are characterized the importance of compared objects. To exclude possibility of casual distribution of ranks and to define degree of coherence of experts estimation can be calculated the coefficient of a konkordation. The first step is to get the average rank of the compared objects (2):

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The second step is calculation deviation degree from the average rank (3):

\[ d_j = \bar{S} - S_j. \]  

Also the quantity of identical ranks – \( t_i \) appointed by experts of \( j \)-objects and the quantity of groups of identical ranks – is defined by \( Z \). As a result the coefficient of a k onkordation can be calculated (4):

\[ K = \frac{12 \sum_{j=1}^{n} d_j}{m^2(n^2-n) - m \sum_{i=1}^{n} r_i}, \]  

where

\[ T_i = \sum_{z=1}^{Z} (t_z^3 - t_z) \]  

Method of consecutive comparisons

The sense of a method of consecutive comparisons consists in systematic check of the received estimates on the basis of their consecutive comparison. The algorithm of definition of degree of the importance of objects consists in the following. At the first stage the expert makes ranging of all objects as reduction of their degree of the importance, based on the especially personal opinion:

After that the expert needs to appropriate to the first element of comparison value equal to 1, \( X_1 = 1 \). Other objects values evaluate as unit fraction. Then the first object needs to be compared with the sum of all other objects. In this case are possible only three variants:

\[ X_1 = X_2 + \ldots + X_n \]  
\[ X_1 < X_2 + \ldots + X_n \]  
\[ X_1 > X_2 + \ldots + X_n \]  

On the basis of possible variants the expert chooses the most suitable and brings into accord with it an assessment of the first element. Then it is necessary to make procedure of comparison of the first element with the sum of all of the subsequent without the last object.

This algorithm of actions repeats until the number of the subsequent after the chosen object doesn't reach quantity equal to 2. After procedure of an assessment of the first element, using the same algorithm of actions, the expert repeats the procedure to make assessment of the second element and the subsequent.

The method of analysis of hierarchies by T. Saaty

According to this method, all of the identified criteria are matched by the expert using the matrix of pairwise comparisons. The grading scale that the author proposes is a simple scale from 1 to 9, the values of which correspond to the importance of the actions or objects with regard to each other. The more the superiority of one object to another, the higher the mark is. For example, a value of 1, reflects the fact that the two actions (projects) make the same contribution to the achievement of objectives [5-7].
Value 5 is attributed, if the experience and judgment give strong preference to one action (object) regarding another, and the value 9 is evidence in favor of choosing one action (object) to another in the highest degree. This scale is a tool for determining the importance of a particular action (object) in relation to the objectives of the study, when it is necessary to make a decision.

Experts are required to make paired comparisons of actions (objects) \( i \) and \( j \). The matrix is filled with pairwise comparisons based on the data. The estimates of experts are put at the top of the matrix (above the diagonal), because if action \( i \) is attributed one of the highest values of scale compared with the action \( j \), the action \( j \) is attributed an inverse value when compared to \( i \).

To determine whether the logical connection between the assessment of the actions (objects) by experts, a consistency index (hereinafter - CI) of the matrix of pairwise comparisons is introduced.

The maximum eigenvalue of the matrix must be calculated to find its index of consistency. The consistency index is calculated by the formula (7):

$$\text{CI} = \frac{\lambda_{\text{max}} - n}{n - 1}$$

where

\( \lambda_{\text{max}} \) – maximum eigenvalue,

\( n \) – shape of the matrix.

If the first effect (object) is preferred more than the second action (object) \( k \) times, and it is \( m \) times more preferable than the third one, and the first is \( k \cdot m \) times preferable regarding the third, such matrix is considered coherent and its consistency index is zero (8).

$$\begin{bmatrix}
1 & \frac{1}{a_{12}} & a_{12} \cdot a_{23} \\
\frac{1}{a_{12}} & 1 & a_{23} \\
a_{12} \cdot a_{23} & \frac{1}{a_{23}} & 1
\end{bmatrix}$$

Typically, the matrix formed on the basis of data obtained by experts, is not perfectly consistent. To evaluate the consistency of the matrix, conformity relation (hereinafter - CR) is introduced. It shows the relationship of CI of the investigated matrix to the average index (hereinafter - AI), calculated for inversely symmetric random matrices of similar dimension. Table 2 shows the AI of matrices formed using a scale from 1 to 9, applied for paired comparisons of actions (objects). The formed experts matrix is considered conventionally compatible if the value of CR is less than or equal to 0.10.

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<td>1.45</td>
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**Table 2**: The average index for random matrices

Existence of the formalized procedure of verification adequacy of the obtained estimates makes possible to use this method for appreciation the correctness of the algorithm embedded in the automated system of generation of expert assessments. The method also provides for the procedure of re-approval already obtained estimates.

Intercomparison of methods

To choose the most appropriate methods the comparison table is made.
Method | Advantages | Disadvantages
---|---|---
The method of simple ranging | Simplicity of procedure of receiving estimates on condition of small amount of the compared objects. The small number of experts is required. Possibility of definition of degree of coherence of opinions of experts. | Distribution of estimates along with reduction of degree of the importance of elements obviously is considered uniform. Impossibility of preservation of objectivity of an assessment at a large number of elements of comparison. |
Method of consecutive comparisons | In the course of an assessment the opinion of the expert isn't limited to purpose of certain coefficients. There is a formation of the unique coefficients defining degree of the importance of each object. | Complexity of perception the expert of procedure of an assessment of the compared elements. At a large number of the compared elements a large number of experts is required. |
The method of analysis of hierarchies by T. Saaty | Allows to make strict, statistically reasonable analysis of coherence of opinions of experts. Allows to develop the mechanism of carrying out questioning allowing to estimate a large number of the compared elements. Demands a small amount of experts. | Complexity of procedure of calculation of weight coefficients of the compared objects. |

Table 3: Intercomparison of methods

2.3. Estimations collection

The organization of collecting expert data can be organized in three stages. At the first stage the list of criteria is offered to each expert group for studying, and also the essence of use of the chosen mathematical method of determination of their weight coefficients is explained. At the second stage open discussion of the criteria offered for acquaintance by results of which carrying out their adjustment according to remarks of experts is possible is carried out. At the third stage there was an individual questioning of each expert. In a method of the analysis of hierarchies estimates of extent of domination of one element of comparison, over another therefore are formed square a matrix which filling is result of questioning act as expert data.

2.4. Definition of criteria weights

In accordance with the selected comparison method weights of criteria are calculated based on collected estimations. With proper calculation the sum of all weights should be equal to 1.
2.5. Analysis of expert evaluations

On the basis of calculations relative importance of all criteria which knowledge allows to carry out an assessment of objects is defined.

3 Conclusions

One of difficulties of expert estimation is the way of experts opinion formalisation. Strict rules of estimation process and using of mathematic methods allow to mitigate risks of human factor [8]. Useful comparison method is the necessary condition of correct decision. The review concludes three methods witch can be used to get a quantitative estimation of quality characteristics of objects. Researches differ in terms, goals, particular characteristics of subject area. That is why is necessary to have considerable experience and to know application features of different approaches to processing of experts estimation.

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References