Using Information-Theoretic Principles to Analyze and Evaluate Complex Adaptive Supply Network Architectures

Joshua Rodewald*, John Colombi, Kyle Oyama, Alan Johnson

*Air Force Institute of Technology, 2950 Hobson Way, WPAFB, OH 45433, USA

Abstract

Information-theoretic principles can be applied to the study of complex adaptive supply networks (CASN). Previous modeling efforts of CASN were impeded by the complex, dynamic nature of the systems. However, information theory provides a model-free approach to the problem removing many of those barriers. Understanding how principles such as transfer entropy, excess entropy/predictive information, information storage, and separable information apply in the context of supply networks opens up new ways of studying these complex systems. Additionally, these principles provide the potential for new business analytics which give managers of CASN new insights into the system’s health, behavior, and eventual control strategies.

1. Introduction

Supply networks exist throughout our society in manufacturing and knowledge-intensive industries as well as many service industries. Examples of these industries include product development, real-estate, healthcare, news/media, and investment services. Thoroughly understanding supply network behavior is critical to managing such systems effectively. Unfortunately, supply networks often take on the behavior of complex adaptive systems making them more difficult to analyze and assess. Being able to fully grasp how material and information flow through a complex adaptive supply network is key to being able to make more informed management decisions and prioritize resources and production throughout the network.

* Corresponding author. Tel.: +1-937-255-3355 x3347; fax: +1-937-255-4981.
E-mail addresses: joshua.rodewald@afit.edu, john.colombi@afit.edu, kyle.oyama@afit.edu, alan.johnson@afit.edu

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Insights from information theory and information dynamics (heavily leveraged in neuroscience) provide a novel way to overcome many of the difficulties associated with modeling complex adaptive supply networks. The notion of transfer entropy is especially useful in mapping the production network merely by observing the nodal output.

2. Background

2.1. Complex Adaptive Supply Networks

A complex adaptive system (CAS) is a network of dynamical elements where the states of both the nodes and the edges can change, and the topology of the network itself often evolves in time in a nonlinear and heterogeneous fashion. A supply network is a network of firms that exist upstream to any one firm in the whole value system where the value stream could represent material or knowledge flow and behaves as a CAS. A complex adaptive supply network (CASN) is a collection of firms that seek to maximize their individual profit and livelihood by exchanging information, products, and services with one another. Supply-chain networks form CASs because they display the following: structures spanning several scales, strongly coupled degrees of freedom and correlations over long length and timescales, coexistence of competition and cooperation, nonlinear dynamics involving interrelated spatial and temporal effects, quasi-equilibrium and combination of regularity and randomness (i.e. interplay of chaos and non-chaos), emergent behavior and self-organization, and adaption and evolution.

Previous researchers have posited that successful modeling efforts of large-scale CASN would require a solid empirical base and that purely abstract mathematical contemplation would be unlikely to lead to useful models. It is also unlikely that a single model can capture all the aspects of supply-chain processes, and therefore the modeling process should occur at multiple levels. Several options exist for modeling CASN: system dynamics, agent-based modeling, deterministic models, and network models. A drawback to system dynamics models is that the structure has to be determined before starting the simulation. Agent-based modeling, from complexity theory, is a bottom-up approach which simulates the underlying processes believed to be responsible for the global pattern. A network view has proven useful in modeling a supply-chain for patterns of interaction. Queuing theory can be used to analyze the steady state operation of a network, and mathematical programming can solve problems of resource allocation.

Li et al. proposed a model for CASN evolution using the principles of CAS and fitness landscape theory. They modeled the evolution of the CASN by modeling the environment, the firm, and the supply network evolution. For simulating the CASN evolution they used a multi-agent architecture, interaction of agents and two different design experiments: the first for structure dynamics of CASNs and the second for dynamic evolution of firm’s fitness. Their work resulted in some interesting managerial implications. Their primary take-away for managers of CASNs was that evolution is a self-organizing process and that any planning and regulation of the market and firm may be undermined by the fact that the outcomes are both open and unknowable.

This result was similar to the implications of applying CAS research to the strategic management of organizations, namely that a manager should not attempt to make sweeping enterprise-wide changes because the system’s nonlinear response is too difficult to predict and control. Instead, the managers should set boundaries or constraints on the system and observe the outcome. Then they would be able to tune the system by modifying the constraints and/or changing the amount of energy allowed into the system. The primary role of a strategic organizational architect is to influence the extent of improvisation, the nature of collaboration, the characteristic rhythm of innovation, and the number and nature of experimental probes by changing structure and demography. One factor complicating the use of CAS models in strategic management is that no theory exists to help managers predict how their actions may cascade through the CAS and affect emergence.

There are at least three major challenges critical to CASN research. First, the complexity of supply networks pushes the limits of researchers’ ability to understand the internal interactions between constructs and mechanisms or larger-scope phenomena. Second, operations management and supply-chain management lack metrics for evolution and dynamism in supply networks. Third, developing robust theories in the presence of adaptation is a formidable task. Supply-chain management theory can be built by identifying CAS phenomena and that future CASN theories show be built by viewing the properties associated with entities, topology, system, and environment as interrelated constructs. Other research issues include system scale and unit of analysis, environmental scope, and leveraging models, measurements, and methodologies for validation.
2.2. Information Dynamics

To address the issue of how information moves between nodes of the supply network, we appeal to the concepts of information dynamics. Information dynamics arises out of concepts in information theory such as conditional entropy, mutual information (Eq. 1), and conditional mutual information (Eq. 2) between processes X, Y, and Z.

\[
I(X; Y) = \sum_{x \in X} \sum_{y \in Y} p(x, y) \log_2 \frac{p(x|y)}{p(x)}
\]

\[
I(X; Y|Z) = \sum_{x \in X} \sum_{y \in Y} \sum_{z \in Z} p(x, y|z) \log_2 \frac{p(x|y, z)}{p(x|z)}
\]

Transfer entropy is perhaps the most applied of the information dynamics and has been used extensively in neuroscience research. Lizier describes transfer entropy (Eq. 3) as the amount of information that a source process provides about a destination (or target) process’ next state in the context of the destination’s past.

\[
T_{Y \rightarrow X}(k, l) = I(Y^{(k)}; X_{n+1}^{(l)} | X_{n}^{(k)})
\]

Excess entropy is another information dynamics measure that is useful in studying CASNs. Excess entropy (Eq. 4) is the mutual information between the semi-infinite past and semi-infinite future of a single process, X. It captures the information in the process’ past that is useful to predicting the process’ future. For this reason, excess entropy is also referred to as predictive information or stored information.

\[
E_X = \lim_{k \rightarrow \infty} I(X_{n}^{(k)}; X_{n+1}^{(k+1)})
\]

Active information storage (Eq. 5) follows from excess entropy except that only the next state of X is considered, rather than the semi-infinite future of X. Active information storage measures how much of the information from the past of process X is actively in use computing the next state of X.

\[
A_X = \lim_{k \rightarrow \infty} I(X_{n}^{(k)}; X_{n+1})
\]

Separable information is one attempt to measure the information modification within a system. This issue remains an open problem, but Lizier has implemented one attempt at capturing information modification (Eq. 6). This separable information is the limit of the linear combination of active information storage and all transfer entropies into source X. It aims to quantify how sources within a process combine to predict the next state of X.

\[
S_X = \lim \left[ I(X_{n}^{(k)}; X_{n+1}) + \sum_{y \in V \setminus X} T_{Y \rightarrow X}(k, l_y) \right]
\]

When considering these information dynamics measures within the context of CASNs, they can provide useful insights into the system behavior, especially for knowledge-intensive CASNs. Transfer entropy is useful for determining dependencies within the network, and by altering the process history lengths, can be used to uncover dependencies which change over time. Excess entropy (or stored information) is information which although not being used for the current process, will be used to determine future process states. This provides insight to managers of CASN which may have previously only considered products “in-hand” rather than those in progress. The active information storage takes on a more immediate role to the managers who want to know what is currently in progress within the CASN. Finally, although information modification is the least understood of the information dynamics, it measures how sources combine to predict future states. This may be useful to managers who want to understand what-if scenarios should a source be delayed or removed.

Transfer entropy methods have been applied in several fields to study complex systems: neuroscience, social networks, medicine, finance, climate, transportation, and others.
3. Methods

To begin applying information-theoretic principles to CASN, the authors devised a simple conceptual supply network made up of two primary supply chains. The network consists of 6 nodes (2 source nodes, 3 intermediary nodes, and 1 final production node), see Fig. 1. In this network, the output of one node feeds into another node which produces its own output based on that input.

The nodes were also given varying production schema as shown in Table 1. The source nodes produced at random time intervals with either high or low frequency production, while the various intermediary nodes had production schema dependent on the preceding node in the network. Nodes were either given random or fixed offset times from the preceding node; fixed offsets represented deterministic production turnaround times while random offsets represented varying production times. The final production node essentially combined the production of the two source streams by employing a fixed production time from the maximum of the two preceding nodes.

Using these simulated production times, the transfer entropy was calculated for every pair of nodes in the network using the JIDT code published by Lizier. The resulting output was then filtered to remove network links of lesser significance. The authors retained edges between nodes with a transfer entropy of 0.03 bits or greater. Again, this value was empirically determined, but the transfer entropy between the random source nodes was a good starting point (since they should be uncorrelated). The resulting network is shown in Fig. 2.

Additionally, the authors investigated the filtering threshold used to generate the network graph and how the threshold changed as the amount of time history data changed. To do this, the authors calculated the transfer entropy on the network for every pair of nodes with the first 10 data points and incrementally added the next 10 data points, observing how the transfer entropy values changed with each addition. The results are shown in Fig. 3. Note that the solid lines are transfer entropy on edges which were in the conceptual supply network while the dashed lines are transfer entropy on edges not in the conceptual network.

<table>
<thead>
<tr>
<th>Node</th>
<th>Node role</th>
<th>Production schema</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Source</td>
<td>Random, high-frequency times  (\text{rand}(10000,1)&gt;0.3)</td>
</tr>
<tr>
<td>B</td>
<td>Source</td>
<td>Random, low-frequency times  (\text{rand}(10000,1)&gt;0.7)</td>
</tr>
<tr>
<td>C</td>
<td>Intermediary</td>
<td>A (every other) + 1</td>
</tr>
<tr>
<td>D</td>
<td>Intermediary</td>
<td>B + \text{randbetween}(1,4)</td>
</tr>
<tr>
<td>E</td>
<td>Intermediary</td>
<td>C + \text{randbetween}(1,2)</td>
</tr>
<tr>
<td>F</td>
<td>Final production</td>
<td>D(\oplus)E + 1</td>
</tr>
</tbody>
</table>
4. Analysis

For this simple supply network, transfer entropy was able to uncover the network structure using only the node production histories. The results were dependent on the filter chosen in the calculations. Once the data leveled off (around 1,000 data points), the filter threshold value did not vary significantly based on the length of time history used to calculate the transfer entropies. However, with fewer data points (especially <200), the TE results would not be able to accurately return the underlying network.

As seen in Fig. 3, there was essentially no distinction between the edges in the conceptual supply network (solid lines) and those not in the conceptual supply network (dashed lines) when the number of data points used was less than 200 and only became clearer when more than 200 data points were used. As more data points were used, the less-significant edges (those not in the original concept network) remained lower than the actual edges in the network. This behavior is expected as the nodes should not be correlated. This could complicate efforts to filter the edges in the network when the network is not known and relatively short time histories of data are available.
An additional way to visualize the threshold is to discretely integrate the data plotted in Fig. 3. Although not shown here, this plot produces a much more apparent “natural” break in the transfer entropy values of the edges and allows visualization of the threshold to lower number of data points.

This analysis leads the authors to propose three heuristics for choosing filter thresholds on transfer entropy data:

- **Heuristic #1**: From two known randomly correlated nodes, calculate the transfer entropy. Filter any edges with transfer entropy less than or equal to this value.
- **Heuristic #2**: From the graph of all transfer entropies, remove the lowest valued edge until a graph of desired (or expected) connectivity is reached.
- **Heuristic #3**: From plots of transfer entropy vs. time history length (or its discreet integral), look for natural breaks in the data and filter any edges below this threshold.

5. Conclusions

CASNs have challenged researchers’ ability to study them because of their dynamic nature and complex behaviors. Information theory and information dynamics provides a way to overcome many of the difficulties associated with modeling CASNs. The notion of transfer entropy is especially useful in mapping the production network merely by observing the nodal output.

A simple conceptual supply network was created using simulated production timing for 6 nodes. Transfer entropy calculations completely revealed the network structure based only on input of the production times. Filtering thresholds were studied for varying lengths of time history and 3 heuristics were proposed to set these thresholds.

Future work will study larger, more complex networks which change over time. Simulations will help determine methods and techniques for analyzing dynamic network structures and revealing links which change with time. Additionally, organizational data will demonstrate these techniques on actual production systems.

References