Engineering Science and Technology, an International Journal 19 (2016) 1826–1835

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Engineering Science and Technology, an International Journal

journal homepage: www.elsevier.com/locate/jestch

Full Length Article

Estimation of solar radiation and optimum tilt angles for south-facing surfaces in Humid Subtropical Climatic Region of India

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ARTICLE INFO

Article history:
Received 12 July 2016
Revised 6 October 2016
Accepted 6 October 2016
Available online 18 October 2016

Keywords:
Solar radiation
Estimation
South-facing surface
Modeling
Optimum tilt angle
India

ABSTRACT

In the present work, availability of solar radiation for south-facing flat surfaces in Humid Subtropical Climatic Region of India has been estimated. Measurements of global and diffuse solar radiation for Aligarh (27.89°N, 78.08°E) which lies in the Humid Subtropical Climatic Region were performed for over a period of three years. Monthly, seasonal and annual optimum tilt angles were estimated. Comparison of total incident solar radiation was performed between Aligarh and New Delhi (Capital of India, 28.61°N, 77.20°E). Annual optimum tilt angle for Aligarh and New Delhi was found as 27.62° and 27.95° respectively (close to the latitude of the respective location). Estimated gains in annual average solar radiation (based on monthly, seasonal and annual optimum tilt angles) in comparison to a horizontal surface were 12.92%, 11.61% and 6.51% (for Aligarh) and 13.13%, 11.80% and 7.58% (for New Delhi) respectively. A loss of 1.16% and 5.68% energy (for Aligarh) and 1.18% and 4.91% (for New Delhi) were estimated with surface at seasonal and annual optimum tilt angles respectively compared to a surface at monthly optimum tilt angle. Based on the study, it was recommended that the inclined surface must be tilted on monthly or seasonal optimum tilt angle for better utilization of solar energy.

1. Introduction

Increasing demand of energy in domestic and industrial processes has burdened the energy resources to be consumed readily releasing huge amounts of pollutants affecting the environment. Solar energy is deemed to be the perfect solution to the world's dependence on conventional fuels. Solar radiations on the surface of the earth at absolute no cost makes it to be the perfect contender to energy crisis.

In India, during the summer season over 90% of the country receives a significant amount of solar radiations of the order of 3.0–6.5 kWh/m^2-day (10.8–23.4 MJ/m^2-day) (as shown in Fig. 1). However, in northern parts of the country this value can reach a maximum value of 7.5 kWh/m^2-day (27 MJ/m^2-day) in the month of May during summers [2]. This solar radiation potential can be utilized in desalination, solar-thermal collectors, building heating and daylighting and Photovoltaic (PV) Cells etc. Researchers are therefore concerned to maximize the amount of useful energy that can be extracted through the incoming solar radiations. It is believed that proper installation of these devices can make a remarkable change in the observed performance. Predominantly climatology, latitude, orientation, tilt angle, azimuth angles and usage over a period of time in a specific geographical region affect the performance of the above mentioned devices [3].

Tilt of a surface (β) is one of the significant factors that considerably affect the availability of solar radiation on a flat surface. Optimization of performance of solar based devices requires option like solar tracking equipment which follows trajectories of Sun's motion to enhance incident radiation [4,5]. However, these options are not always economical. As estimated by Vermaak [6], the inclined tracking system requires 6.94 ha (550% more) and the 2-axis tracking system requires 4.81 ha (almost 350% more) in comparison to a plant equipped with static PV panels (which requires an area of 1.07 ha). Also, trackers needs periodic maintenance and calibration, requires input energy for their operation which is in the range of 5–10% of the energy produced [7]. Further, trackers are made up of sophisticated mechanical parts which add to capital cost and an increase in cost of absolute power produced from solar PV panels [8]. Other method readily suggested by researchers is to optimize the orientation of flat surfaces at optimum tilt inclination (β_{opt}) [3]. Vieira et al. [9] performed an experimental study which suggested that the sun tracking panel exhibited a low average gain in power generated relative to the fixed panel. In another study performed by Sinha & Chandel [8],

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Peer review under responsibility of Karabuk University.

http://dx.doi.org/10.1016/j.jestch.2016.10.004
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Nomenclature

$\bar{H}_B$	beam component of solar radiation on an inclined surface (MJ/m^2-day)
$\bar{H}_d$	diffuse solar radiation on a horizontal surface (MJ/m^2-day)
$\bar{H}_D$	diffuse component of solar radiation on an inclined surface (MJ/m^2-day)
$\bar{H}_g$	global solar radiation on a horizontal surface (MJ/m^2-day)
$\bar{H}_R$	reflected component of solar radiation on an inclined surface (MJ/m^2-day)
$\bar{H}_T$	total solar radiation on an inclined surface (MJ/m^2-day)
k_d	cloudiness index
k_t	sky-clearness index
n	day of the year
$\bar{R}_b$	beam conversion factor
$\bar{R}_d$	diffuse conversion factor = $(1+\cos(\beta))/2$
$\bar{R}_r$	reflected conversion factor = $\rho(1-\cos(\beta))/2$

Greek letters

β	tilt angle (degrees)
β_{opt}	optimum tilt angle (degrees)
γ	surface azimuth angle (degrees)
δ	angle of declination (degrees)
θ	angle of incidence (degrees)
θ_z	zenith angle (degrees)
ρ	ground reflectivity
ϕ	latitude (degrees)
ω	hour angle (degrees)
ω_s	sunset hour angle (degrees)

it was reported that the horizontal axis weekly adjustment and vertical axis continuous adjustment tracking systems produced less energy annually than the existing PV system at fixed tilt. Optimum tilt inclination can be adjusted daily, monthly, seasonally, bi-annually or annually for maximizing the performance of the device in use [10].

It is evident that for a region in northern hemisphere, the surface orientation should be set due south i.e. $\gamma = 0$, where γ is the surface azimuth angle. It has been readily reported that the relation of optimum tilt inclination i.e. β_{opt} is purely with latitude and azimuth [11,12]. In the scenario of unavailability of information on optimization of tilt, various rules of thumb are generally adopted [13]. According to these, maximum solar radiation is achieved by making changes in azimuth angles in range of 10–20° and equalizing the tilt angle to latitude for the region [3]. However such rules are not reliable as estimates can be far from actual values. Different authors have suggested an extensive range of optimum tilt angle correlations in literature.

Optimization of tilt angle has been performed for various location in European countries like Bakirci [14] and Ertekin [15] for Turkey; Stanciu et al. [16] for Romania; Hartner et al. [17] for Austria and Germany; Calabrò [18] for Italy; Mehleri et al.[19] for Greece. For Middle Eastern countries Kazem et al. [20] for Oman; Jafari & Javaran [21], Jafarkazemi et al. [22] and Moghadam et al. [23] for Iran; Jafarkazemi & Saadabadi [24] for Abu Dhabi, UAE; Tamimi & Sowayan [25] for Riyadh, Saudi Arabia; Benghanem [26] for Madinah, Saudi Arabia; Elminir et al. [27] for Egypt; Altarawneh et al. [28] and Shariah et al. [29] for Jordan; Kamal Skeiker [30] for Syria. For Asian Countries, by Khahro et al. [31] for Pakistan; Krishna et al. [32] for Bangkok; Handoyo et al. [33] for Indonesia; Li & Lam [34] for Hong Kong; Tang & Wu [35] for China; for Nigeria by Akachukwu Ben Eke [36]; and Siraki & Pillay [37] for Canada.

For India, Ahmad & Tiwari [10,12] estimated optimum tilt angles using Liu & Jordan model and suggested that yearly optimum tilt angle is approximately equal to the latitude. For winters, optimum tilt angle was proposed as $\phi + 19°$ and for summers as $\phi - 16°$. Shukla et al. [38] compared the measured solar radiation data with six different isotropic and anisotropic models to estimate solar radiation on inclined surfaces in Bhopal, India (23°17′ N, 77°36′ E). They concluded Badescu model gives the best estimation of solar radiation on inclined surfaces with least statistical errors. Pandey and Katiyar [39,40] performed comparative study of diffuse solar radiation on tilted surface using different models for Lucknow, India (26.75°N, 80.50°E) and compared with the measurements for the location. Based on the statistical analysis performed they concluded that Klucher's model gives best estimation. It is observed from the literature that Liu & Jordan isotropic model is the most widely used approach for estimation of solar radiation on inclined surfaces.

The present work aims at exploring the availability of solar radiation on south-facing inclined flat surfaces (non-concentrating) in humid subtropical climatic region of India. Horizontal solar radiation measurements were performed for the city of Aligarh. The estimation procedure is based on Liu & Jordan (isotropic) model. The optimum values for tilt angles and solar radiation availability for New Delhi (the capital of India) have also been reported and compared with the estimations made for the city of Aligarh. Monthly, seasonal and annual optimum tilt angles have been suggested for both the locations. Gain in the availability of solar radiation at monthly, seasonal and annual optimum tilt angle as compared to horizontal surface has been stated. Also losses in solar energy at seasonal and annual optimum tilt angles in comparison to a surface at monthly optimum tilt angle have been reported.

1.1. Location under study

Aligarh city (27.89°N, 78.08°E, and 178 m above msl) is located in Northern part of India in the state of Uttar Pradesh 140 km southwest of Capital of India, New Delhi (28.61°N, 77.20°E, 216 m above msl). This region comes under the humid-subtropical climatic region with dry-winters (classified as "*CWa*" according to Köppen-Geiger climate classification system) [41] and receives a good amount of solar radiation throughout the year. Although the potential of solar energy utilization in this region is high, but the precise information on solar radiation on horizontal and inclined surfaces is rarely available.

2. Methodology

2.1. Solar radiation data

For Aligarh, the solar radiation data (global and diffuse radiation) used in this study were observed at Heat Transfer and Solar Energy Laboratory, Department of Mechanical Engineering, Aligarh Muslim University, Aligarh. The data were recorded using high quality meteorological equipment (two Kipp & Zonen CMP-11 secondary standard pyranometers, shadow ring CMP121-B and a Logbox-SD datalogger) as shown in Fig. 2. The method of solar radiation measurement using pyranometers is quite popular for

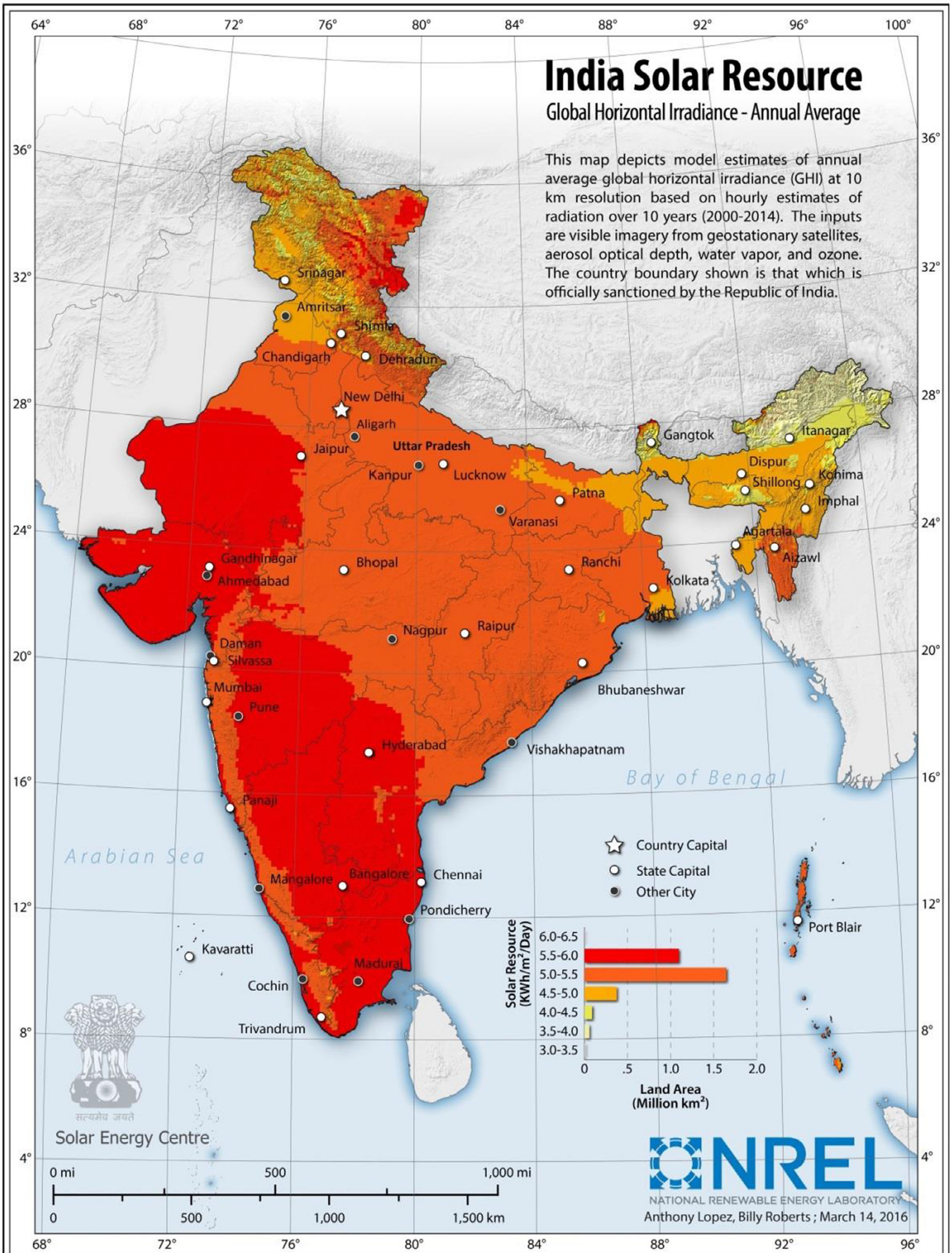

Fig. 1. Annual average global solar radiation in India (Source: [1]).

ground based solar radiation assessment. Further, measurement of diffuse solar radiation using the shadow-ring device over a pyranometer has advantages of simplicity of operation, direct estimation of diffuse component value and no requirement of tracking system, consequently reducing the cost of installation and operation [42].

Solar radiation measurements were performed and analyzed for over three years (year 2013 to present) with the aim of establishing

Fig. 2. Pyranometers for global and diffuse solar radiation measurements.

a solar radiation database for Aligarh city. Observed values from the datalogger were available as one-minute instantaneous solar radiation values (in W/m^2) on a horizontal surface from which the monthly average daily values of global and diffuse solar radiation (in MJ/m^2-day) were calculated. Beam radiation was found from the difference of global and diffuse solar radiation.

The monthly average solar radiation data obtained for three consecutive years of measurement for Aligarh city are shown in Fig. 3 where the subscripts 1, 2 and 3 represents the year of measurement respectively. It is seen that summer season in this region receives a huge amount of solar radiation with a maximum value of global solar radiation observed in the month of May (8.27 kWhr/m^2-day) for first year of measurement, April (8.18 kWhr/m^2-day) for second year of measurement and again in the month of April (7.63 kWhr/m^2-day) for third year of measurement. January receives the least amount of solar radiation during first and second year of measurement with values as 4.2 kWhr/m^2-day and 3.9 kWhr/m^2-day respectively. However, for the third year of measurement the month of July observes a minimum global solar radiation value of 2.73 kWhr/m^2-day. The maximum and minimum values of beam solar radiations observed are 5.87 kWhr/m^2-day (in April) and 2.27 kWhr/m^2-day (in January) for first year of measurement; 5.84 kWhr/m^2-day (in April) and 2.27 kWhr/m^2-day in November for second year of measurement; 5.38 kWhr/m^2-day (in April) and 1.22 kWhr/m^2-day in July for third year of measurement. However, diffuse solar radiation have maximum values of 3.65 kWhr/m^2-day in the month of July for first year of measurement, 2.54 kWhr/m^2-day in the month of June for second year of measurement, 2.44 kWhr/m^2-day in the month of August for third year of measurement. This is due to heavy rain in northern India during the monsoon season of June–August consequently adversely affecting the availability of beam radiation. Least values of diffuse solar radiation are 1.50 kWhr/m^2-day (in December), 1.61 kWhr/m^2-day (in January) and 1.34 kWhr/m^2-day in September for first, second and third year of measurement respectively.

The monthly average daily values of global, beam and diffuse solar radiation for Aligarh city are illustrated in Fig. 4(a). While, the long-term solar radiation data for New Delhi were available from various published resources in literature [10,43,44] and is represented in Fig. 4(b). Annual average global, beam and diffuse solar radiations calculated from the observed data were 22.12 MJ/m^2-day, 14.20 MJ/m^2-day and 7.92 MJ/m^2-day (or 6.14 kWhr/m^2-day, 3.94 kWhr/m^2-day, 2.20 kWhr/m^2-day) respectively for Aligarh against the available values of 18.72 MJ/m^2-day, 10.90 MJ/m^2-day and 7.82 MJ/m^2-day (or 5.20 kWhr/m^2-day, 3.03 kWhr/m^2-day, 2.17 kWhr/m^2-day) respectively for New Delhi. The measured solar radiation availability for Aligarh city is

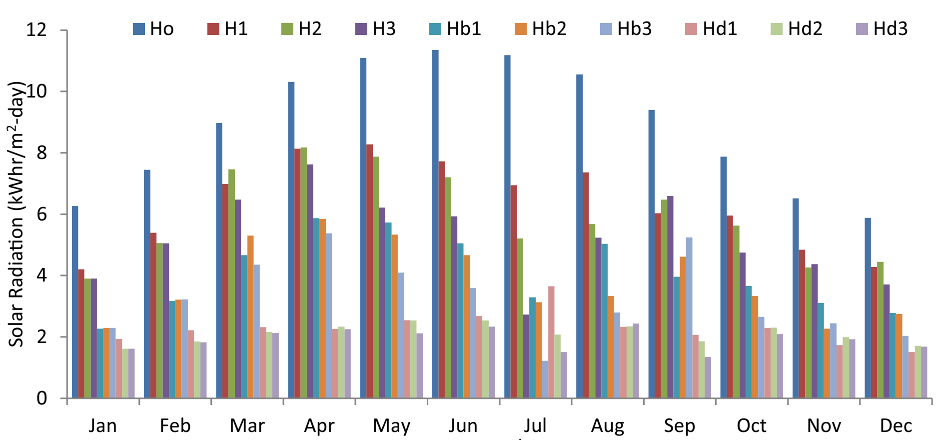

Fig. 3. Observed monthly average solar radiation data for three years of measurement at Aligarh.

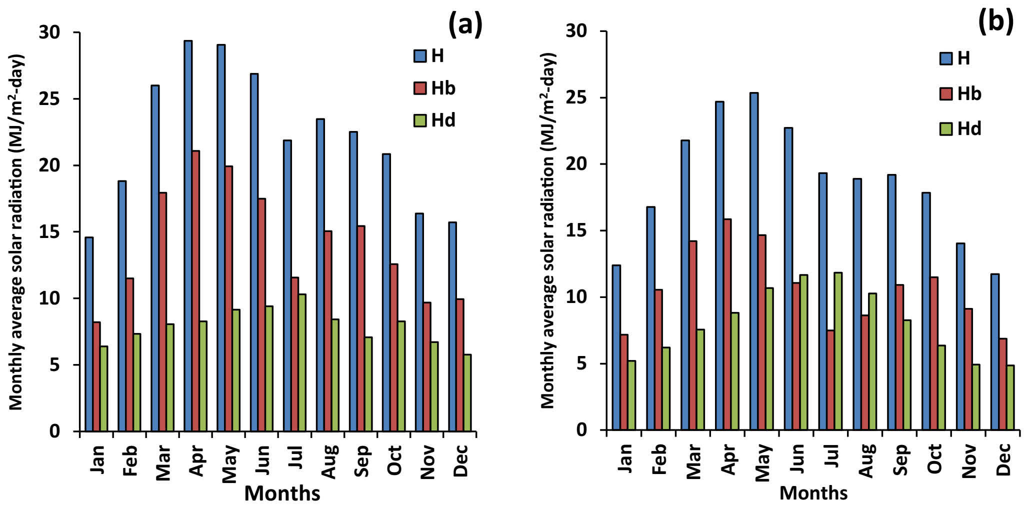

Fig. 4. Monthly average solar radiation data for (a) Aligarh and (b) New Delhi.

found to be better in comparison to its neighboring capital city New Delhi. Aligarh city has reasonably clear atmosphere with an annual average sky-clearness index (k_t) of 0.69 in comparison to metro-capital city of New Delhi which has a lower value of 0.60. This is due to the fact that pollution levels are severely higher in New Delhi.

2.2. Modelling

The parameters generally involved in the estimation of solar radiation on an inclined surface are declination (δ), latitude (ϕ), angle of incidence (θ), surface azimuth angle (γ), hour angle (ω), day of the year (n), global, beam, diffuse and reflected solar radiation, orientation towards equator, tilt angle (β), sky-clearness (k_t) and cloudiness index (k_d). The procedure followed under the present study is described below.

Declination angle (δ) ranges from a maximum value of +23.45° on June 21st–22nd to a minimum value −23.45° in December 20th–21st. Its value is zero on March 22nd and September 22nd of the year. According to Cooper [45], declination angle is calculated using the following relation:

$$\delta = 23.45 sin\left[360\frac{(284+n)}{365}\right] \tag{1}$$

where n is the day of the year.

The angle of incidence for a surface oriented in any direction can be mathematically expressed by following relation [13]:

$$\cos\theta = \sin\delta\sin\phi\cos\beta - \sin\delta\cos\phi\sin\beta\cos\gamma + \cos\delta\cos\phi\cos\beta\cos\omega + \cos\delta\sin\phi\sin\beta\cos\gamma\cos\omega + \cos\delta\sin\beta\sin\gamma\sin\omega \tag{2}$$

For a surface in northern hemisphere facing south (i.e. γ = 0°), Eq. (2) can be simplified as:

$$\cos\theta = \sin(\phi-\beta)\sin\delta + \cos(\phi-\beta)\cos\delta\cos\omega \tag{3}$$

For a horizontal surface (β = 0°), the angle of incidence (θ) becomes equal to zenith angle (θ_z). Substituting this value in Eq. (3), zenith angle can be written as:

$$\cos\theta_z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega \tag{4}$$

The total solar energy received on an inclined surface is the sum of beam and diffuse radiations directly incident on a surface and reflected radiations (reflected by the surroundings). Thus, the monthly average total solar radiation (in MJ/m^2-day) for an inclined surface is expressed as:

$$\bar{H}_T = \bar{H}_B + \bar{H}_D + \bar{H}_R \tag{5}$$

Mathematically, the optimum tilt angle (β_{opt}) is obtained by differentiating Eq. (5) with respect to the tilt angle (β) and equating to zero, i.e.

$$\left(\frac{d}{d\beta}(\bar{H}_T)\right) = 0 \tag{6}$$

Thus, the value of optimum tilt angle (β_{opt}) is estimated.

In Eq. (5), the beam radiations falling on a tilted surface is given as [46]:

$$\bar{H}_B = (\bar{H}_g - \bar{H}_d)\bar{R}_b \tag{7}$$

where $\bar{H}_g$ and $\bar{H}_d$ are the monthly average daily global and diffuse radiation on a horizontal surface.

Various researchers have proposed numerous models which are classified as isotropic: (Liu & Jordan [47], Tian [48], Koronakis [49] and Badescu [50]) and anisotropic (Hay [51], Reindl et al. [52], Hay and Davies, Klucher and Reindl models [53], Skartveit and Olseth [54], and Steven & Unsworth [55]) to estimate solar radiation on inclined surfaces. However, according to Tang &Wu [35], measured diffuse solar radiation data gives better estimates of optimum tilt angle. In this study, measured data of global and diffuse solar radiation has been utilized to calculate optimum tilt angles. Liu & Jordan (isotropic) model has been used to estimate total solar radiation on inclined flat surfaces facing south.

According to Liu & Jordan [47], the beam conversion factor ($\bar{R}_b$) is given as:

$$\bar{R}_b = \frac{\omega_s\sin\delta\sin(\phi-\beta) + \cos\delta\sin\omega_s\cos(\phi-\beta)}{\omega_s\sin\phi\sin\delta + \cos\phi\cos\delta\sin\omega_s} \tag{8}$$

where $\beta$is the tilt angle and $\omega_s$is sunset hour angle, given as:

$$\omega_s = \cos^{-1}(-\tan\phi\tan\delta) \tag{9}$$

Reflected radiation falling on tilted surface is given by:

$$\bar{H}_R = \bar{H}_g\bar{R}_r, \quad \text{where} \quad \bar{R}_r = \rho\left(\frac{1-\cos\beta}{2}\right) \tag{10}$$

where $\bar{R}_r$ is the reflected conversion factor, ρ is ground reflectivity assumed as 0.2 for hot and humid tropical location as suggested by Muneer & Saluja [56].

Table 1
A comparison of β_{opt} (deg) of the present study with those of [10] for New Delhi.

Months	β_{opt} (deg)		Seasons	β_{opt} (deg)		Annual	β_{opt} (deg)	
	Ahmad & Tiwari [10]	Present study		Ahmad & Tiwari [10]	Present study		Ahmad & Tiwari [10]	Present study
Jan	56	56		56	56	Annual	30	28
Feb	45	47		30	31			
Mar	32	32	Spring	30	31			
Apr	14	13		30	31			
May	0	0		0	0			
Jun	0	0	Summer	0	0			
Jul	0	0		0	0			
Aug	6	6		24	25			
Sep	26	25	Autumn	24	25			
Oct	40	43		24	25			
Nov	53	54	Winter	56	56			
Dec	58	59		56	56			

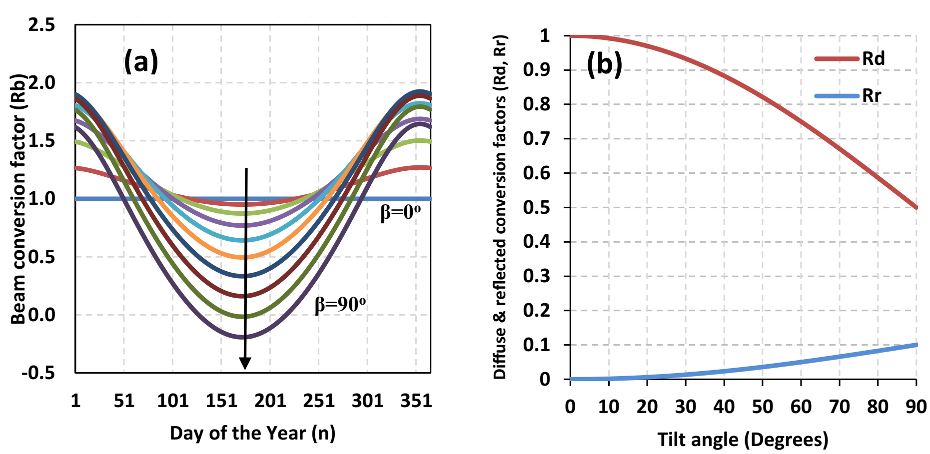

Fig. 5. (a) Beam conversion factor at different tilt angles for Aligarh (b) Diffuse and reflected conversion factors.

Diffuse radiation falling on a tilted surface is given by:

$$\bar{H}_D = \bar{H}_d \bar{R}_d = \bar{H}_d \left(\frac{1 + \cos\beta}{2} \right) \quad (11)$$

where $\bar{R}_d$ is the diffuse conversion factor representing the ratio of diffuse solar radiation on an inclined surface to diffuse solar radiation on a horizontal surface. The classification of models for diffuse solar radiation on an inclined surface is performed based on the method of estimation of $\bar{R}_d$. For the purpose of validation of procedure employed under the present study, the values of optimum tilt angles for the capital of India, New Delhi were estimated and compared with the work of Ahmad & Tiwari [10]. Table 1 compares monthly, seasonal and annual optimum tilt angles for New Delhi. The values are similar and thus the results are validated.

3. Results and discussion

Using the procedure described in the previous Section 2.2 based on the Liu & Jordan model, estimates have been made to obtain optimum tilt angles and corresponding total solar radiation on inclined surfaces.

Fig. 5(a) describes the variation of beam conversion factor with day of the year at various tilt angles for the city of Aligarh. Also, Fig. 5(b) shows the variation of diffuse and reflected conversion factors at various tilt angles.

For all the months and different tilt angles, monthly average total solar radiations were calculated using Eq. (5). The results have been plotted and are shown in Fig. 6 for Aligarh city where the tilt angle has been varied in the range of 0–90° (in steps of 10°). Fig. 6 (a) shows the total solar radiation versus tilt angle for the months

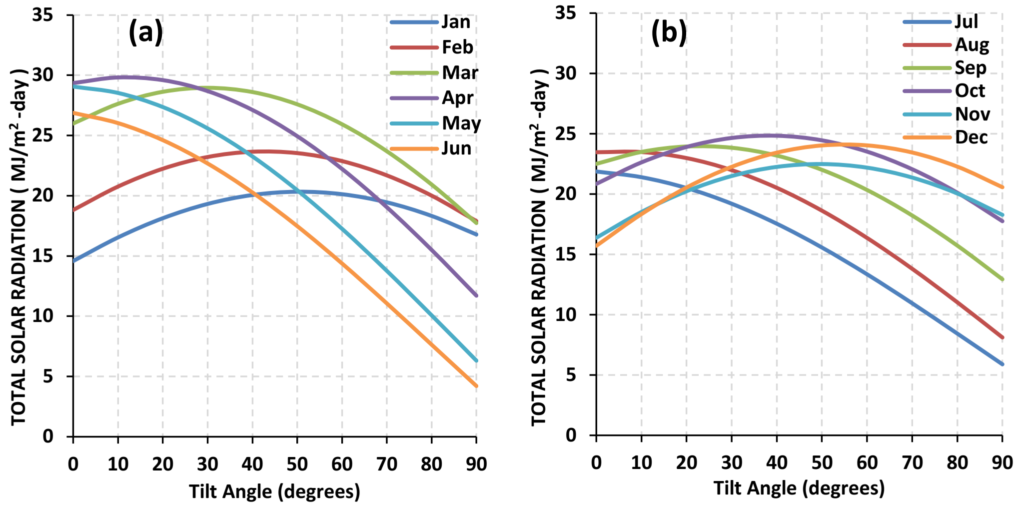

Fig. 6. Monthly average daily total solar radiation versus tilt for south-facing surface in Aligarh.

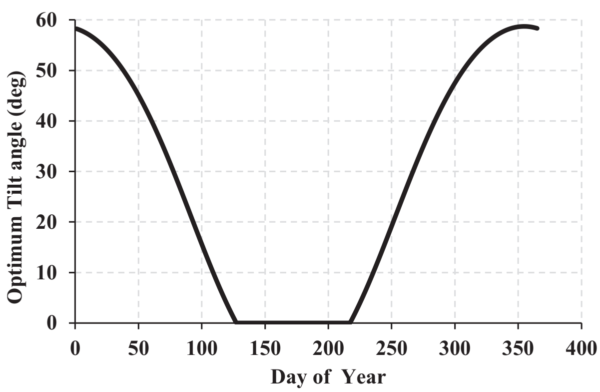

Fig. 7. Optimum tilt angle versus day of year for south facing surface in Aligarh.

Table 2
Monthly, seasonal and annual optimum tilt angles for south facing surfaces in Aligarh City.

Months	β_{opt} (deg)	Seasons	β_{opt} (deg)	Annual	β_{opt} (deg)
Jan	55.86		56.00	Annual	27.62
Feb	46.83		30.19		
Mar	31.45	Spring	30.19		
Apr	12.29		30.19		
May	0.00		0.00		
Jun	0.00	Summer	0.00		
Jul	0.00		0.00		
Aug	6.05		24.29		
Sep	24.62	Autumn	24.29		
Oct	42.21		24.29		
Nov	53.81	Winter	56.00		
Dec	58.33		56.00		

from January to June while Fig. 6(b) shows the same for the remaining months of July to December. It is apparent from the figure, that the solar radiation is an intensive function of tilt angle. The calculated solar energy radiation incident on the flat surface increased from horizontal position of 0° to a certain angle of inclination, after which, further increment in angle of tilt of the flat surface resulted to fall of the solar energy radiation received. The result also indicated that the optimum angle for catching maximum solar energy radiation on a flat surface varies with the months of the year. The maximum solar radiation is achieved for every month with unique optimum tilt angle. The optimum tilt angle increases in winter months and decreases to minimum value in summer & autumn months.

Fig. 7 shows the variation of daily optimum tilt angle (β_{opt}) for a south-facing surface in Aligarh. The optimum tilt angle varies from 58.24° on first day of the year (January 1st) to 0.17° on 131st day of the year (May 07). Between 132nd day to 224th day of the year, the calculated values of the optimum tilt angle were found to be negative. However, zero value is suggested for such days since negative tilt has no significance. From 225th day to the end of the year variations of tilt angle is between 0.41° and 58.32°. Maximum tilt angle was observed for 20th–22nd December as 58.71°.

The values of monthly, seasonal and annual optimum tilt angle obtained for Aligarh are presented in Table 2. These are also plotted against the day of the year in Fig. 8. In the figure, the average optimum tilt angle for different seasons was evaluated by averaging the value of the optimum tilt angle for each month falling within a particular season. Therefore, four seasonal optimum tilt angles were obtained (corresponding to each season) as 56.00° in winter, 30.19° in spring, 0° in summer, and 24.29° in autumn. Thus, winters have a higher value of optimum tilt angle while summer observes a lower tilt angle value. Annual optimum tilt angle was calculated by averaging the value of optimum tilt angles for all months of a year and was found to be 27.62° for Aligarh. This value of tilt angle is sufficiently close to the latitude of the location considered. Surface at this tilt value will remain fixed throughout the year.

A comparison of total incident solar radiation for the city of Aligarh and New Delhi is presented in Table 3. The measured values of solar radiation at horizontal surface ($\beta = 0^{\circ}$) are presented along with the values of total solar radiation ($\bar{H}_T$) at monthly, seasonal

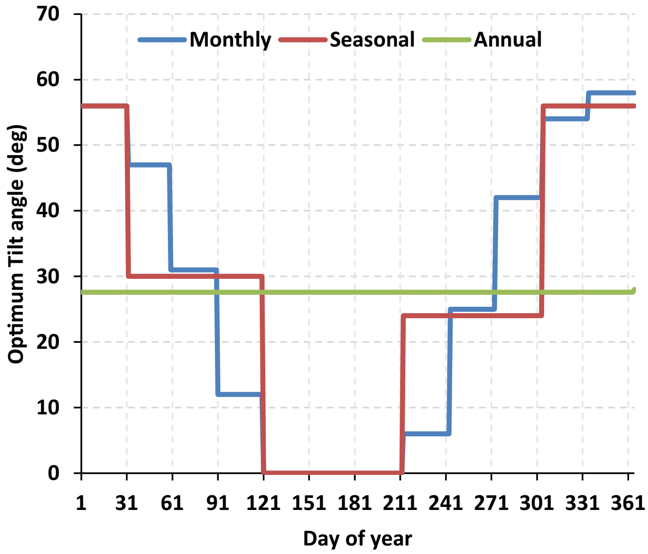

Fig. 8. Variations of monthly, seasonal & annual optimum tilt angles for south-facing surface in Aligarh.

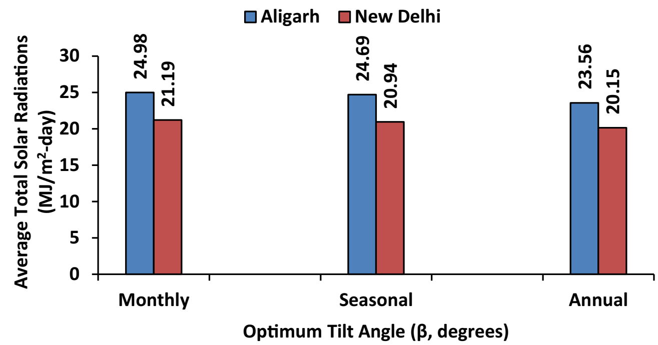

Fig. 9. Comparison of average maximum total solar radiation for Aligarh and New Delhi.

and annual optimum tilt angles. The monthly value of total solar radiation was maximum during the period of March to May with the peak value estimated as 29.92 MJ/m^2-day for Aligarh (in April) and 25.35 MJ/m^2-day for New Delhi (in May). Total solar radiation calculated at seasonal optimum tilt angle was found to be maximum in summer for both the locations and the estimated values were 29.06 MJ/m^2-day for Aligarh and 25.34 MJ/m^2-day for New Delhi. The minimum values observed in winter season were 20.27 MJ/m^2-day for Aligarh and 17.53 MJ/m^2-day for New Delhi. The maximum and minimum values of incident solar radiation on annual optimum tilt angle are 28.97 MJ/m^2-day (in April) and 19.08 MJ/m^2-day (in January) respectively for Aligarh. The maximum values of incident solar radiation for New Delhi was also estimated in April as 24.31 MJ/m^2-day, while the minimum value was estimated in the month of December as 16.10 MJ/m^2-day.

Percentage gain in the availability of solar radiation on an inclined surface is defined as:

$$Percent\ Gain\ (\%) = \left(\frac{\bar{H}_T|_{\beta=\beta_{opt_i}}}{\bar{H}_T|_{\beta=0}} - 1 \right) \times 100 \tag{12}$$

where $i = monthly,\ seasonal\ and\ annual$

The percent gain in annual average total solar radiation on optimally inclined surface in comparison to a horizontal surface is 12.92% (monthly optimum tilt angle), 11.61% (seasonal optimum tilt angle) and 6.51% (annual optimum tilt angle) for Aligarh. For New Delhi, the gains are estimated as 13.13% (monthly optimum tilt angle), 11.80% (seasonal optimum tilt angle) and 7.58% (annual optimum tilt angle).

Fig. 9 compares the average annual solar radiation on a tilted surface fixed at monthly, seasonal and annual optimum tilt angles for Aligarh and New Delhi. As observed from the figure, the annual average total solar radiation estimated at monthly optimum tilt angle is found to be the highest, followed by average annual solar radiation estimated at seasonal and annual optimum tilt angle values.

The difference between the total solar radiation at monthly, seasonal and annual optimum tilt angles for both the locations is small. However, there is a loss in available solar radiation as surface is fixed on seasonal or annual optimum tilt angle in comparison to solar radiation at monthly optimum tilt angle. Mathematically:

$$Percent\ Loss\ (\%) = \left(1 - \frac{\bar{H}_T|_{\beta=\beta_{opt_j}}}{\bar{H}_T|_{\beta_{opt}(monthly)}} \right) \times 100 \tag{13}$$

where $j = seasonal,\ annual$

A loss of 1.16% and 5.68% energy is estimated with surface at seasonal and annual optimum tilt angle in comparison to a surface at monthly optimum tilt angle for Aligarh. Similarly, a loss of 1.18% and 4.91% energy is estimated with surface at seasonal and annual

Table 3
Total solar radiation (in MJ/m^2-day) for south facing surfaces at monthly, seasonal and annual optimum tilt angles.

Months	Aligarh				New Delhi			
	$(\bar{H}_T)_{\beta=0}$	$(\bar{H}_T)_{\beta opt=monthly}$	$(\bar{H}_T)_{\beta opt=seasonal}$	$(\bar{H}_T)_{\beta opt=annual}$	$(\bar{H}_T)_{\beta=0}$	$(\bar{H}_T)_{\beta opt=monthly}$	$(\bar{H}_T)_{\beta opt=seasonal}$	$(\bar{H}_T)_{\beta opt=annual}$
Jan	14.59	20.28	20.27	19.08	12.38	17.56	17.56	16.45
Feb	18.82	23.66	23.22	23.03	16.78	21.41	21.00	20.78
Mar	26.00	29.04	28.96	28.94	21.78	24.20	24.14	24.14
Apr	29.35	29.92	28.69	28.97	24.70	25.13	24.02	24.31
May	29.06	29.06	29.06	26.07	25.34	25.35	25.34	23.27
Jun	26.88	26.88	26.88	23.19	22.72	22.72	22.72	20.51
Jul	21.87	21.87	21.87	19.55	19.33	19.33	19.33	18.04
Aug	23.47	23.60	22.65	22.27	18.90	18.97	18.19	17.96
Sep	22.51	24.05	23.97	23.92	19.19	20.20	20.14	20.08
Oct	20.85	24.85	24.27	24.52	17.86	21.75	21.17	21.37
Nov	16.38	22.45	22.37	21.24	14.04	20.15	20.10	18.82
Dec	15.71	24.08	24.11	21.89	11.74	17.50	17.53	16.10
Average	22.12	24.98	24.69	23.56	18.73	21.19	20.94	20.15
% Gain	–	12.92	11.61	6.51	–	13.13	11.80	7.58

optimum tilt angle in comparison to a surface at monthly optimum tilt angle for New Delhi.

The difference in annual average solar radiation potential for Aligarh and New Delhi is 3.79 MJ/m^2-day (based on monthly optimum tilt angle), 3.75 MJ/m^2-day (based on seasonal optimum tilt angle) and 3.41 MJ/m^2-day (based on annual optimum tilt angle). It is therefore found that the city of Aligarh shows a considerable greater potential of solar radiation availability than New Delhi. This is a consequence of better availability of horizontal solar radiation throughout the year in Aligarh city which is further magnified when projected on optimally inclined flat surfaces.

It is therefore recommended that wherever adjustment of tilt angle is possible, the facade of the solar based devices must be shifted according to the monthly or seasonal tilt angles to gain the maximum available solar energy. However, for a static inclination of south-facing surfaces, tilt angle need to be fixed at annual optimum value which is sufficiently close to the latitude of the location.

4. Conclusion

Energy falling on a surface can significantly be improved by making adjustments in the surface inclination. Different months have different optimum tilt angle value due to the change in the Sun's position. In this study, an estimation of monthly, seasonal and annual optimum tilt angles for Aligarh (which lies in the Humid Subtropical Climatic Region of India) were performed and compared with the corresponding values of New Delhi (Capital of India). The following conclusions are drawn:

1. Winters have higher value of optimum tilt angle than summers. For Aligarh, seasonal optimum tilt angles were 56.00° (winter), 30.19° (spring), 0° (summer), and 24.29° (autumn). For New Delhi, 56.42° (winter), 30.64° (spring), 0° (summer), and 24.73° (autumn) values were estimated.
2. Annual optimum tilt angle (β_{opt}) for Aligarh is 27.62° and New Delhi is 27.95°. These values are close to the latitude of the locations.
3. Maximum and minimum values of total solar radiation at annual optimum tilt angles were 28.97 MJ/m^2-day (April) and 19.08 MJ/m^2-day (January) respectively for Aligarh and 24.31 MJ/m^2-day (April) and 16.10 MJ/m^2-day (December) for New Delhi.
4. Gains in annual average total solar radiation on optimally inclined surface in comparison to a horizontal surface were 12.92% (monthly optimum tilt angle), 11.61% (seasonal optimum tilt angle) and 6.51% (annual optimum tilt angle) for Aligarh. For New Delhi, the gains were estimated as 13.13% (monthly optimum tilt angle), 11.80% (seasonal optimum tilt angle) and 7.58% (annual optimum tilt angle).
5. A loss of 1.16% and 5.68% solar energy for Aligarh and 1.18% and 4.91% for New Delhi were estimated with surface at seasonal and annual optimum tilt angle in comparison to a surface at monthly optimum tilt angle.
6. The difference in annual average solar radiation potential for Aligarh and New Delhi was 3.79 MJ/m^2-day (based on monthly optimum tilt angle), 3.75 MJ/m^2-day (based on seasonal optimum tilt angle) and 3.41 MJ/m^2-day (based on annual optimum tilt angle).

It is recommended that where possible the surface must be inclined on monthly or seasonal optimum tilt angle for better solar radiation utilization. Annual optimum tilt angle can be used for surfaces with fixed inclination where varying the tilt angle is not possible. This also helps reduce installation and operation cost.

Acknowledgments

The authors would like to acknowledge the financial support in procurement of meteorological equipment provided by the Mechanical Engineering Department, Aligarh Muslim University, Aligarh, India under XII Five Year Development Program-Equipment grant. The authors are also thankful to the comments provided by the reviewers for improvement on the content of the manuscript.

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