



On stability of electroweak vacuum during inflation

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ARTICLE INFO

Article history:

Received 20 March 2015

Accepted 7 May 2015

Available online 11 May 2015

Editor: A. Ringwald

ABSTRACT

We study Coleman–De Luccia tunneling of the Standard Model Higgs field during inflation in the case when the electroweak vacuum is metastable. We verify that the tunneling rate is exponentially suppressed. The main contribution to the suppression is the same as in flat space–time. We analytically estimate the corrections due to the expansion of the universe and an effective mass term in the Higgs potential that can be present at inflation.

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1. Introduction

At tree level the Standard Model (SM) Higgs potential has an absolute minimum corresponding to the electroweak (EW) vacuum. The loop corrections change the picture drastically. They modify the effective potential for the Higgs field through the renormalization group (RG) running of the Higgs quartic coupling λ [1,2]. The precise evolution of λ strongly depends on the values of the Higgs and top-quark masses. It is still possible, within uncertainties of the top mass, that λ stays positive all the way up to the Planck scale [3]. However, for the current best-fit values of the SM parameters, λ changes sign at large RG scale $\mu_0 \sim 10^{10}$ GeV and reaches a negative minimum at $\mu_* \sim 10^{16}–10^{18}$ GeV, see Fig. 1. It is worth stressing that this RG evolution is obtained under the assumption of no new physics interfering with the running of λ . As a result, the effective Higgs potential¹

$$V_h = \frac{\lambda(h)h^4}{4} \quad (1)$$

goes much below the EW vacuum at large values of the field, as shown schematically in Fig. 2. This makes the EW vacuum metastable.

While in a low density, low temperature environment characteristic of the present-day universe the SM vacuum is safely long-lived [2], the situation may be different during primordial inflation.

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¹ We neglect the SM mass term which is tiny compared to all contributions appearing below.

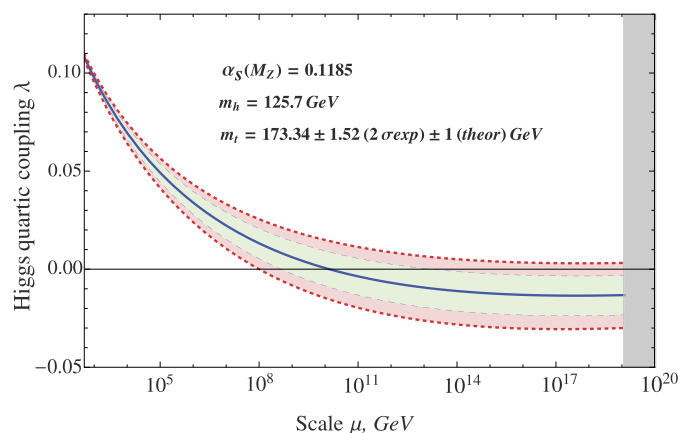


Fig. 1. Running of the Higgs quartic coupling in the Standard Model at NNLO in the \overline{MS} scheme. The RG equations are solved using the code available at [4] based on [5,1]. Blue solid line corresponds to the best-fit values of the Standard Model parameters [6]. Blue dashed lines correspond to 2σ experimental uncertainty in the measurement of the top-quark mass [7] and red dotted lines – to the theoretical uncertainty discussed in [3]. The plot is restricted to the scales smaller than the Planck mass $M_p = 1.22 \cdot 10^{19}$ GeV.

Indeed, most inflationary models predict the Hubble expansion rate during inflation H_{inf} to be much higher than the measured Higgs mass. Thus, if the Higgs does not have any other couplings besides those present in SM, it behaves at inflation as an essentially massless field and develops fluctuations of order H_{inf} . Denote by h_{max} the value of h corresponding to the top of the barrier separating the EW vacuum from the run-away region. Then, even if

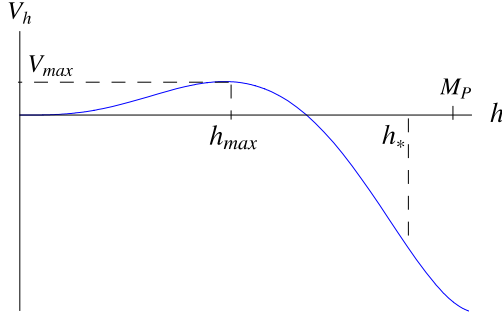


Fig. 2. Schematic form of the effective Higgs potential (not to scale).

h is originally placed close to the origin, it will roll beyond the barrier with order-one probability for $H_{inf} > h_{max}$ [8–13].

A simple cure to the problem is to endow the Higgs with an effective mass $m_{eff} \gtrsim H_{inf}$ during the inflationary stage. This can be due, for example, to a non-minimal coupling to gravity,² $V_{hR} = \xi R h^2/2$ [8,14], or a coupling between h and the inflaton field³ ϕ of the form $V_{h\phi} = f(\phi)h^2/2$ [9,11]. This raises the potential barrier and suppresses the over-barrier transitions. In this situation the EW vacuum is still able to decay via quantum tunneling.

Tunneling from a false vacuum in (quasi-) de Sitter space-time can proceed in two distinct regimes: via the Hawking–Moss (HM) instanton [16] which describes quantum jumps on top of the potential barrier, or via Coleman–De Luccia (CDL) bounce [17] corresponding to genuinely under-barrier penetration. While HM transitions have been extensively discussed in connection with the Higgs behavior during inflation (see e.g. [8,10,12,13]), the CDL tunneling is usually discarded with the common lore that it is sufficiently suppressed. However, to the best of our knowledge, a verification of this assertion is missing in the literature.⁴ Moreover, Ref. [10] which explicitly addressed this question has reported an opposite result that the CDL decay of the EW vacuum is enhanced, instead of being exponentially suppressed. If true, this would pose a serious challenge for the stability of the EW vacuum during inflation.

The purpose of this *letter* is to clarify the above issue. We will estimate the CDL tunneling rate and confirm that it is exponentially suppressed. The suppression exponent will be found to be essentially the same as in flat space-time, up to small corrections which we will estimate analytically.

2. Bounces in de Sitter space

In this section we assume that the energy density of the universe is dominated by the inflaton with negligible back-reaction of the Higgs field on the metric. The validity of this assumption will be discussed later. Then, neglecting the slow-roll corrections, we arrive to the problem of a false vacuum decay in external de Sitter space-time. This process is described by the Euclidean version of the Higgs action

$$S_E = \int d^4x \sqrt{g_E} \left(\frac{1}{2} g_E^{\mu\nu} \partial_\mu h \partial_\nu h + V_h(h) \right), \quad (2)$$

where $g_{E\mu\nu}$ is the metric of a 4-dimensional sphere, which is the analytic continuation of the de Sitter metric [17] (see also [18]),

² We work in the signature $(-, +, +, +)$, so that the curvature of de Sitter space is positive, $R = 12H_{inf}^2$.

³ We assume that the inflaton is distinct from the Higgs, unlike the case of Higgs inflation [15].

⁴ Note that the thin-wall approximation, which is often invoked in the analysis of the CDL tunneling and which makes the exponential suppression manifest, is not applicable in the case of the Higgs field.

$$ds_E^2 = d\chi^2 + \rho^2(\chi) d\Omega_3^2, \quad \rho = \frac{1}{H_{inf}} \sin(H_{inf} \chi),$$

$$0 \leq \chi \leq \frac{\pi}{H_{inf}}. \quad (3)$$

Here $d\Omega_3$ is the line element on a unit 3-sphere. We search for a smooth solution of the Higgs equations of motion following from (2). Assuming $O(4)$ symmetry, one reduces the action to

$$S_E = 2\pi^2 \int_0^{\pi/H_{inf}} d\chi \rho^3 \left(\frac{h'^2}{2} + V_h \right), \quad (4)$$

which yields the equation for the bounce $h_b(\chi)$,

$$h_b'' + 3H_{inf} \text{ctg}(H_{inf} \chi) h_b' = \frac{dV_h}{dh}. \quad (5a)$$

To be regular, the solution must obey the boundary conditions,

$$h_b'(0) = h_b'(\pi/H_{inf}) = 0. \quad (5b)$$

The probability of false vacuum decay per unit time per unit volume scales as

$$\frac{dP}{dt dV} \propto \exp(-S_E), \quad (6)$$

where the action is evaluated on the solution $h_b(\chi)$.

Hawking–Moss instanton. Eqs. (5) always have a constant solution with the Higgs field sitting on top of the potential barrier, $h_b = h_{max}$ (see Fig. 2). This instanton can be interpreted as describing the over-barrier jumps of the Higgs field due to non-zero de Sitter temperature, $T_{dS} = H_{inf}/(2\pi)$ [19]. The rate of such transitions is given by (6) with the action

$$S_E^{(HM)} = \frac{8\pi^2}{3} \frac{V_{max}}{H_{inf}^4}. \quad (7)$$

The transition rate is exponentially suppressed if $H_{inf} \lesssim V_{max}^{1/4}$. In the pure SM $V_{max}^{1/4}$ is of order 10^9 GeV [2] implying that the EW vacuum is stable with respect to HM transitions whenever $H_{inf} < 10^9$ GeV and unstable otherwise. In the latter case new contributions into the Higgs potential that raise V_{max} are required to stabilize the SM vacuum. A simple option is to endow h with an effective mass m_{eff} during inflation. The potential becomes

$$V_h = \frac{\lambda(h) h^4}{4} + \frac{m_{eff}^2 h^2}{2}. \quad (8)$$

For $H_{inf} \gtrsim 10^{10}$ GeV the qualitative picture is captured by neglecting the slow logarithmic dependence of the coupling on the field and normalizing it at a fixed scale above μ_0 , so that λ is negative and is of order 0.01 in the absolute value. This gives for the position and height of the potential barrier,

$$h_{max} = \frac{m_{eff}}{\sqrt{|\lambda|}}, \quad V_{max} = \frac{m_{eff}^4}{4|\lambda|} \quad (9)$$

leading to the instanton action,

$$S_E^{(HM)} = \frac{8\pi^2}{3|\lambda|} \left(\frac{m_{eff}}{H_{inf}} \right)^4. \quad (10)$$

As expected, the transitions are strongly suppressed provided the mass is bigger than $|\lambda|^{1/4} H_{inf}$. Note that for these values of the mass h_{max} lies above μ_0 , which justifies our approximation of constant negative λ . For the case when the Higgs mass is due to

non-minimal coupling to gravity one has $m_{\text{eff}}^2 = 12\xi H_{\text{inf}}^2$, so that the suppression (10) does not depend on the Hubble parameter and is large already for $\xi \gtrsim 0.1$ [8,14,20].

Coleman–De Luccia bounce. Another decay channel is described by inhomogeneous solutions of (5) which interpolate between the false vacuum and a value h_* in the run-away region. These correspond to genuinely under-barrier tunneling. To understand their properties, let us first neglect the running of λ normalizing it at a high enough scale, so that $\lambda < 0$. If we further neglect the mass and space–time curvature, we obtain the setup of tunneling from the top of an inverted quartic potential in flat space. This is described by a family of bounces,

$$h_{\bar{\chi}}(\chi) = \sqrt{\frac{8}{|\lambda|}} \frac{\bar{\chi}}{\chi^2 + \bar{\chi}^2}, \quad (11)$$

parameterized by their size $\bar{\chi}$. The action of these solutions is independent of $\bar{\chi}$ due to the classical scale invariance of the setup,

$$S_E = \frac{8\pi^2}{3|\lambda|}. \quad (12)$$

The mass and finite Hubble rate break the degeneracy. Assuming that the size of the instanton is small compared to the length

$$l = \min(m_{\text{eff}}^{-1}, H_{\text{inf}}^{-1}) \quad (13)$$

characterizing the breaking of scale invariance, one can estimate the corrections to the bounce action perturbatively. Substituting (11) into (4) and expanding to the order $O(l/\bar{\chi}^2)$ we obtain,

$$S_E^{(\text{CDL})}(\bar{\chi}) = \frac{8\pi^2}{3|\lambda|} \left[1 + 3(m_{\text{eff}}^2 - 2H_{\text{inf}}^2) \bar{\chi}^2 \log(l/\bar{\chi}) \right], \quad (14)$$

where we have kept only the log-enhanced contributions. The tunneling rate is given by the configuration minimizing the action. If $m_{\text{eff}}^2 > 2H_{\text{inf}}^2$ the minimal suppression is reached at the configuration of zero size,⁵ $\bar{\chi} = 0$, and coincides with the flat-space result (12). One observes that in this case the assumption $\bar{\chi} \ll l$ is justified. In the opposite case, $m_{\text{eff}}^2 < 2H_{\text{inf}}^2$, the correction due to the expansion of the universe dominates and makes the solution spread over the whole 4-sphere. We have checked numerically that the only solution in this case is the HM instanton.

We now restore the running of couplings which provides additional source of the scale invariance breaking. This enters into the calculations through the loop corrections in the instanton background. For instantons of the size smaller than l these corrections can be evaluated neglecting both the mass m_{eff} and the Hubble H_{inf} . Thus, they are the same as in the flat space [22] and roughly amount to substituting in (14) the coupling constant evaluated at the scale of inverse instanton size, $\mu = \bar{\chi}^{-1}$. Numerically, for the best-fit values of the SM parameters, this dependence on $\bar{\chi}$ turns out to be much stronger than the one introduced by the effective mass and the Hubble expansion. This freezes the size of the instanton at the value corresponding to the minimum of the running coupling constant, $\bar{\chi}_*^{-1} \approx \mu_* \sim 10^{16} - 10^{18}$ GeV. The total answer for the suppression is then given by (14) evaluated at $\bar{\chi}_*$. The corrections due to m_{eff} and H_{inf} are small as long as⁶ $m_{\text{eff}}, H_{\text{inf}} \lesssim 10^{15} - 10^{17}$ GeV.

3. Discussion of approximations

We have obtained the formula (14) under the assumption that the transition happens in an external de Sitter space–time. Let us check its validity. First, the Hubble rate during inflation is not exactly constant, but slowly varies. We have seen that the size of the bounce is much smaller than the horizon size. This implies that the formation of the bubble of the new phase inside the false vacuum occurs very fast.⁷ Thus neglecting the change in the Hubble rate during the formation of the bubble is justified.

Second, in the case when the effective Higgs mass is given by the coupling to the inflaton, the Higgs exerts a force on the inflaton during tunneling. This force should not lead to large displacements of ϕ that could change its energy density. One estimates the shift of ϕ due to the Higgs force as

$$\square \delta\phi = \frac{h^2}{2} \frac{dm_{\text{eff}}^2}{d\phi} \implies \delta\phi \sim \frac{h_*^2}{H^2} \frac{dm_{\text{eff}}^2}{d\phi}, \quad (15)$$

where \square stands for the Laplacian on the 4-sphere and $h_* = \sqrt{8/|\lambda(\bar{\chi}_*^{-1})|} \bar{\chi}_*^{-1}$ is the value of the Higgs in the center of the instanton. Requiring $V'_{\text{inf}} \delta\phi \ll V_{\text{inf}}$ we obtain the condition

$$\frac{dm_{\text{eff}}^2}{d\phi} \ll \frac{V'_{\text{inf}}}{6\epsilon h_*^2}, \quad (16)$$

where $\epsilon = (M_p V'_{\text{inf}})^2 / (16\pi V_{\text{inf}}^2)$ is the slow-roll parameter. This condition is satisfied if the dependence of m_{eff} on the inflaton is weak enough.

Last, but not least, one should check if the energy density of the Higgs field is smaller than that of the inflaton. This requirement turns out to be violated in the center of the CDL bounce for realistic values of H_{inf} . What saves the day is the fact that the size of the region where this violation occurs is of order $\bar{\chi}_*$. On the other hand, the log-enhanced corrections in (14) come from the region of order $\sim l$, which is much larger. Thus they are not modified by the back-reaction of the Higgs field on the geometry.

The effects of the back-reaction can be taken into account neglecting completely the inflaton energy density, i.e. in the same way as in the case of the false vacuum decay in the flat space [24]. They give an additional contribution to the bounce action,⁸

$$\Delta S_E^{(\text{CDL})} = \frac{256\pi^3(1 - 12\xi)}{45(M_p \bar{\chi} \lambda)^2}. \quad (17)$$

For moderate values of ξ these corrections are small as long as $\bar{\chi}_*^{-1} < 5 \cdot 10^{16}$ GeV. Finally, further corrections to the bounce action can come from Planck-suppressed higher-order operators in the Higgs action. The analysis of these corrections is the same as in flat space–time. Note that they can be quite significant due to the fact that the size of the instanton is close to Planckian [25].

Acknowledgements

We thank Fedor Bezrukov, Archil Kobakhidze, Alexander Spencer-Smith and Arttu Rajantie for stimulating discussions. S.S. is supported by the Swiss National Science Foundation, grant No. 156749.

⁵ A proper interpretation of this singular bounce is given within the formalism of constrained instantons [21].

⁶ The current bound on the primordial tensor perturbations [23] constrains $H_{\text{inf}} \lesssim 10^{14}$ GeV during last ~ 60 e-folds of inflation.

⁷ The time of the bubble formation should not be confused with the vacuum decay time, which is exponentially long.

⁸ Here we assume that gravity is described by Einstein's general relativity at least up to the scale $\bar{\chi}^{-1}$.

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