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Scalings of elliptic flow for a fluid at finite shear viscosity

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ABSTRACT

Within a parton cascade approach we investigate the scaling of the differential elliptic flow $v_2(p_T)$ with eccentricity ϵ_x and system size and its sensitivity to finite shear viscosity. We present calculations for shear viscosity to entropy density ratio η/s in the range from $1/4\pi$ up to $1/\pi$, finding that the v_2 saturation value varies by about a factor 2. Scaling of $v_2(p_T)/\epsilon_x$ is seen also for finite η/s which indicates that it does not prove a perfect hydrodynamical behavior, but is compatible with a plasma at finite η/s. Introducing a suitable freeze-out condition, we see a significant reduction of $v_2(p_T)$ especially at intermediate p_T and for more peripheral collisions. This causes a breaking of the scaling for both $v_2(p_T)$ and the p_T-averaged v_2, while keeping the scaling of $v_2(p_T)/\langle v_2\rangle$. This is in better agreement with the experimental observations and shows as a first indication that the η/s should be significantly lower than the pQCD estimates. We finally point out the necessity to include the hadronization via coalescence for a definite evaluation of η/s from intermediate p_T data.

The Relativistic Heavy Ion Collider (RHIC) has successfully shown that a transient state of matter at initial temperature T and energy density ϵ well above the one expected at the phase transition ($T_c \sim 170$ MeV and $\epsilon_c \sim 0.7$ GeV/fm^3) has been created. In particular the large value of the elliptic flow v_2 indicates that such a matter, called quark–gluon plasma (QGP), behaves like a nearly perfect fluid. In fact the dynamics of the bulk of plasma (i.e. for transverse momentum $p_T < 1.5$ GeV) is successfully described by ideal hydrodynamics [1], at least for the most central collisions [2]. At higher transverse momenta, due to incomplete equilibration, the hydrodynamical behavior breaks down as confirmed by the saturation of the baryon to meson ratio and by the quark number scaling of elliptic flow v_2 [3–7]. In this intermediate p_T region ($1.5 < p_T < 5$ GeV) kinetic theory provides the most reliable approach and indeed parton cascade has successfully predicted the $v_2(p_T)$ saturation pattern for $p_T \geqslant 1.5$ GeV [8]. Furthermore the cascade approach hints at a parton cross section significantly larger than estimated in perturbative QCD (pQCD) in general consistency with the observed nearly hydrodynamical behavior. On the other hand a minimum viscosity is imposed by quantum mechanical considerations [9] and more recently a study of supersymmetric gauge theory in infinite coupling limit [10] has given a lower bound for the shear viscosity to entropy density ratio $\eta/s \geqslant 1/4\pi$. All known substances, from water to a meson gas, obey this bound and indeed all of them are significantly above it [11]. A first recent evaluation of shear viscosity in lattice QCD (lQCD) is consistent with the lower bound [12,13] and shows a mild evolution in the range of temperature covered by RHIC [13]. The description of the elliptic flow, the strong scattering of heavy quarks, the measurements of p_T fluctuations, all point [11] to a η/s that should be close to the bound and much smaller than the one expected in a pQCD regime [14] (about 5–10 times the lower bound). A first attempt based on Knudsen number analysis of v_2/ϵ_x has lead to estimate $\eta/s \sim 0.11$–0.19 depending on initial conditions [15]. Moreover, since the pioneering work of [16], it has been started an effort in developing viscous hydrodynamics [17,18] which is indicating a significant reduction of v_2 even for η/s at the lower bound. The effect appears to be quite strong in the calculations of Ref. [18] while is less pronounced in Ref. [17] where data on p_T averaged elliptic flow, $\langle v_2\rangle$, are fairly well reproduced as a function of the collision centrality with $\eta/s \sim 0.1$. On the other hand the low p_T minimum bias $v_2(p_T)$ seems to favor calculations with an even smaller η/s. Similar findings for the dependence of the average elliptic flow $\langle v_2\rangle$ on shear viscosity have been reported also in the context of a parton cascade [19]. However a detailed investigation of viscous effects on differential elliptic flow $v_2(p_T)$ within a transport theory is still pending.

In this Letter we present a study of the elliptic flow and its scaling properties as a function of p_T at finite η/s in the range $(4\pi)^{-1} < \eta/s < \pi^{-1}$. The analysis is based on a parton cascade approach for massless particles and it is mainly focused on the intermediate p_T region, where kinetic theory automatically ac-

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counts for non-equilibrium effects. The main idea is to keep the η/s of the medium constant during the collision dynamics. The parton cross section is rearranged according to the local density and mean momentum values. Simulations have been carried out for a large range of impact parameters in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Some simulations have been performed also for Cu + Cu for a first investigation of the system size dependence. A first issue that we discuss is the scaling of the v_2 with the spatial eccentricity $\epsilon_x = \langle y^2 - x^2\rangle/\langle x^2 + y^2\rangle$ and the system size. We show that if η/s is kept constant down to the thermal freeze-out ($\epsilon \sim 0.2$ GeV/fm^3) a parton cascade exhibits a v_2/ϵ_x scaling in the whole p_T range investigated (up to 3.5 GeV). Therefore the prediction of the scaling is not a unique feature of ideal hydrodynamics [20]. However experimentally the (in)dependence of v_2/ϵ_x on the centrality of the collision and on the system size is indeed a delicate issue as raised by recent publications from PHENIX [21] and STAR [22]. We point out that once a suitable freeze-out condition is introduced at $\epsilon_c \sim 0.7$ GeV/fm^3 a cascade approach at finite viscosity can account for the breaking of the scaling for $v_2(p_T)/\epsilon_x$ together with a persisting scaling for $v_2(p_T)/\langle v_2\rangle$, as experimentally observed.

A first attempt of our investigation has been also to put a reasonable constraint on the η/s value of the RHIC fluid through the observed $v_2(p_T)$ pattern for 1 GeV $< p_T <$ 3 GeV. A significant dependence of $v_2(p_T)$ on shear viscosity is found with a reduction of the saturation value of nearly a factor 2 going from the lower bound to $\eta/s = \pi^{-1}$. However a definitive evaluation of η/s is entangled with the observation of quark number scaling in the same p_T range and hadronization by coalescence plus fragmentation has to be self-consistently included.

The partonic transport approach at the present stage does not contain the different aspects of the dynamics and in particular it misses the effects of the fields which are important to incorporate the phase transition behavior of the speed of sound [24]. However it is certainly a powerful approach whenever a finite mean free path has to be considered and in particular at intermediate p_T where the hydrodynamical behavior breaks down. We have developed a (3 + 1)-dimensional Monte Carlo cascade for on-shell partons based on the stochastic interpretation of the transition rate. Such an interpretation is free from several unphysical drawbacks and particularly suitable for an extension to multiparticle collisions as pointed out by Xu and Greiner [25]. The evolution of parton distribution function from initial conditions through elastic scatterings is followed by propagating particles along straight lines and sampling possible transitions in a certain volume and time interval according to the Boltzmann equation for two-body scatterings

$$p_\mu \partial^\mu f_1 = \int\limits_2 \int\limits_{1'} \int\limits_{2'} (f_{1'} f_{2'} - f_1 f_2) |\mathcal{M}_{1'2' \to 12}|^2 \delta^4(p_1 + p_2 - p'_1 - p'_2), \tag{1}$$

where $\int_j = \int_j d^3p_j/(2\pi)^3 2E_j$, $\mathcal{M}$ denotes the transition matrix for the elastic processes and f_j are the particle distribution functions.

For the numerical implementation, we discretize the space into cells small respect to the system size and we use such cells to calculate all the local quantities. In particular we evaluate at each timestep the local collision probability and decide whether or not a collision can occur by means of a Monte Carlo algorithm. We have performed several checks to test the validity of the code similarly to what thoroughly discussed in Ref. [25], obtaining the same results. More specifically we have performed tests to choose a good discretization for convergency of the results for the elliptic flow that is the main observable analyzed in the present Letter. The calculations shown are performed with cells of transverse area 0.5 fm^2 and a longitudinal size of $\Delta\eta_s = 0.1$, where η_s is the space–time rapidity. Furthermore we have implemented the subdivision (or test particles) technique which allows for a better mapping of the phase space. This is indeed necessary due to the smallness of the cell volume. Our tests have indicated that, for large (small) systems as Au (Cu), $N = 6(25)$ test particles for each real parton are sufficient to achieve stable results for the collision rate and the elliptic flow.

In kinetic theory in ultra-relativistic conditions the shear viscosity can be expressed as [26]

$$\eta = \frac{4}{15}\rho\langle p\rangle\lambda, \tag{2}$$

with ρ the parton density, λ the mean free path and $\langle p\rangle$ the average momentum. Therefore considering that the entropy density for a massless gas is $s = \rho(4 - \mu/T)$, μ being the chemical potential, we get

$$\frac{\eta}{s} = \frac{4\langle p\rangle}{15\,\sigma_{\mathrm{tr}}\rho(4 - \mu/T)} \tag{3}$$

where σ_{tr} is the transport cross section, defined as

$$\sigma^{\mathrm{tr}} = \int d\theta \frac{d\sigma}{d\theta} \sin^2\theta.$$

We use a pQCD inspired cross section with the infrared singularity regularized by Debye thermal mass m_D [8]

$$\frac{d\sigma}{dt} = \frac{9\pi\alpha_s^2}{(t + m_D^2)^2}\left(\frac{1}{2} + \frac{m_D^2}{2s}\right), \tag{4}$$

where s, t are the Mandelstam variables and $m_D = 0.7$ GeV.

Our approach is to artificially keep the η/s of the medium constant during the dynamics of the collisions in a way similar to D. Molnar in [27]. This is achieved by evaluating locally in space and time the strength of the cross section needed to keep the η/s constant. From Eq. (3) we see that assuming locally the thermal equilibrium this can be obtained evaluating in each cell the cross section according to

$$\sigma_{tr} = \frac{4}{15}\frac{\langle p\rangle}{\rho(4 - \mu/T)}\frac{1}{\eta/s}, \tag{5}$$

with η/s set from 1 to 4 in units of the minimum value.

We notice that, once the cross section for the massless partons is renormalized to keep the η/s fixed, distinction between quarks and gluons is irrelevant. So in the following we will refer generically to partons.

Partons are initially distributed according to a mixture of the density of participant nucleons (80%) and of binary collisions (20%) calculated with a Glauber model. The eccentricity of the system is therefore similar to the one used in standard calculations.[1] We also start our simulation like in hydrodynamics at a time $t = 0.6$ fm, assuming free-streaming evolution from $t = t_0$ to $t = 0.6$ fm. Partons with $p_T < p_0 = 2$ GeV are distributed according to a thermalized spectrum, while for $p_T > p_0$ we take the spectrum of non-quenched minijets as calculated in [28]. In principle the transport approach allows for an investigation of the important issue of thermalization which should be strictly related to three-body scatterings [25]. Here looking at collective modes like the elliptic flow we implicitly assume that the results do not depend significantly on the details of the collision kinematics once the η/s value has been fixed. So inelastic processes can be expected to affect at most the absolute value of $v_2(p_T)$ but not the scalings we discuss in the following.

[1] We note that our eccentricity is usually quoted in literature as ϵ_{std}. However, we use a standard Glauber model with no fluctuations (except for the numerical noise) and hence ϵ_{std} does not significantly differ from the so-called participant eccentricity ϵ_{part} like in hydro calculations.

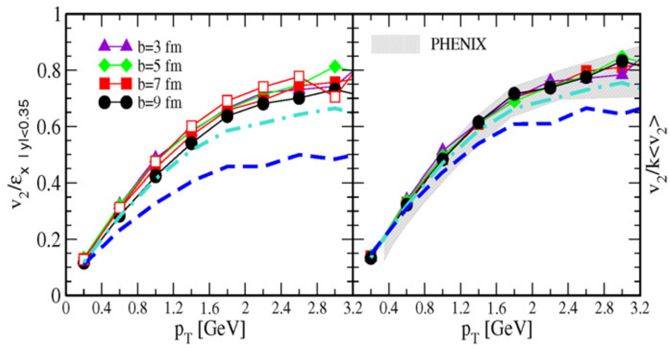

Fig. 1. Parton $\frac{v_2}{\epsilon_x}$ (left panel) and $\frac{v_2}{k\langle v_2\rangle}$ (right panel) in the central rapidity region ($|y| < 0.35$) for Au + Au (filled symbols) and Cu + Cu (open symbols) collisions at $\sqrt{s} = 200A$ GeV. Different symbols refer to cascade simulations at various impact parameters for $\eta/s = 1/4\pi$. In both panels also results for Au + Au at $b = 7$ fm with $\eta/s = 1/2\pi$ (dot-dashed line) and $\eta/s = 1/\pi$ (dashed line) are shown. The grey band in the right panel refers to charged particles data in the pseudorapidity range $|\eta| \leqslant 0.35$ with the constant $k = 3.1$ as in [21].

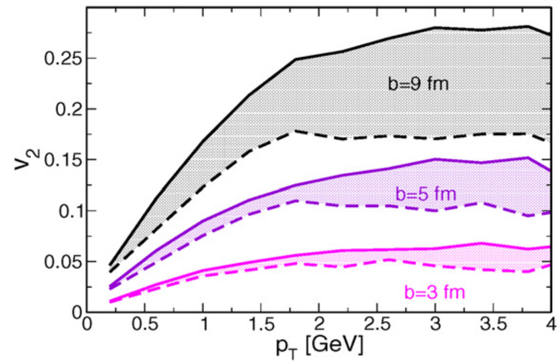

Fig. 2. Parton differential elliptic flow for Au + Au at different impact parameters with (dashed curves) and without (solid curves) a freeze-out condition ($\epsilon_{crit} = 0.7$ GeV fm^{-3}).

A first objective of our study is to investigate the scaling behavior of the elliptic flow with the initial eccentricity and the system size to see if such a scaling, typical of a hydrodynamical behavior [20], persists also in a cascade approach. The interest for such a behavior is triggered by the recent observation by the PHENIX Collaboration [21] of a scaling of $v_2(p_T)/\langle v_2\rangle$ up to $p_T \sim 3$ GeV, a region usually considered out of the range where hydrodynamics should work. In order to allow for a comparison with the results from ideal hydrodynamics of Ref. [20] we have followed the evolution of the system considering a finite constant viscosity to entropy density ratio, as in hydrodynamical studies the zero mean free path condition is implicit for the entire evolution of the system. The hadronic re-scattering together with the formation and decay of the resonances are neglected. This is justified by the fact that the bulk of $v_2(p_T)$ develops in the early stage of the reaction, i.e. well before hadronization sets in, as found by several theoretical approaches [1,29,30] and more recently confirmed experimentally [23,31]. In Fig. 1 the parton v_2/ϵ_x (left panel) and $v_2/k\langle v_2\rangle$ (right panel) in the central rapidity region ($|y| < 0.35$) are shown as a function of transverse momentum p_T for different impact parameters and the two systems Au + Au (filled symbols) and Cu + Cu (open symbols) at $200A$ GeV with our cascade approach when η/s is kept constant at $1/4\pi$. The dot-dashed and dashed lines are the results for Au + Au at $b = 7$ fm with $\eta/s = 1/2\pi$ and $\eta/s = 1/\pi$, respectively. The value of the constant in the right panel is set to $k = 3.1$, as in [21], where $k\langle v_2\rangle$ is assumed to be equivalent to the initial eccentricity ϵ_x.

A first important result is the clear observation of a scaling as a function of centrality and system size for the $v_2(p_T)/\langle v_2\rangle$ which approximately holds also for $v_2(p_T)/\epsilon_x$. In fact, since the parton cross section is re-normalized in order to keep a small constant η/s, dynamical effects related to the different density and temperature conditions that are reached at the different impact parameters, are damped. This indicates that the scaling $v_2(p_T)/\langle v_2\rangle$, which is advocated as a signature of the hydrodynamical behavior [21] is a more general property that holds also at finite mean free path or shear viscosity at least for values close to the lower bound. Moreover the scaling is shown to persist also at higher p_T (~ 3 GeV) where not only the scaling but also the saturation shape is correctly reproduced by the parton cascade approach. Recently $v_2(p_T)$ has been investigated also with viscous hydrodynamics [17,18]. It is interesting that a similar (but weaker) p_T dependence is found with a quantitative agreement with minimum bias data for $\eta/s \sim 0.1$. This is in general agreement with our calculations, performed at various impact parameters, as we can see in Fig. 1 (right) comparing our results (symbols) with the shaded area (PHENIX data). We however are not aware of an explicit investigation of $v_2(p_T)/\epsilon_x$ and $v_2(p_T)/\langle v_2\rangle$ scaling within hydrodynamics. Our simulations show a good sensitivity to the shear viscosity especially at intermediate p_T where $v_2(p_T)/\epsilon_x$ drops of about 40% when increasing η/s by a factor 4 above the lower bound. Obviously a smaller sensitivity is observed for the ratio $v_2/\langle v_2\rangle$, being both quantities affected by the η/s.

While the $v_2/\langle v_2\rangle$ scaling was initially considered to stand for the v_2/ϵ_x scaling and it is quoted as a further validation of ideal hydrodynamics [21], latest results from STAR [22] show that the $v_2(p_T)$ scaled by the participant eccentricity ϵ_x is not independent on centrality. In fact the build up of a stronger collective motion in more central Au + Au collisions is observed. On the other hand, a good scaling with centrality is observed for $v_2/\langle v_2\rangle$ ratio. This feature will be further clarified in the following.

Effect of QGP freeze-out. We investigate the effect of a freeze-out condition on the elliptic flow. Freeze-out conditions are justified by the fact that, at a critical value for energy density, hadronization sets in and parton dynamics is no longer acting. To take into account such an effect we stop the interactions among partons as the local energy density drops below 0.7 GeV/fm^3, an intermediate value in the range corresponding to a mixed quark-hadron phase [1]. Previous calculations were performed with a freeze-out condition at $\epsilon = 0.2$ GeV/fm^3 which corresponds to the end of a mixed phase or roughly to a hadronic-thermal freeze-out. We have checked that a freeze-out at 0.2 GeV/fm^3 is practically identical to consider no energy density freeze-out at all. This is in agreement with the observation that both theoretically and experimentally the elliptic flow does not develop significantly during the hadronic stage [1,23,29–31], hence we will refer to such a calculation as the one without freeze-out.

When the freeze-out condition is implemented a sizeable reduction of the elliptic flow is observed (see Fig. 2), especially for the most peripheral collisions and at larger p_T. Correspondingly our results show that the scaling of elliptic flow with the initial spatial eccentricity is broken (see filled symbols in left panel of Fig. 3). In particular v_2/ϵ_x varies of nearly 40–50% from $b = 3$ fm to $b = 9$ fm in the intermediate p_T region (< 3 GeV). The amount of such a spreading is consistent with the data reported by [22] for the centrality selections 0–10% and 10–40%, with central collisions exhibiting a bigger elliptic flow to eccentricity ratio than the peripheral ones. On the other hand, the scaling of $v_2/\langle v_2\rangle$ with the impact parameter is still observed (see right panel in Fig. 3). We are therefore driven to the conclusion that the breaking of the $v_2(p_T)/\epsilon$ scaling, as observed in Fig. 3, traces back to the freeze out physics, which deserves a deeper investigation.

We are mainly focused on the p_T dependence but it is of course interesting to look at the behavior of the averaged $\langle v_2\rangle$. We how-

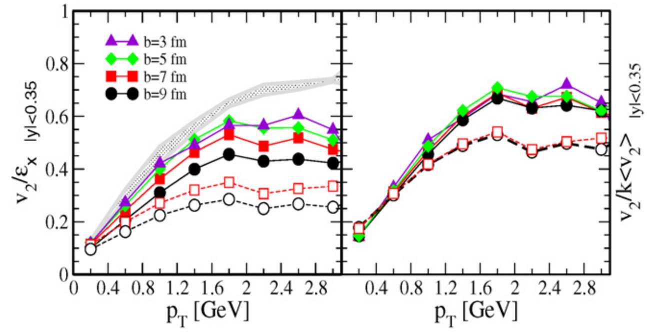

Fig. 3. Same as in Fig. 1, but with freeze-out, filled symbols refer to calculations at $\eta/s = 1/4\pi$; open symbols are for $b = 7$ fm (squares) and $b = 9$ fm (circles) calculations at $\eta/s = 1/\pi$. The grey band in the left panel refers to the results of simulations without freeze out (see left panel in Fig. 1).

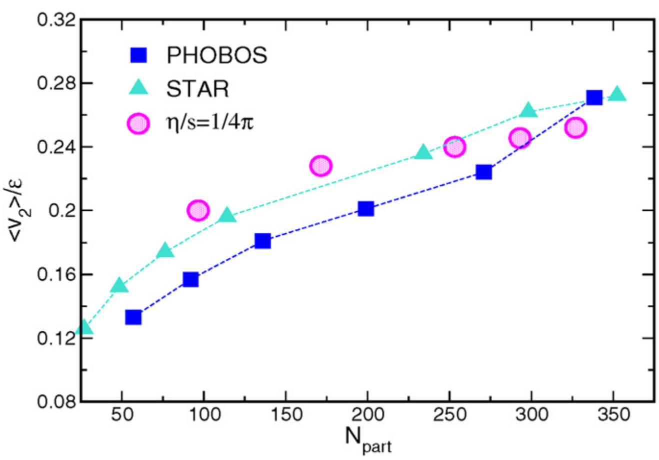

Fig. 4. $\frac{\langle v_2 \rangle}{\epsilon}$ as a function of participant number for Au + Au collisions at $\sqrt{s} = 200$ GeV in the central rapidity region $|y| \leqslant 1$ for $\eta/s = 1/4\pi$ (circles). Triangles and squares are the corresponding data from [22] and [32], respectively.

ever note that a comparison of the $\langle v_2 \rangle$ in our parton cascade with experimental data should face two main limitations. One is the lack of finite quark mass effect that are known to reduce the value of v_2 up to a p_T of the order of the mass, an effect known in a hydrodynamical picture as mass ordering. The other limitation is due to the absence of resonance formation and decay which are known to affect the elliptic flow especially for pions. Both effects are relevant at $p_T < 1$ GeV and therefore should be taken into account to estimate the absolute value of $\langle v_2 \rangle$, as appropriately done in hydrodynamical models. Nevertheless we are mainly interested in showing that a cascade approach is able to approximately reproduce the observed trend of $\langle v_2 \rangle/\epsilon_x$ with the number of participant N_{part}. We also mention that the last translates into the scaling of v_2/ϵ_x with $1/S dN/dy$ as one would expect from general arguments [33].

Within the ideal hydrodynamics picture, the p_T averaged elliptic flow is practically proportional to the initial spatial eccentricity ϵ_x, leading to a centrality independent value of the $\langle v_2 \rangle/\epsilon_x$ ratio. However recent measurements performed by STAR and PHOBOS [22,32] have pointed out significant deviations from the scaling. Such deviations are confirmed by our cascade approach at finite viscosity which also reproduces the scaling of $v_2(p_T)/\langle v_2 \rangle$ as discussed above (see Fig. 4).

We mention that a breaking of the scaling for the average $\langle v_2 \rangle$ is observed also without freeze-out condition, even if the freeze-out reduces the absolute value and enhances the breaking.

As a last point we discuss how information on the viscosity to entropy density ratio of the RHIC plasma can be inferred from the

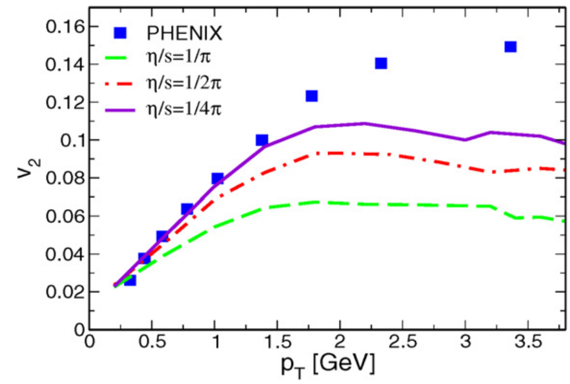

Fig. 5. Differential elliptic flow for Au + Au collisions at $b = 5$ fm, $|y| \leqslant 0.35$ and $\eta/s = 1/4\pi$ (solid curve), $\eta/s = 1/2\pi$ (dot-dashed curve) and $\eta/s = 1/\pi$ (dashed curve). Results from cascade are compared with data from [21] (squares).

$v_2(p_T)$ absolute value. One has to consider that at intermediate p_T there are several evidences for hadronization via coalescence and it has been shown that due to a coalescence mechanism the parton v_2 translates into a nearly doubled hadron v_2 [34,35]. Therefore a definite evaluation of η/s from $v_2(p_T)$ data needs to include the coalescence plus fragmentation mechanism that should account for the baryon–meson quark number scaling. However we refrain from using simple naive coalescence formula [36] here, considering that it has been shown that space-momentum correlation and the freeze-out hypersurface can significantly affect the relation between quark and hadron v_2 [37–39]. It is therefore necessary a further development of the parton cascade approach that includes self-consistently the coalescence and fragmentation process. Nonetheless from Fig. 5 we notice that for $\eta/s = \pi^{-1}$ the parton elliptic flow, with a quite small slope at low p_T, saturates at about 6%. Even assuming a coalescence mechanism in the hadronization phase, this value appears to be too low to reproduce the baryon and meson v_2. This provides anyway an indication that a η/s as high as 4 times the minimum value should be ruled out for the RHIC fluid and the viscosity is therefore quite smaller than pQCD estimates [14]. On the other hand the results with both $\eta/s = 1/4\pi$ and $\eta/s = 1/2\pi$ could be quite close to the experimental data within a coalescence picture. Such a range of values, to be narrowed in the next future, is slightly larger than the first estimates with viscous hydrodynamics [17,18], slightly below to the one based on Knudsen number analysis [15] and contains the best present evaluation in lQCD [13].

Summary and conclusions. We have investigated the dependence on the shear viscosity of the elliptic flow $v_2(p_T)$ and its scaling properties. Our analysis through a parton cascade approach automatically accounts for non-equilibrium effects. However the present study starts from the assumption of local equilibrium for the parton distribution (at $p_T < 2$ GeV) and for the evaluation of η/s through Eq. (5). This generally limits the investigation of non-equilibrium effects but allows a more direct comparison with the studies in the context of hydrodynamics. As a first result we find that the approximate scaling of $v_2(p_T)/\epsilon_x$ advocated as a signature of the perfect hydrodynamical behavior [21] can still hold also at finite viscosity and in a parton cascade approach. However such a scaling versus centrality and system size is present only if one makes the fireball evolve down to energy density $\epsilon \sim 0.2$ GeV/fm^3 corresponding typically to the end of a mixed phase. If a freeze-out condition for the partonic dynamics is put at $\epsilon \sim 0.7$ GeV/fm^3 then a sizeable breaking of the $v_2(p_T)/\epsilon_x$ scaling is seen while $v_2(p_T)/\langle v_2 \rangle$ still scales. This is in qualitative agreement with the recent experimental data from STAR [22] indicating that freeze-out of QGP dynamics with the consequent change of η/s should be more thoroughly investigated. As a final remark we notice that without any freeze-out condition the $v_2(p_T)$ at parton level would be close to the data for $\eta/s = 1/4\pi$, see Fig. 1. On the other hand we consider such an agreement misleading because it would not be compatible with the enhancement of v_2 due to coalescence and the observation of quark number scaling [35]. Instead the freeze-out condition seems to pave the way for a consistency among the different available observables on elliptic flow: the breaking of $v_2(p_T)/\epsilon_x$, the persistence of $v_2(p_T)/\langle v_2 \rangle$ scaling and the presence of a coalescence plus fragmentation hadronization mechanism acting at intermediate p_T. We therefore conclude that a safe evaluation of the shear viscosity to entropy density ratio from the available data on $v_2(p_T)$ necessitates a cascade approach that includes self-consistently hadronization by coalescence and fragmentation. Finally we mention that such an investigation can be strengthened by a study of the fourth harmonic in the azimuthal anisotropy, i.e. the $v_4 = \langle \cos(4\phi) \rangle$. A first analysis shows a stronger sensitivity to η/s and especially a more critical dependence on the freeze-out dynamics respect to v_2 [40].

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