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Probabilistic Strain Energy Life Assessment Model

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Abstract

Under the "safe-life" methodology used by the United States Navy the service life of aircraft fleet are determined by monitoring the aircraft's usage and estimating the respective fatigue damage. The calculated fatigue life expenditure representing cumulative damage is then used to determine if it is possible for an aircraft or a fleet of aircrafts to reach the point of crack initiation and whether or not it should be retired. This research focuses on the development and initial application of a probabilistic strain-energy model to augment the empirical-based fatigue life expended approach. Experimental fatigue data obtained in this research is used to determine the relation between the number of cycles-to-failure and the cumulative total strain energy. A Bayesian framework for regression, including consideration of the model error is used to develop a probabilistic model of life that includes parameter uncertainties due to the limitation and scatter observed in the experimental data.

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Keywords: strain energy; probability; crack growth

1. Introduction

The safe-life methodology full-scale fatigue testing (FSFT) has been an established method of aircraft service life estimation that has been used by the United States Navy for the past thirty years. The FSFT procedure requires data acquisition from either in-flight or static simulation conditions. The program acquires data that includes the count and state of loads, strain, deflection, cracks, and crack growth. It then determines the rate of crack growth and subsequent failure based on the cumulative damage (i.e., Miner's Rule) which involves determining where cracks exist assuming a standard crack initiation size of 0.254 mm or larger. Reverse calculation is then used to obtain the number of cycles to crack initiation. In many cases, the calculated life by this approach can be much shorter than reality, which results in fleets of aircraft and rotorcraft being prematurely retired [1]. Additionally, this leads to purchases of new fleets to replace the retired ones, often prematurely based on the actual service life remaining. Based on the highly conservative methodology of the safe-life method, the University of Maryland is developing an extension of this method to estimate the service life of fleets of aircraft and rotorcraft. This methodology is based primarily on mechanistic and engineering methodologies including testing and probabilistic assessment of test data using Bayesian estimation. This paper will discuss the one aspect of the extended methodology involving

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development of a probabilistic strain energy-based structural life assessment model. In Section 2 the experimental setup will be defined as well as the analytical aspects of the data analysis. The results will be discussed in Section 3, as the estimated and actual crack growth results are compared and the parameters based on the Bayesian inference are defined.

Nomenclature

da/dN	crack length growth per cycle
ΔK	stress intensity factor
C	material energy absorption capacity
m	fatigue exponent
W_p	plastic strain energy
W_e	elastic strain energy
D	damage
N	test cycle
N_i	cycle of crack initiation
ΔW_{tot}	cumulative strain energy
μ_β	mean vector of parameters
Σ_β	covariance matrix of parameters

2. Life Model Development

The purpose of the experiment is to propose and assess a model through which the remaining life of a structure can be estimated. The model proposed relies on the relationship between damage and absorbed strain energy. To develop and probabilistically estimate the parameters, a number of fatigue experiments have been performed.

2.1. Description of Experiments

This study focuses on the fatigue and crack initiation behavior of Al 7075-T651 samples. Seven samples were tested under constant tensile force at a loading ratio of zero. The design specifications for these samples are listed in Table 1.

Table 1: Design Specifications for Al 7075-T651 Samples

	Single Hole	Three Hole Samples
Length	508 mm	508 mm
Width	27.7 mm	38.1 mm
Thickness	12.7 mm	12.7 mm
Hole Diameter	5.08 mm	6.35 mm
Hole Location	The hole was located in the center of the coupon, 254 mm from the top and bottom	152.4 mm from the top and bottom of the coupon with 101.6 mm of spacing between each hole

An MTS 810 uni-axial fatigue testing machine was used for all tests. Stress amplitudes ranging from 165 to 279 MPa were used. Each test was conducted at room temperature and a frequency of 2 Hz. Fig. 1 shows the MTS 810 testing machine and pictures of both the single-hole and three-hole specimens.

Periodically during the test, the areas around the hole were visually inspected for the presence of surface cracks. Upon the observation of a crack, rather than waiting for the sample to fail due to full fracture, the sample was removed from the MTS 810, and the edges were back-cut to a position just short of the crack length. Upon completion of back cutting, samples were then placed back into MTS 810 and pulled apart. This allowed for the fracture surface to be observed without requiring the extra time needed to allow the sample to fully fail due to fatigue, or contributing any additional uncertainty in determining the point of crack initiation by allowing the crack to propagate further.

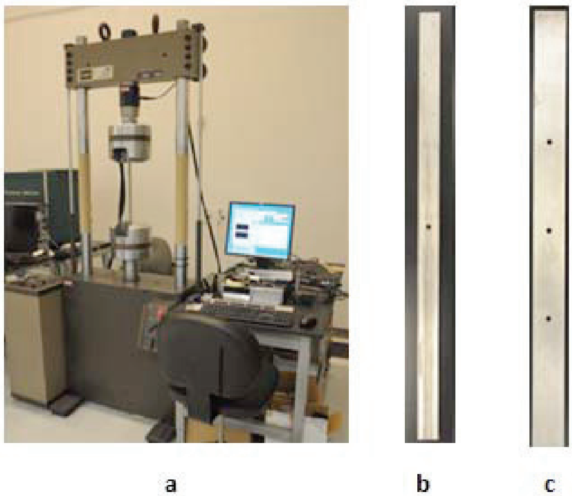

Fig 1: (a) MTS 810 uni-axial fatigue testing machine, (b) single-hole specimen, (c) three-hole specimen

The tested samples were then analyzed using a Buehler ViewMet Inverted optical microscope to observe the crack initiation surface. Using image analysis software, the observed cracks were measured. An example of a crack surface used for measurement is shown in Fig. 2. These values were used in the Walker equation [2] to make an estimation of the number of cycles leading to crack initiation (assumed to be 0.254 mm).

$$da/dN = C\left(\frac{\Delta K}{(1-R)^{w}}\right)^{m} \quad (1)$$

where, da/dN is the crack length growth per cycle, ΔK is the stress intensity factor, R is the stress ratio and C, m, and w are material constants. As mentioned above, in this initial work, each experiment was performed at a loading ratio of zero. The Walker equation is used in anticipation of future tests of non-zero stress ratios.

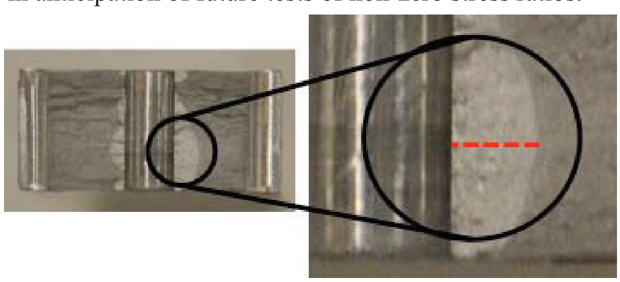

Fig 2: Crack surface measurements

2.2. Proposed Model and Data Analysis

Processing the data per specimen was done through an intricate set of MATLAB programs built upon a series of physical expressions. First, the hole-stress and strain per cycle was calculated from the MTS data using a combination of the Ramberg-Osgood Relationship and Neuber's Rule [3].

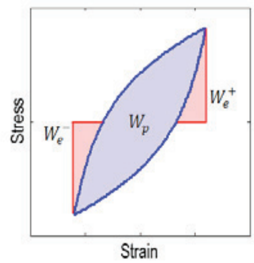

Fig 3: Hysteresis Loop and the breakdown of total strain energy

Hysteresis loops, as shown in Fig. 3, were calculated for all test cycles at the hole. In each loop, the total dissipated strain energy was calculated by obtaining the area within the loop, which is W_p plastic strain energy, in addition to the elastic strain energy $W_{e,}$ which is a sum the two adjoining areas outside of the loop[4].

$$W_{tot} = W_p + W_e = W_p + W_e^+ + W_e^- \tag{2}$$

The energy for the model is treated as cumulative, so the cumulative strain energy was calculated for each specimen. Damage, D, was calculated as a ratio of the number of test cycles N at a given point over the calculated number of cycles to crack initiation N_i.

$$D = (N_i - N/N_i)100 \tag{3}$$

Finally, the proposed model correlated D and ΔW_{tot} is represented as [5]:

$$\Delta W_{tot} = CD^{-m} \text{ or } D = \sqrt[-m]{\Delta W_{tot}/C} \tag{4a-b}$$

To probabilistically estimate parameters C and m, a Bayesian parameter estimation procedure was developed. Equation (4a) was transformed into a general linear regression model to account for all of the tests within the same series [6]. Therefore,

$$\log \Delta W_{tot\,ij} = \log C_i - m_i \log D_{ij} + \varepsilon_{ij} \tag{5}$$

Here ϵ is the residual deviation for specimen i at time t_{ij} which is modeled as a normal distribution (0, $\sigma\varepsilon$). Using this form, the likelihood function becomes:

$$L\left(\log \Delta W_{tot\,ij}, \log D_{ij} \,|\Theta_\beta,\right) = \prod_{i=1}^{q} \int_{\Theta_\beta} \prod_{j=1}^{r_i} \frac{\phi_{NOR}(Z_{ij})}{\sigma_\epsilon} f\left(\Theta_\beta\right) d\Theta_\beta \,, Z_{ij} = \frac{\log \Delta W_{tot\,ij} - \left(\log C_i - m_i \log D_{ij}\right)}{\sigma_\epsilon} \tag{6a-b}$$

Variable Θ_β represents the vector of parameters (i.e., C, m, and $\sigma\varepsilon$) with mean and covariance matrix of parameters (μ_β Σ_β) [6]. Using a prior estimation of parameters from two sets of fatigue tests at low-cycle and high-cycle fatigue, to obtain the joint probability density functions of the parameters C, m, and σ_ε the Bayesian inference was solved using the program WinBUGS [7].

3. Experimental Results

3.1. Crack Growth Results

As discussed above, Walker's equation was used to estimate the number of cycles required to reach the point of crack initiation. For this study the point of crack initiation was assumed to have a length of 0.254 mm. General properties for the material constants *C, m* and *w,* as well as estimated stress intensity factors were determined using eFatigue calculators [8]. The results of this analysis are presented in Table 2. These results are then used to determine the damage amount by way of Equation (3).

Table 2: Experimental results and analysis.

Sample Type	Stress (MPa)	A_f (mm)	N_f (experimental)	$N_{Region\ II}$ (Estimated)	$N_{Region\ I}$ (Calculated)
Three Hole	191	4.89	55487	28578	26909
Three Hole	165	0.68	64046	27223	36823
Single Hole	188	1.91	46687	25403	21284
Single Hole	248	1.98	10422	9948	474
Single Hole	248	4.26	14637	11188	3449
Single Hole	248	3.46	16800	10917	5883
Single Hole	279	1.53	12210	6227	5983

3.2. Parameter Estimation Results

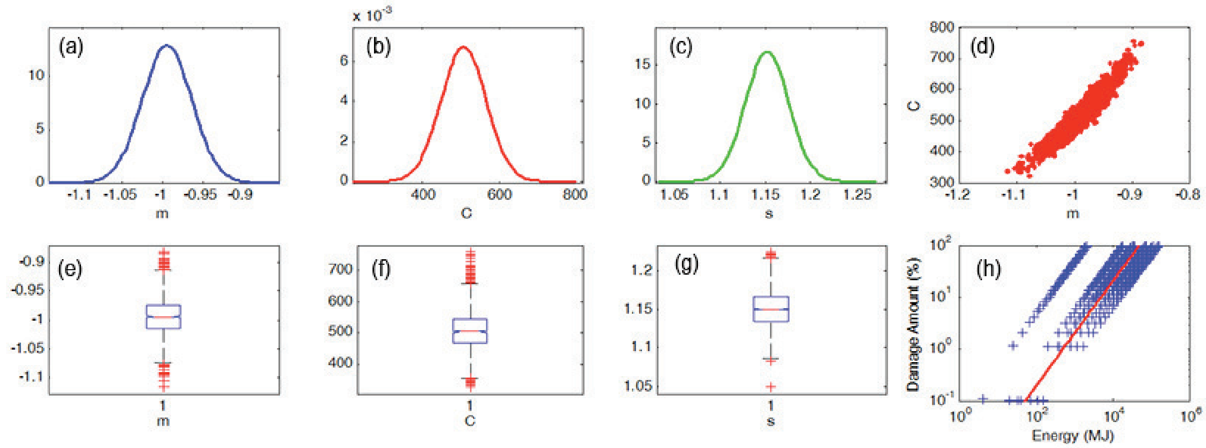

Fig 4: (a-c) Marginal PDF of parameters, (d) parameter spread, (e-g) box and whiskers parameters, and (h) best fit model from program execution.

Fig. 4 contains the results of a run of the code, where 5000 iterations were performed and the following maximum life estimation applies. The following are the numeric results of this run of the code.

$$\mu_{\Theta_\beta} = \begin{bmatrix} -0.99 \\ 506.91 \\ 1.15 \end{bmatrix}, \; \Sigma_{\Theta_\beta} = \begin{bmatrix} 0.0010 & 1.73 & 0.0000058 \\ 1.73 & 3467 & 0.0045 \\ 0.0000058 & 0.0045 & 0.00056 \end{bmatrix} \quad (7a\text{-}b)$$

Table 3: Mean, standard deviation (std), confidence intervals, and median of the parameters.

	mean	std	2.50%	Median	97.50%
m	-0.99	0.031	-1.05	-0.99	-0.93
C	506.91	58.88	403.83	502.95	636.51
σ	1.15	0.02	1.11	1.15	1.20

The following mean life estimation model is formed based on the current output:

$$FLR = 1 - 0.01(D) = 1 - 0.01\left(\frac{\Delta W_{tot}}{505.73e^{\varepsilon}}\right)^{1.01} \approx 1 - (1.977 \times 10^{-5})\Delta W_{tot}e^{-\varepsilon}, \varepsilon = \mathrm{NOR}(0, \sigma_{\varepsilon}) \quad (8)$$

where FLR stands for the fraction of life remaining.

4. Future Work and Concluding Remarks

In the current and future work, efforts are being placed on further model refinement. Experimental results for non-zero stress rations of R = 0.1 and 0.4 are already under way. Additionally, testing scenarios that generate marker-bands, a technique used to create a specific pattern on the fracture surface and can be used to better determine the number of cycles after crack initiation will be used.

All new generated data will be used to update and improve the accuracy of the model presented in this paper. As more data is added, autocorrelation, an instance in which neighboring observations are non-independent, which results in biased ordinary least squared estimates are biased.

The process described in this paper was used to obtain a new life estimation model that is based on modern advances in reliability engineering and fatigue testing. As further tests are conducted the model described in Equation (4b) will be updated and adjusted until enough tests have been performed to fit the needs of the model.

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