Classical bouncing Universes from vector fields

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A B S T R A C T

For the anisotropic Universe filled with massless vector field in the General Relativity frame we obtain bouncing solution for one of scale factors. We obtain the Universe with finite maximal energy density, finite value of R, $R^{\mu\nu}R_{\mu\nu}$, $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ and non-zero value of a scale factor for directions transverse to a vector field. Such a bounce can be also obtained for a massive vector field with kinetic initial conditions, which gives isotropic low energy limit. We discuss the existence of a bounce for a massless vector field with additional matter fields, such as cosmological constant or dust. We also discuss bouncing solution for massless vector field domination in $n + 2$-dimensional space–time.

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0. Introduction

The evolution of Abelian vector fields in cosmology has been discussed in the context of generating isotropic inflation [1], large scale structure [2–6], dark energy and background anisotropies [2,7]. One shall note, that unlike scalar fields, we observe elementary vector fields in accelerator experiments. Thus, we expect them to appear in the early Universe, for instance as a gauge bosons. Since then, they are considered to be a viable alternative for scalar fields in solving problems of classical cosmology. As we shall show in this Letter, vector fields may also result in a finiteness of the energy density of the Universe and generate bouncing evolution of one of scale factors in the GR frame.

Our original motivation to study the evolution of a massless vector field in the GR frame was to find the classical limit for its evolution in Loop Quantum Cosmology [8]. However, it turned out that the massless vector field domination can generate a bounce already in the GR frame. Let us note, that bouncing solutions were found long before LQC [9–11], for example in $f(R)$ theories [12], in the cycling Universe [13] or as a result of quantum corrections [14]. In the present Letter we study the detailed model mentioned already in the previous paper [8]. It turns out, that the model has several advantages: one does not need to introduce any modification of General Relativity (like in $f(R)$ theory) or to introduce any exotic kinetic terms for fields, which would dominate the Universe during a bounce (like in ekpyrotic/cyclic Universe). Solutions, that we have obtained here may form an interesting background for particle physics models of the early Universe.

In this Letter we will use convention $8\pi G = M_{pl}^2 = 1$.

This Letter is organised as follows: In Section 1 we discuss the evolution of space–time for a massless vector field domination, for which we find bouncing solution of the scale factor $a(t)$. In Section 2 we discuss a massless vector field domination in $n + 2$-dimensional space–time. In Section 3 we introduce more realistic (i.e. with isotropic low energy limit) scenarios with a bounce, such as the domination of a massive vector field with kinetic initial conditions or the Universe filled with a massless vector field, cosmological constant and/or dust. Conclusions are presented in Section 4.

1. Massless vector field domination

1.1. Canonical vector field

The evolution of a subdominant vector field $\mathcal{A}_\mu$ in FRW space–time has been analysed in e.g. [2,4]. It has been shown, that the homogeneous (background) component of a vector field lies in the space-like part of a four-vector. Then one may choose one of axes to point in the direction of a background vector field. Let us choose it to be the z axis, which gives

$$\mathcal{A}_\mu = A_\mu(t) + \delta A_\mu(\vec{x}, t), \qquad A_\mu = A(t)\delta^z_\mu,$$
$$\delta A_\mu = (\delta A_t, \delta \vec{A}), \tag{1.1}$$

where δA_μ is a vector field perturbation, which may contribute to the generation of the large scale structure. In this Letter we will consider the Universe filled with a background vector field and a perfect fluid. A vector field breaks the isotropy of space, thus the metric tensor takes the form of

$$g_{\mu\nu} = \text{Diag}\big(1, -a(t)^2, -a(t)^2, -b(t)^2\big), \tag{1.2}$$

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where $a(t), b(t)$ are scale factors. The action of an abelian, massive vector field is of the form

$$S_A = \int d^4x \sqrt{-g} \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + m^2 A^\mu A_\nu \right)$$
$$= \int dt \, \frac{a^2}{2b} (\dot{A}^2 - m^2 A^2), \tag{1.3}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Then the Einstein equation looks as follows

$$H(H + 2\mathcal{H}) = \rho = \rho_f + \rho_A, \tag{1.4}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + H\mathcal{H} = -p_\perp = -p_f - p_A, \tag{1.5}$$

$$2\frac{\ddot{a}}{a} + H^2 = -p_\parallel = -p_f + p_A, \tag{1.6}$$

where ρ_f, p_f are the energy density and pressure of a perfect fluid and $H = \frac{\dot{a}}{a}$, $\mathcal{H} = \frac{\dot{b}}{b}$ are Hubble parameters along x, y (H) and z ($\mathcal{H}$) axis. The $p_\perp$ and $p_\parallel$ are components of the pressure transverse and parallel to the z axis. Note, that the above decomposition is anisotropic since $p_\perp \neq p_\parallel$. The ρ_A is an energy density of a vector field and p_A is a vector component of transverse pressure defined by

$$\rho_A = \frac{1}{2b^2}(\dot{A}^2 + m^2 A^2), \qquad p_A = \frac{1}{2b^2}(\dot{A}^2 - m^2 A^2). \tag{1.7}$$

Equations of motion of a vector field and a perfect fluid look as follow

$$\ddot{A} + (2H - \mathcal{H})\dot{A} + m^2 A = 0,$$
$$\dot{\rho}_f + (2H + \mathcal{H})(\rho_f + p_f) = 0. \tag{1.8}$$

First of all let us consider a massless vector field domination. This scenario has been briefly analysed in [8]. When the energy-stress tensor consists only of the massless vector field, then $\rho_A = \rho = p$. From Eqs. (1.4), (1.6) one obtains

$$H^2 + 2H\mathcal{H} = 2\frac{\ddot{a}}{a} + H^2 \quad \Rightarrow \quad \frac{\ddot{a}}{\dot{a}} = \frac{\dot{b}}{b} \quad \Rightarrow \quad b = E\dot{a}, \tag{1.9}$$

where $E = const > 0$ has a dimension of time. This shows, that the Universe shrinks along the z axis if only $\ddot{a} < 0$. The equation of motion for a massless vector field looks as follows

$$\dot{\rho}_A + 4H\rho_A = 0 \quad \Rightarrow \quad \rho_A = \rho_{Ao} \left(\frac{a_o}{a} \right)^4, \tag{1.10}$$

where $\rho_o = \rho(t_o)$ and t_o is any fixed moment in the history of the Universe. To calculate $a(t)$ let us combine Eqs. (1.6), (1.10) which together give

$$2\ddot{a}a + \dot{a}^2 = \rho_o a_o^4 a^{-2}. \tag{1.11}$$

The Eq. (1.11) can be simplified to

$$\dot{x} = \frac{\sqrt{\rho_I}}{x} \sqrt{x - 1}, \tag{1.12}$$

where ρ_I is a constant of integration, which has a dimensionality of energy density. The $x(t)$ is defined by $x = a(t)/a_I$, where $\rho(a_I) = \rho_I$. From Eq. (1.12) one can see that $x \geqslant 1$, so ρ_I is the maximal allowed energy density of the Universe [8]. So far we have made no assumptions about ρ_I, but in this case we restrict ourselves to consider $\rho_I \ll M_{pl}^4$. Under this assumption GR could be the correct theory of gravity in the whole energy range. The exact solution of Eq. (1.12) is

$$x(t) = \left[\frac{2^{1/3}}{(2 + w^2 + \sqrt{4w^2 + w^4})^{1/3}} + \frac{(2 + w^2 + \sqrt{4w^2 + w^4})^{1/3}}{2^{1/3}} - 1 \right], \tag{1.13}$$

where $w^2 = \frac{9}{4} \rho_I (t - t_I)^2$. From Eqs. (1.9), (1.13) one obtains $R = 0$ for any t. For $t \to t_I$ one also obtains $R^{\mu\nu} R_{\mu\nu} \to 4\rho_I^2$ and $R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \to 20\rho_I^2$. Thus the massless vector field domination gives no initial curvature singularity of the Universe,[1] which together with $a_I \neq 0$, $H(t_I) = 0$ are features of bouncing solutions, like e.g. the Big Bounce. On the other hand, from (1.9), (1.12) one obtains $\mathcal{H} \to \infty$, $b \to 0$ for $t \to t_I$, which looks alike the Big Bang scenario. From now on we will set t_I to be equal to zero. When we are very close to the initial value of the energy density one obtains $w \ll 1$, which gives

$$x(t) \simeq \left(1 + \frac{1}{9} w^2 \right) = \left(1 + \frac{1}{4} \rho_I t^2 \right), \qquad b \propto t. \tag{1.14}$$

The Universe expands along the z direction for $w \in (0, 4)$. For $\rho \ll \rho_I$ one obtains $w \gg 1$. Then

$$x(t) \simeq w^{2/3} = \left(\frac{9}{4} \rho_I \right)^{1/3} t^{2/3}, \qquad b \propto t^{-1/3} \propto a^{-1/2}. \tag{1.15}$$

The low energy limit gives the Kasner-like solution of diagonal Bianchi I model, with one direction shrinking and two expanding. Such predictions are obviously excluded by the astronomical observations, but in Section 3 we will consider realistic extensions of this model. The evolution of $a(w)$ and $b(w)$ is shown in Fig. 1.

1.2. Vector field non-minimally coupled to gravity

Let us investigate the case of a vector field with non-minimal coupling to gravity. This kind of coupling has been introduced to provide the slow-roll evolution during inflation for the physical vector field defined by $V(t) = A(t)/b$ [1,2,15]. In such a case a vector field may survive inflation and dominate the Universe after reheating. On the other hand, the non-minimal coupling may cause certain instabilities on the level of vector field's perturbations, such as ghosts [16]. The action for a non-minimally coupled background vector field looks as follows [17]

$$S_A = \int d^4x \sqrt{|g|} \left[\frac{1}{2} R + \frac{1}{2b^2} \dot{A}^2 - \frac{1}{2b^2} (m^2 + \xi R) A^2 \right],$$
$$S = S_A + S_f, \tag{1.16}$$

where $g = \det(g_{\mu\nu})$. Let us assume, that the perfect fluid is minimally coupled to gravity. Then one obtains the effective Einstein equation in the form

$$H(H + 2\mathcal{H}) = \rho_f + \frac{\dot{A}^2}{2b^2} + \frac{2\xi}{b^2} (\mathcal{H} + 2H) A\dot{A}$$
$$+ \left(m^2 + 2\xi (-2\mathcal{H}^2 - 2\mathcal{H}H + H^2) \right) \frac{A^2}{2b^2}, \tag{1.17}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + H\mathcal{H} = -p_\perp$$
$$= -p_f - (1 - 4\xi) \frac{\dot{A}^2}{2b^2} - \frac{2\xi}{b^2} A \left((3\mathcal{H} - H)\dot{A} - \ddot{A} \right)$$
$$+ \frac{A^2}{2b^2} \left(m^2 + 2\xi (3\mathcal{H}^2 - \mathcal{H}H + H^2 - \dot{\mathcal{H}} + \dot{H}) \right), \tag{1.18}$$

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[1] The energy conditions for the vector field domination require $\rho > 0$, $p > 0$ and $\rho \geqslant |p|$. The massless vector field satisfies all of them, since $p = \rho = \dot{A}^2/2b^2 > 0$.
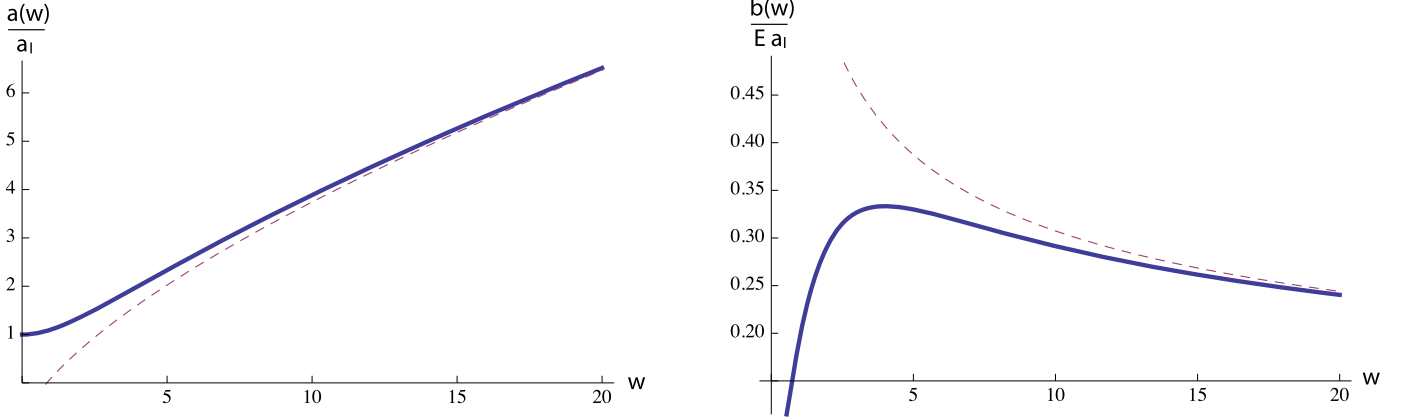
Fig. 1. Left and right panels show the evolution of $a(w)/a_I$ and $b(w)/(a_I E)$ respectively for massless vector field domination. Let us note, that $b(0) = 0$. In both panels solid blue lines and dashed red lines represent the evolution of scale factors and its low energy limit respectively.

$$2\frac{\ddot{a}}{a} + H^2 = -p_\parallel$$

$$= -p_f + (1+4\xi)\frac{\dot{A}^2}{2b^2} - \frac{2\xi}{b^2}A\big((4\mathcal{H}-2H)\dot{A} - \ddot{A}\big)$$

$$- \frac{A^2}{2b^2}\big(m^2 + 2\xi\big(-2\mathcal{H}^2 + 8\mathcal{H}H + 3H^2 + 4\dot{\mathcal{H}} + 2\dot{H}\big)\big). \quad (1.19)$$

The bouncing solution appears only if initially energy of a vector field's kinetic term dominates, since we want to avoid $b(t)$ dependence of the parallel pressure and the energy density. From Eqs. (1.17), (1.19) for $\dot{A}^2 \gg \max\{m^2 A^2, RA^2\}$ one obtains approximate equations of motions of scale factors

$$\dot{x} \simeq \frac{1}{x}\sqrt{(1+4\xi)\rho_I}\sqrt{x-1}, \qquad \frac{\ddot{x}}{x} - \frac{\dot{b}}{b}\frac{\dot{x}}{x} \simeq 2\xi\rho_I x^{-4}. \quad (1.20)$$

Thus, from Eq. (1.12) one recovers (1.13) as a solution for $x(w)$ around a bounce, but the definition of w variable is changed to $w^2 = \frac{9}{4}\rho_I(1+4\xi)t^2$. We have obtained bouncing solution for x with slightly changed effective time variable, but we need to assume, that $1 + 4\xi > 0$ to keep w real. Let us note, that the non-minimal coupling violates Eq. (1.9). To find the evolution of b for energies close to ρ_I let us consider $b = b(x)$. Then from Eq. (1.20) one obtains

$$b \propto (x-1)^{\frac{1}{2(1+4\xi)}} x^{-\frac{1+2\xi}{1+4\xi}} \to 0 \quad \text{for } x \to 1, \quad (1.21)$$

so one recovers all features of bounce generated by a minimally coupled massless vector field.

1.3. Vector field with non-canonical kinetic term

Another way to obtain the slow-roll evolution of ρ_A during inflation is to consider a vector field with a non-canonical kinetic term. Such a field may dominate the Universe after inflaton and become the vector curvaton [5,6]. This model does not produce instabilities, such as ghosts, which is its advantage over the non-minimally coupled vector field [5]. Let us define the action as

$$S = \int d^4x \sqrt{-g}\left(\frac{1}{2}R - f\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right), \quad (1.22)$$

where $f = f(a,b)$, $m = m(a,b)$. Time dependence of the mass term is necessary to obtain a constant value of $\mathcal{M} = m/\sqrt{f}$, which is a mass term of a canonically normalised vector field $U = \sqrt{f}A/b$. Einstein equations, together with the equation of motion of U, look as follows

$$H(H + 2\mathcal{H}) = \rho_f + \rho_A = \rho_f + \frac{1}{2b^2}\big(f\dot{A}^2 + m^2 A^2\big), \quad (1.23)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \mathcal{H}H = -p_\perp$$

$$= -p_f + \frac{A^2}{2b^2}\left(m^2 + \frac{a}{2}m_{,a}^2\right) - \frac{\dot{A}^2}{2b^2}\left(f + \frac{a}{2}f_{,a}\right), \quad (1.24)$$

$$2\frac{\ddot{a}}{a} + H^2 = -p_\parallel$$

$$= -p_f - \frac{\dot{A}^2}{2b^2}(f - bf_{,b}) + \frac{A^2}{2b^2}\big(m^2 - b\, m_{,b}^2\big), \quad (1.25)$$

$$\ddot{U} + (2H + \mathcal{H})\dot{U}$$

$$+ \left(\mathcal{M}^2 + 2\mathcal{H}H + \dot{\mathcal{H}} + \frac{1}{4}\frac{\dot{f}^2}{f^2} - \frac{1}{2}\frac{\ddot{f}}{f} - \frac{1}{2}(2H - \mathcal{H})\frac{\dot{f}}{f}\right)U = 0. \quad (1.26)$$

To obtain cancellation of all time dependent mass terms in Eq. (1.26) one needs $f \propto b^2$, $m \propto b$. For $f \propto a^{-4}$, $m \propto a^{-2}$ the time dependent part of the mass term is equal to $2\dot{H} + \dot{\mathcal{H}}$, so it is negligible during inflation. Let us note, that for both sets of f, m functions one obtains $p_\perp = p_\parallel$, so T^μ_ν is fully isotropic.[2] The continuity equation for a vector field is of the form

$$\dot{\rho}_A + (2H + \mathcal{H})(\rho_A + p_A) = 0 \quad \text{for } f \propto b^2,$$

$$\dot{\rho}_A + (2H + \mathcal{H})(\rho_A - p_A) = 0 \quad \text{for } f \propto a^{-4}. \quad (1.27)$$

One can see, that $f \propto b^2$ gives T^μ_ν identical with a scalar field domination scenario. On the other hand, for $f \propto a^{-4}$ one obtains surprising results. For the kinetic term domination one obtains $\rho_A \simeq p_A$, which means that $\dot{\rho}_A \ll H\rho_A$. This case is similar to the slow-roll phase of a scalar field. The potential term domination brings us to $\rho_A \simeq -p_A$, so $\dot{\rho}_A \simeq -6H\rho_A$, which is the evolution similar to the massless scalar field scenario. In general, the behaviour of kinetic and potential terms has been swapped, comparing to a massive scalar field scenario.

For example, for a massless vector field (or massive vector field with kinetic initial conditions) with $f \propto a^{-4}$ one obtains $\dot{\rho}_A = 0 \Rightarrow \rho_A = const$. Then, from Eq. (1.25), one obtains

$$2\frac{\ddot{a}}{a} + H^2 = \rho_A \quad \Rightarrow \quad \frac{\dot{a}}{a_I} = \dot{x} = \sqrt{\frac{\rho_A}{3x}}\sqrt{x^3 - 1}. \quad (1.28)$$

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[2] In [5,6] Dimopoulos et al. consider $m \propto a$ and $f \propto a^{-4}$ or $f \propto a^2$ to obtain flat power spectra of initial inhomogeneities.
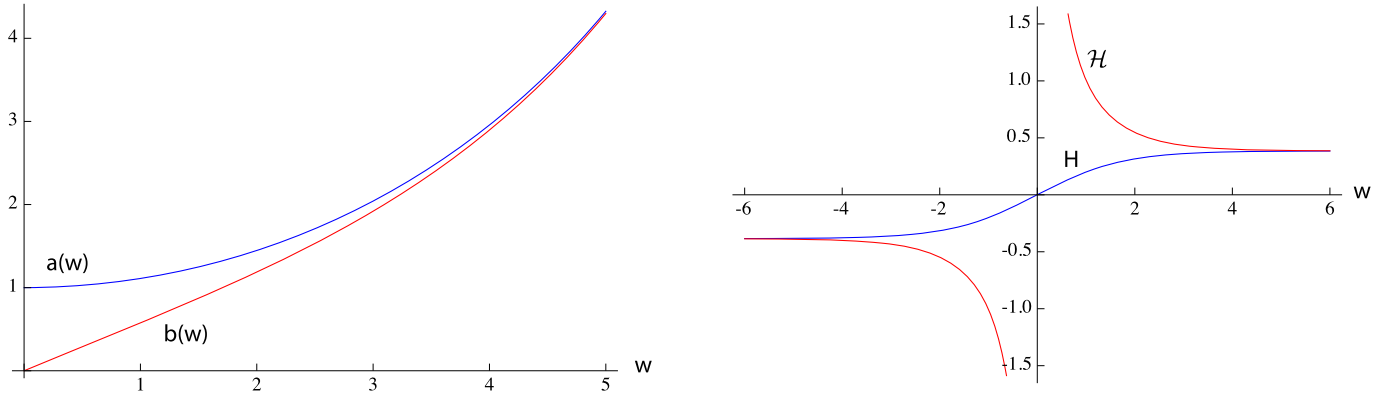
Fig. 2. The left panel shows the evolution of scale factors for the domination of a massless vector field with non-canonical kinetic term and $f \propto a^{-4}$. The right panel presents the evolution of Hubble parameters in the same model. One can see, that after $w \sim t\sqrt{\rho_I} \sim 5$ the Universe becomes isotropic and enters the era of exponential expansion.

We have obtained a bounce for $a = a_I$. From now on we shall replace ρ_A by ρ_I, since ρ_A does not need to be constant in the massive case. The solution of Eq. (1.28) looks as follows

$$x(t) = \cosh^{2/3}\left[\frac{1}{2}\sqrt{3\rho_I}(t - t_I)\right] = \cosh^{2/3}\left[\frac{w}{\sqrt{3}}\right], \qquad (1.29)$$

where $w = 3\sqrt{\rho_I}(t - t_I)/2$. For a massless vector field, from Eqs. (1.23), (1.25) one obtains $b \propto \dot{a} \propto \dot{x}$, so we have found analytical solutions for both scale factors. For such an evolution of scale factors one obtains no initial curvature singularity, since $R \to -51\rho_I/4$, $R^{\mu\nu}R_{\mu\nu} \to 1413\rho_I^2/32$, and $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} \to 549\rho_I^2/16$ for $x \to 1$. Let us note, that for big values of w one obtains

$$a(t) \propto b(t) \propto e^{\sqrt{\rho_I/3}t}. \qquad (1.30)$$

This exponential expansion is identical with the one of the FRW Universe filled with a cosmological constant. Thus, such a vector field can be a good candidate to play the role of Dark Energy. The Universe becomes isotropic very fast, even without a vector field's mass term, or with the negligible influence of the mass term on the evolution of the Universe. Let us note, that for $a = b$ the $a(t)$ from Eq. (1.29) does not satisfy $3H^2 = p_I = const$. Thus, anisotropic initial conditions for scale factors are necessary to obtain bouncing solution for $a(t)$.

On the other hand, one can ask about the naturalness of a kinetic term's domination at early times. For a scalar field it is natural from the point of view of the phase space to consider the kinetic term domination around a bounce. Thus, at early times, due to analogy with scalar fields, one shall expect the domination of a vector field's potential term for $f \propto a^{-4}$. This means, that the bouncing solution is strongly fine tuned if $\mathcal{M} \neq 0$. Let us also note, that the bouncing solution described by Eq. (1.29) may be also obtained with a cosmological constant. In particular, it may be obtained for a scalar field with potential initial conditions. Thus, such a bouncing solution is not a unique feature of vector fields.

The evolution of scale factors and Hubble parameters for a massless vector field with non-canonical kinetic term is shown in Fig. 2.

2. Vector field in $n + 2$ dimensions

As we have proven, a massless vector field generates a bounce for $a(t)$ and gives the Kasner-like evolution at the low energy limit. As an interesting theoretical exercise let us consider more generic case of a massless vector field domination in $n + 2$ dimensions,

where n is a number of dimensions with a rotation symmetry.[3] Let us assume, that the vector field points x_{n+1} axis, which gives the metric tensor of the form $g_{lk} = \text{Diag}(1, -a^2, -a^2, \ldots, -a^2, -b^2)$, where $l, k = 0, 1, \ldots, n + 1$. Then from Einstein equations one obtains

$$\frac{1}{2}n(n-1)H^2 + nH\mathcal{H} = n\frac{\ddot{a}}{a} + \frac{1}{2}n(n-1)H^2$$

$$\Rightarrow \quad \frac{\ddot{a}}{a} = \frac{\dot{b}}{b} \quad \Rightarrow \quad b = E\dot{a}. \qquad (2.1)$$

It is worth to note, that Eq. (2.1) is valid for any n. From the continuity equation one obtains $\rho_A \propto a^{-2n}$, so from $G^{n+1}_{n+1} = p_A = \rho_A$ one obtains the equation of motion for a scale factor $a(t)$

$$\ddot{x}x + \frac{1}{2}(n-1)\dot{x}^2 = \frac{\rho_I}{n}x^{2(n-1)}, \qquad (2.2)$$

where $x(t) = a(t)/a_I$, $a_I = a(t_I)$ and t_I is the moment of a bounce. Let us assume, that $n \neq 1$. Then, from Eq. (2.2) one finds

$$\dot{x} = x^{1-n}\sqrt{\frac{2\rho_I}{n(n-1)}}\sqrt{x^{n-1} - 1}. \qquad (2.3)$$

Eq. (2.3) has the solution of the form of

$$\frac{\sqrt{2(n-1)n}}{(1+n)\sqrt{\rho_I}}x^{(1+n)/2}\,{}_2F_1\left[\frac{1+n}{2(1-n)}, \frac{1}{2}, \frac{n-3}{2(n-1)}, x^{1-n}\right]$$
$$= t - t_I, \qquad (2.4)$$

where ${}_2F_1$ is a hypergeometric function. Analogously to the 4-dimensional case a bounce appears for $t = t_I$. The ρ_I is a constant of integration and the maximal energy density of the Universe. Eq. (2.4) can be simplified in two limits. Let us consider the evolution of scale factors around a bounce (which means, that $x - 1 \ll 1$) and in the low energy limit (for which $x \gg 1$). Then from Eq. (2.3) one obtains

$$x \simeq 1 + \frac{\rho_I}{2n}(t - t_I)^2 \quad \text{for } x - 1 \ll 1,$$

$$x \simeq t^{\frac{2}{n+1}} \quad \text{for } x \gg 1. \qquad (2.5)$$

Those simplified solutions are valid for any n. The $n = 1$ case gives the equation of motion of the form of

$$\dot{x} = \sqrt{2\rho_I \ln\left[x(t - t_I)\right]}. \qquad (2.6)$$

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[3] This means, that the 4-dimensional space–time is described by $n = 2$.

Thus, for $x = 1$ one obtains $\dot{x} = 0$. In such a case there is no analytical solution for $x(t)$.

2.1. The initial curvature singularity of the Universe

To discuss the issue of initial singularity in $n + 2$ dimensions let us express R, $R^{\mu\nu}R_{\mu\nu}$ and $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ us a function of x. From Eqs. (2.1), (2.3) one obtains

$$R = \frac{2}{n}(n-2)x^{-2n}\rho_I, \tag{2.7}$$

$$R^{\mu\nu}R_{\mu\nu} = 7\frac{4}{n^2}(2n^2 - 3n + 2)x^{-4n}\rho_I^2, \tag{2.8}$$

$$\begin{aligned} R^{\mu\nu\alpha\beta} & R_{\mu\nu\alpha\beta} \\ &= \frac{4\rho_I^2 x^{-4n}}{(n-1)n^2}\big(n^2(n+1)x^{2(n-1)} - 4n^2(2n-1)x^{n-1} \\ &\quad + 2(8n^3 - 12n^2 + 7n - 2)\big). \end{aligned} \tag{2.9}$$

In the limit of a bounce, i.e. for $x \to 1$, one obtains

$$R \to \frac{2}{n}(n-2)\rho_I, \qquad R^{\mu\nu}R_{\mu\nu} \to \frac{4}{n^2}(2n^2 - 3n + 2)\rho_I^2,$$

$$R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} \to \frac{4}{n^2}(9n^2 - 10n + 4)\rho_I^2. \tag{2.10}$$

Let us note, that for $n \neq 0$ all invariants mentioned above are finite around a bounce. Thus, for any $n \neq 0$ one obtains no initial curvature singularity of the Universe.

3. Isotropic models with classical bounce

3.1. Massive vector field domination

The fact, that a massive vector field with kinetic initial conditions can give bouncing solution for $a(t)$ has been proven in [8]. During a bounce the kinetic term dominates over the potential one. Nevertheless, $A(t)$ grows with time, which leads to vector field's oscillations. During that period the background anisotropy $(H - \mathcal{H})/(H + \mathcal{H})$ decreases like t^{-1} and the Universe becomes isotropic. To find more about the evolution of space–time in such a model see [8].

3.2. Massless vector field and cosmological constant

Analytical solutions of Einstein equations for the massless vector field domination scenario may be obtained also for a massless vector field with a perfect fluid as long as p_f is not a function of $b(t)$. Then both sides of Eq. (1.6) are functions of $a(t)$ and its time derivatives. The simplest case is a cosmological constant, which gives $\rho_f = -p_f = \rho_\Lambda = -\Lambda = const$. Then, with a certain choose of t_I, one can simplify Eq. (1.6) to

$$\dot{x}(x) = \frac{1}{x}\sqrt{\left(\rho_{AI} - \frac{\rho_\Lambda}{3}\right)x + \frac{\rho_\Lambda}{3}x^4 - \rho_{AI}}. \tag{3.1}$$

From Eq. (3.1) one obtains $\dot{x} \to 0$ for $x \to 1$, so a_I is a minimal value of $a(t)$. Thus, one obtains a bounce similar to the one from a massless vector field domination. The $\rho_I = \rho_\Lambda + \rho_{AI}$ is the maximal value of the energy density, since $\rho_\Lambda = const$ and $\rho_A = \rho_{AI}x^{-4}$. Let us note, that a cosmological constant, together with a massless vector field, has along the z axis the effective equation of state $p = -\rho$. Then, form Eqs. (1.4), (1.6) one obtains $b = E\dot{a}$, so Eq. (1.9) is still valid. Thus, there is a clear analogy between cases with and without a cosmological constant. Simply, one expects the cosmological constant to dominate the Universe for energies much lower than ρ_I, so it shall not have significant influence on the evolution of space–time around the bounce. To find the low energy limit of that model let us assume, that $x \gg 1$ and $\rho_\Lambda > 0$. Then one finds

$$x(t) \propto \sinh^{2/3}\left(\sqrt{\frac{3\rho_\Lambda}{4}}t\right) \to e^{\sqrt{\rho_\Lambda/3}t},$$

$$b \propto \dot{x} \to e^{\sqrt{\rho_\Lambda/3}t}, \tag{3.2}$$

so for $x \gg 1$ one obtains isotropic Universe even without a vector field's oscillations.

For the Universe filled with massive vector field with kinetic initial conditions and with a cosmological constant one obtains the following low energy limit for the evolution of space–time: When a vector field starts to oscillate the Universe becomes isotropic. Vector field's oscillations generate particles and radiation, so in the presence of Λ one obtains the observed Universe. This realistic low energy limit brings us a chance to consider a situation, in which there is no need for any theory of gravity wider than GR. Let us also note, that a cosmological constant may come from the massless vector field with non-canonical kinetic term (see Section 1.3). Thus, one can obtain a bounce with a good low energy evolution of the Universe using only vector fields.

3.3. Massless vector field and dust

Eq. (1.13), which is the analytical solution for $a(t)$ obtained for the massless vector field domination, is also valid for the Universe filled with a massless vector field and dust, since dust does not contribute to $p_\parallel$. Thus, Eq. (1.12) remains unchanged. On the other hand, from the continuity equation, one obtains $\rho_A \sim a^{-4}$, $\rho_m \sim a^{-2}b^{-1}$, so the energy density is a function of both scale factors. This means, that even if one defines ρ_{AI} such as $\forall_a \rho_A < \rho_{AI}$, there may be no ρ_{mI} such that $\forall_{a,b} \rho_m < \rho_{mI}$, so the total energy density does not need to be finite at all times. Let us note, that Eq. (1.9) is not valid, since from Eqs. (1.4), (1.6) one obtains

$$H\mathcal{H} - \frac{\ddot{a}}{a} = \frac{1}{2}\rho_m \neq 0. \tag{3.3}$$

This equation may be written as an equation of motion of $b(t)$. Let us define $x = a/a_I$, $y = b/b_o$, $x_o = a_o/a_I$ where $a_I = a(t_I)$, $b_o = b(t_o)$, $t_o \neq t_I$ is a certain moment and t_I is the moment of a bounce of $a(t)$. The t_o needs to be different than t_I since we do not know, if ρ_m is finite at $t = t_I$. From Eq. (3.3) one obtains

$$\dot{x}\dot{y} - \ddot{x}y = \rho_{mo}\frac{x_o^2}{2x}. \tag{3.4}$$

From Eqs. (1.12), (3.4) one finds the evolution of y as a function of x

$$y(x) = \frac{\rho_{mo}}{3\rho_{AI}}\frac{x_o^2}{x}(x^2 + 4x - 8) + C\frac{1}{x}\sqrt{x-1}, \tag{3.5}$$

where C is the constant of integration. Thus, for $x \to 1$ one obtains $(x^2 + 4x - 8)/x \to -3$ and $\sqrt{x-1}/x \to 0$, which means, that for x close enough to 1 one obtains $y < 0$. This means, that the minimal allowed value of x need to be larger that 1. Let us define x_p such, as $y(x_p) = 0$. Let us note, that $H(x_p) > 0$, since $x_p > 1$, which together with $\mathcal{H}(x_p) = \infty$ gives $H(H + 2\mathcal{H})(x = x_p) = \infty$. This seems to be natural, since $\rho_m \propto (x^2 y)^{-1} \Rightarrow \rho_m(x_p) = \infty$. One cannot extend the evolution of the Universe for $t < t_p$, so the considered model has all features of the Big Bang. The only difference is, that the beginning of space–time is not a point, but a circle described by $y = 0$, $x = x_p$.

Similar results can be found for the Universe filled with massless vector field, cosmological constant and dust. There for $x \to 1$ one obtains $y \to -\rho_{m_o} x_0^2/(\rho_{AI} + \Lambda)$, so for $\Lambda < 0$ and $\rho_A < -\Lambda$ one obtains $y > 0$ for $x < x_p$. Thus, one can solve the problem of negative y at the moment of bounce. However, this generates another problem, since y becomes negative for $x > x_p$. In such a case the Universe would start with a bounce to immediately end up in the Big Crunch. This means, that $y(x)$ may be positive for $x < x_p$ or for $x > x_p$, but one cannot obtain $y > 0$ at all times.

4. Conclusions

In the initial section of this Letter we have shown, that a massless vector field domination generates bouncing evolution of space–time in a plane transverse to a vector field's direction. The bouncing solution has many features of other bounces known in cosmology, such as a non-zero initial value of the scale factor $a(t)$, lack of initial curvature singularity and finite initial (maximal) energy density. Along the vector field's direction one obtains the evolution of the scale factor and the Hubble parameter similar to the Big Bang model, i.e. $b \to 0$ and $\mathcal{H} \to -\infty$ for $t \to t_I$. The Kasner-like solution with $a \sim t^{2/3}$, $b \sim t^{-1/3}$ is the low energy limit of the evolution of space–time.

In Section 2 we have considered bouncing solution for a massless vector field domination in the general $n + 2$-dimensional case, where n is a number of dimensions with isotropic evolution. We have found the analytical solution for the scale factor $a(t)$, as well as its simple form in the high and low energy limit. We have also proven, that R, $R^{\mu\nu}R_{\mu\nu}$ and $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ do not diverge for $t \to t_I$ for all $n \neq 0$.

In Section 3, to obtain isotropic low energy limit of the evolution of space–time, we have discussed several extensions of the massless vector field domination scenario. All of them obtain bounce at (x, y) plane for $t = t_I$. First of all we have considered massive vector field with kinetic initial conditions. In such a case the Universe has bouncing solution for $\rho \sim \rho_I$ and isotropic low energy limit, which is generated by the vector field's oscillations. Secondly, we have considered the Universe filled with a massless vector field and a cosmological constant. In this case a cosmological constant contributes to ρ_I and generates isotropic expansion for $a \gg a_I$. The massive vector field with cosmological constant

may give realistic low energy limit with particles and radiation originating from the A_μ and with a Dark Energy coming from Λ. We have also considered the Universe filled with massless vector field and dust, for which the bounce appears in the forbidden region $b < 0$. The true beginning of space–time appears at $b = 0$, $\mathcal{H} = \infty$, $a > a_I$, $H \neq 0$, so the initial energy density diverges and one obtains all features of the Big Bang. Similar results are given by a massless vector field, dust and cosmological constant domination.

Let us also note, that the presented model has several advantages comparing to bounces from e.g. $f(R)$ theories or ekpyrotic/cyclic Universe. GR remains the only theory of gravity needed and the matter sector has the standard form known from particle physics.

Acknowledgements

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