Quantum states with negative energy density
in the Dirac field and quantum inequalities

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Abstract

Energy densities of the quantum states that are superposition of two multi-electron–positron states are examined. It is shown that the energy densities can be negative when two multi-particle states have the same number of electrons and positrons or when one state has one more electron–positron pair than the other. In the cases in which negative energy could arise, we find that the energy is that of a positive constant plus a propagating part which oscillates between positive and negative, and the energy can dip to negative at some places for a certain period of time if the quantum states are properly manipulated. It is demonstrated that the negative energy densities satisfy the quantum inequality. Our results also reveal that for a given particle content, the detection of negative energy is an operation that depends on the frame where any measurement is to be performed. This suggests that the sign of energy density for a quantum state may be a coordinate-dependent quantity in quantum theory.
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1. Introduction

Although the energy density of a field in classical physics is strictly positive, the local energy density in quantum field theory can be negative due to quantum coherence effects [1]. The Casimir effect [2] and squeezed states of light [3] are two familiar examples which have been studied experimentally. As a result, all the known pointwise energy conditions in classical general relativity, such as the weak energy condition

and null energy condition, are allowed to be violated. However, if the laws of quantum field theory place no restrictions on negative energy, then it might be possible to produce gross macroscopic effects such as violation of the second law of thermodynamics [4,5], traversable wormholes [6,7], "warp drive" [8], and even time machines [7,9]. Therefore, a lot of effort has been made toward determining the extent to which these violations of local energy are permitted in quantum field theory. One powerful approach is that of the quantum inequalities constraining the magnitude and duration of negative energy regions [4,10–15]. Quantum inequalities have been derived for scalar and electro-

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magnetic fields in flat as well as curved spacetimes [13,16–18] and they have also been examined in the background of evaporating black holes [19,20]. However, as far as the Dirac field is concerned, not as much work has been done. In this respect, Vollick has shown that the superposition of two single particle electron states can give rise to negative energy densities and demonstrated that the resulting energy densities obey quantum inequalities which are derived for scalar and electromagnetic fields [21]. He has also given a quantum inequality for Dirac fields in two-dimensional spacetimes [22] using arguments similar to those of Flanagan's [23]. However, there does not seem much hope of generalizing this argument beyond the two dimensions. It is worth noting that the existence of quantum inequalities for the Dirac (and Majorana) field in general 4-dimensional globally hyperbolic spacetimes was recently established [24].

In this Letter, we will examine the negative energy densities for more general states that are the superposition of two multi-electron–positron states, and discuss whether there are any inequalities constraining the magnitude of negative energy when it appears and its life time. We will work in the units where $c = \hbar = 1$ and take the signature of the metric to be $(-+++)$.

2. Quantum states with negative energy densities

For the Dirac field Lagrange density is

$$L = \frac{i}{2}\bar{\psi}\gamma^{\mu}(\overleftarrow{\partial_{\mu}} + \overrightarrow{\partial_{\mu}})\psi - m\bar{\psi}\psi. \tag{1}$$

The symmetrized stress tensor is given by

$$T_{\mu\nu} = \frac{i}{4}\big[\bar{\psi}\gamma^{\mu}\big(\overleftarrow{\partial^{\nu}} + \overrightarrow{\partial^{\nu}}\big)\psi + \bar{\psi}\gamma^{\nu}\big(\overleftarrow{\partial^{\mu}} + \overrightarrow{\partial^{\mu}}\big)\psi\big]. \tag{2}$$

The field operator can be expanded as

$$\psi(x) = \sum_{k}\sum_{\alpha=1,2}\big[b_{\alpha}(k)u^{\alpha}(k)e^{ik\cdot x} + d_{\alpha}^{\dagger}(k)v^{\alpha}(k)e^{-ik\cdot x}\big], \tag{3}$$

where the mode functions are taken to be

$$u^{\alpha}(k) = \begin{pmatrix} \sqrt{\frac{\omega+m}{2\omega V}}\phi^{\alpha} \\ \frac{\sigma\cdot\mathbf{k}}{\sqrt{2\omega(\omega+m)V}}\phi^{\alpha} \end{pmatrix}, \tag{4}$$

$$v^{\alpha}(k) = \begin{pmatrix} \frac{\sigma\cdot\mathbf{k}}{\sqrt{2\omega(\omega+m)V}}\phi^{\alpha} \\ \sqrt{\frac{\omega+m}{2\omega V}}\phi^{\alpha} \end{pmatrix}, \tag{5}$$

and $\phi^{1\dagger} = (1, 0)$, $\phi^{2\dagger} = (0, 1)$. Here $b_{\alpha}(k)$ and $b_{\alpha}^{\dagger}(k)$ are the annihilation and creation operators for the electron, respectively, while $d_{\alpha}(k)$ and $d_{\alpha}^{\dagger}(k)$ are the respective annihilation and creation operators for the positron. The four operators anticommute except in the cases $\{b_{\alpha}(k), b_{\alpha'}(k')\} = \{d_{\alpha}(k), d_{\alpha'}(k')\} = \delta_{\alpha,\alpha'}\delta_{k,k'}$. The renormalized expectation value of the energy density, i.e., $\langle :T_{00}: \rangle$, in an arbitrary quantum state, is

$$\begin{aligned}
\langle\rho\rangle = {}& \frac{1}{2}\sum_{k,k'}\sum_{\alpha,\alpha'}(\omega_{k} + \omega_{k'}) \\
&\times \big[\langle b_{\alpha}^{\dagger}(k)b_{\alpha'}(k')\rangle u^{\dagger\alpha}(k)u^{\alpha'}(k')e^{-i(k-k')\cdot x} \\
&\quad + \langle d_{\alpha'}^{\dagger}(k')d_{\alpha}(k)\rangle v^{\dagger\alpha}(k)v^{\alpha'}(k')e^{i(k-k')\cdot x}\big] \\
&+ \frac{1}{2}\sum_{k,k'}\sum_{\alpha,\alpha'}(\omega_{k'} - \omega_{k}) \\
&\times \big[\langle d_{\alpha}(k)b_{\alpha'}(k')\rangle v^{\dagger\alpha}(k)u^{\alpha'}(k')e^{i(k+k')\cdot x} \\
&\quad - \langle b_{\alpha}^{\dagger}(k)d_{\alpha'}^{\dagger}(k')\rangle u^{\dagger\alpha}(k)v^{\alpha'}(k')e^{-i(k+k')\cdot x}\big].
\end{aligned} \tag{6}$$

Now consider a state vector of the form

$$|\Psi\rangle = \frac{1}{\sqrt{1+\lambda^{2}}}\big[|a(q; j)\rangle + \lambda|b(l; n)\rangle\big], \tag{7}$$

where $|a(q; j)\rangle$ and $|b(l; n)\rangle$ are two multi-particle states with the first symbol in the bracket indicating the number of electrons and the second symbol the number of positrons. For example, we can write $|a(q; j)\rangle = |k_{1}s_{1}, k_{2}s_{2}, \ldots, k_{q}s_{q}; \overline{k_{1}'s_{1}}, \overline{k_{2}'s_{2}}, \ldots, \overline{k_{j}'s_{j}}\rangle$ and $|b(l; n)\rangle = |k_{1}'s_{1}', k_{2}'s_{2}', \ldots, k_{l}'s_{l}'; \overline{k_{1}s_{1}'}, \overline{k_{2}s_{2}'}, \ldots, \overline{k_{n}s_{n}'}\rangle$. Plugging Eqs. (7), (4) and (5) into Eq. (6), we find

$$\begin{aligned}
\langle\rho\rangle = {}& \frac{1}{1+\lambda^{2}}\Bigg[\frac{1}{V}\Bigg(\sum_{r=1}^{q}\omega_{k_{r}} + \sum_{t=1}^{j}\omega_{\bar{k}_{t}}\Bigg) \\
&+ (f_{1} + f_{2} + f_{3} + f_{4})\lambda \\
&+ \frac{1}{V}\Bigg(\sum_{f=1}^{l}\omega_{k_{f}} + \sum_{g=1}^{n}\omega_{\bar{k}_{g}}\Bigg)\lambda^{2}\Bigg],
\end{aligned} \tag{8}$$

where

$$
\begin{aligned}
f_1 = \frac{1}{2} \sum_{k,k'} \sum_{\alpha,\alpha'} & (\omega_k + \omega_{k'}) \\
\times & \left[\langle a(q,j) | b_\alpha^\dagger(k) b_{\alpha'}(k') | b(l,n) \rangle \right. \\
& + \left. \langle b(l,n) | b_\alpha^\dagger(k) b_{\alpha'}(k') | a(q,j) \rangle \right] \\
& \times u^{\dagger\alpha}(k) u^{\alpha'}(k') e^{-i(k-k')\cdot x},
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
f_2 = \frac{1}{2} \sum_{k,k'} \sum_{\alpha,\alpha'} & (\omega_k + \omega_{k'}) \\
\times & \left[\langle a(q,j) | d_{\alpha'}^\dagger(k') d_\alpha(k) | b(l,n) \rangle \right. \\
& + \left. \langle b(l,n) | d_{\alpha'}^\dagger(k') d_\alpha(k) | a(q,j) \rangle \right] \\
& \times v^{\dagger\alpha}(k) v^{\alpha'}(k') e^{i(k-k')\cdot x},
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
f_3 = \frac{1}{2} \sum_{k,k'} \sum_{\alpha,\alpha'} & (\omega_{k'} - \omega_k) \\
\times & \left[\langle a(q,j) | d_\alpha(k) b_{\alpha'}(k') | b(l,n) \rangle \right. \\
& \times v^{\dagger\alpha}(k) u^{\alpha'}(k') e^{i(k+k')\cdot x} \\
& - \langle b(l,n) | b_\alpha^\dagger(k) d_{\alpha'}^\dagger(k') | a(q,j) \rangle \\
& \left. \times u^{\dagger\alpha}(k) v^{\alpha'}(k') e^{-i(k+k')\cdot x} \right],
\end{aligned} \tag{11}
$$

and

$$
\begin{aligned}
f_4 = \frac{1}{2} \sum_{k,k'} \sum_{\alpha,\alpha'} & (\omega_{k'} - \omega_k) \\
\times & \left[\langle b(l,n) | d_\alpha(k) b_{\alpha'}(k') | a(q,j) \rangle \right. \\
& \times v^{\dagger\alpha}(k) u^{\alpha'}(k') e^{i(k+k')\cdot x} \\
& - \langle a(q,j) | b_\alpha^\dagger(k) d_{\alpha'}^\dagger(k') | b(l,n) \rangle \\
& \left. \times u^{\dagger\alpha}(k) v^{\alpha'}(k') e^{-i(k+k')\cdot x} \right].
\end{aligned} \tag{12}
$$

Obviously, the first and the last term in Eq. (8) are always positive. Therefore, $\langle \rho \rangle$ can be negative only when the second term is nonvanishing. There are only four such cases.

Case 1. The number of electrons and the number of positrons in $|a\rangle$ are the same as those in $|b\rangle$, respectively. And there is only one different single electron state in these two states. Here only f_1 is nonzero.

Case 2. The number of electrons and the number of positrons in $|a\rangle$ are the same as those in $|b\rangle$, respectively. And there is only one different single

positron state in these two states. Here only f_2 survives.

Case 3. Two states are the same except for that there is one more single electron state and one more single positron state in $|b\rangle$. Here only f_3 does not vanish.

Case 4. Two states are the same except for that there is one more single electron state and one more single positron state in $|a\rangle$. Here only f_4 is not equal to zero.

Only in these four cases can the energy density of the superposition state be negative and all other possible cases all give rise to positive results. Now we will discuss Case 1 and Case 3 in detail to see how negative energy can arise and if certain quantum inequalities can be satisfied. It is easy to see that Case 2 and Case 4 are similar to Case 1 and Case 3, respectively.

(a) Case 1. Let the two different single electron states in $|a(q;j)\rangle$ and $|b(l;n)\rangle$ to be characterized by (k_ϵ, s_ϵ) and (k_τ, s_τ), respectively, and for simplicity, take $k_\epsilon = k_{\epsilon_y}$, $k_\tau = k_{\tau_z}$, $s_\epsilon = 2$, $s_\tau = 1$. Eq. (8) now reads

$$
\begin{aligned}
\langle \rho \rangle = \frac{1}{(1+\lambda^2)V} \\
\times \left[\lambda^2 (E_0 + \omega_{k_{\tau_z}}) + \lambda\beta_1 + (E_0 + \omega_{k_{\epsilon_y}}) \right],
\end{aligned} \tag{13}
$$

where

$$
E_0 = \sum_{r=1}^{q-1} \omega_{k_r} + \sum_{t=1}^{j} \omega_{\bar{k}_t}, \tag{14}
$$

$$
\beta_1 = \frac{k_{\epsilon_y} k_{\tau_z} (\omega_{k_{\epsilon_y}} + \omega_{k_{\tau_z}}) \sin\theta_1}{2\sqrt{\omega_{k_{\epsilon_y}} \omega_{k_{\tau_z}} (\omega_{k_{\epsilon_y}} + m)(\omega_{k_{\tau_z}} + m)}}, \tag{15}
$$

and $\theta_1 = (k_{\epsilon_y} - k_{\tau_z})x$. E_0 is the total energy of $q - 1$ electrons and j positrons. Note that the energy density $\langle \rho \rangle$ is that of a positive constant part plus a part propagating at the speed of light in the spacetime. Therefore, the sign of the energy could depend on the location and time where any measurement is to be taken. From Eq. (13) we know that $\langle \rho \rangle$ will be negative if

$$
\beta_1^2 > 4(E_0 + \omega_{k_{\tau_z}})(E_0 + \omega_{k_{\epsilon_y}}), \tag{16}
$$

and if

$$\frac{-\beta_1 - \sqrt{\beta_1^2 - 4(E_0 + \omega_{k_{\tau_z}})(E_0 + \omega_{k_{\epsilon_y}})}}{2(E_0 + \omega_{k_{\tau_z}})} < \lambda$$

$$< \frac{-\beta_1 + \sqrt{\beta_1^2 - 4(E_0 + \omega_{k_{\tau_z}})(E_0 + \omega_{k_{\epsilon_y}})}}{2(E_0 + \omega_{k_{\tau_z}})}. \quad (17)$$

Let us now discuss if the quantum states could be manipulated to satisfy Eq. (16). In order to show that this is possible, consider the ultrarelativistic limit, $k_{\tau_z}, k_{\epsilon_y} \gg m$. It then follows that

$$\beta_1 = \frac{1}{2}(\omega_{k_{\tau_z}} + \omega_{k_{\epsilon_y}}) \sin\theta_1. \quad (18)$$

Substituting Eq. (18) into Eq. (16), we have

$$\sin^2\theta_1 > \frac{16(E_0 + \omega_{k_{\tau_z}})(E_0 + \omega_{k_{\epsilon_y}})}{(\omega_{k_{\tau_z}} + \omega_{k_{\epsilon_y}})^2}. \quad (19)$$

For Eq. (19) to hold, it is necessary that

$$16(E_0 + \omega_{k_{\tau_z}})(E_0 + \omega_{k_{\epsilon_y}}) \leqslant (\omega_{k_{\tau_z}} + \omega_{k_{\epsilon_y}})^2. \quad (20)$$

Eq. (20) is satisfied if

$$\omega_{k_{\epsilon_y}} \leqslant 7\omega_{k_{\tau_z}} + 8E_0$$
$$- 4\sqrt{(3\omega_{k_{\tau_z}} + 5E_0)(\omega_{k_{\tau_z}} + E_0)}, \quad (21)$$

or

$$\omega_{k_{\epsilon_y}} \geqslant 7\omega_{k_{\tau_z}} + 8E_0$$
$$+ 4\sqrt{(3\omega_{k_{\tau_z}} + 5E_0)(\omega_{k_{\tau_z}} + E_0)}. \quad (22)$$

Therefore, if the quantum states are manipulated in such a way that the above conditions are met and λ is chosen according to Eq. (17), then the energy density for a quantum state of the form (7) can be made negative at some places in space at some time.

(b) Case 3. Let the single positron and electron states in $|b(l, n)\rangle$ that do not exist in $|a(q, j)\rangle$ with be characterized by $(\bar{k}_\tau, \bar{s}_\tau)$ and $(k_\varepsilon, s_\varepsilon)$ respectively and further take $k_\varepsilon = k_{\varepsilon_y}, \bar{k}_\tau = \bar{k}_{\tau_z}, s_\varepsilon = 1$ and $\bar{s}_\tau = 2$ as an example to study how negative energy density arises in this case. Now, the energy density becomes

$$\langle \rho \rangle = \frac{1}{(1 + \lambda^2)V}$$
$$\times \left[\lambda^2 (E_a + \omega_{k_{\varepsilon_y}} + \omega_{\bar{k}_{\tau_z}}) + \lambda\beta_3 + E_a \right], \quad (23)$$

where

$$E_a = \sum_{r=1}^{q} \omega_{k_r} + \sum_{t=1}^{j} \omega_{\bar{k}_t}, \quad (24)$$

$$\beta_3 = \frac{(\omega_{\bar{k}_{\tau_z}} - \omega_{k_{\varepsilon_y}})k_{\varepsilon_y}}{2\sqrt{\omega_{k_{\varepsilon_y}}(\omega_{k_{\varepsilon_y}} + m)}}\sqrt{\frac{\omega_{\bar{k}_{\tau_z}} + m}{\omega_{\bar{k}_{\tau_z}}}}\sin\theta_3, \quad (25)$$

and $\theta_3 = (\bar{k}_{\tau_z} + k_{\varepsilon_y})x$. Note that here again the energy density $\langle \rho \rangle$ is that of a positive constant part plus a part propagating at the speed of light in the spacetime. It is easy to see that $\langle \rho \rangle$ will be negative if

$$\beta_3^2 > 4(E_a + \omega_{k_{\varepsilon_y}} + \omega_{\bar{k}_{\tau_z}})E_a \quad (26)$$

and

$$\frac{-\beta_3 - \sqrt{\beta_3^2 - 4(E_a + \omega_{k_{\varepsilon_y}} + \omega_{\bar{k}_{\tau_z}})E_a}}{2(E_a + \omega_{k_{\varepsilon_y}} + \omega_{\bar{k}_{\tau_z}})} < \lambda$$

$$< \frac{-\beta_3 + \sqrt{\beta_3^2 - 4(E_a + \omega_{k_{\varepsilon_y}} + \omega_{\bar{k}_{\tau_z}})E_a}}{2(E_a + \omega_{k_{\varepsilon_y}} + \omega_{\bar{k}_{\tau_z}})}. \quad (27)$$

In the ultrarelativistic limit,

$$\beta_3 = \frac{1}{2}(\omega_{\bar{k}_{\tau_z}} - \omega_{k_{\varepsilon_y}}) \sin\theta_3. \quad (28)$$

Substituting Eq. (28) into Eq. (26) yields

$$\sin^2\theta_3 > \frac{16(E_a + \omega_{k_{\varepsilon_y}} + \omega_{\bar{k}_{\tau_z}})E_a}{(\omega_{\bar{k}_{\tau_z}} - \omega_{k_{\varepsilon_y}})^2}. \quad (29)$$

For the above inequality to admit a solution, we must require that

$$16(E_a + \omega_{k_{\varepsilon_y}} + \omega_{\bar{k}_{\tau_z}})E_a \leqslant (\omega_{\bar{k}_{\tau_z}} - \omega_{k_{\varepsilon_y}})^2. \quad (30)$$

And this is satisfied if

$$\omega_{\bar{k}_{\tau_z}} \leqslant \omega_{k_{\varepsilon_y}} + 8E_a 4\sqrt{2\omega_{k_{\varepsilon_y}}E_a + 5E_a^2}, \quad (31)$$

or

$$\omega_{\bar{k}_{\tau_z}} \geqslant \omega_{k_{\varepsilon_y}} + 8E_a 4\sqrt{2\omega_{k_{\varepsilon_y}}E_a + 5E_a^2}. \quad (32)$$

Henceforth, if the quantum states are manipulated in such a way that the above conditions are met and λ is chosen according to Eq. (27), then it is possible to produce energy density for a quantum state of the form (7) at some places in space at some time.

It is interesting to note that the conditions derived above do not apply when $\omega_{\bar{k}_{\tau_z}} = \omega_{k_{\varepsilon_y}}$, and when this happens β_3 is zero, thus the energy density is positive. This reveals that in the center of mass frame of the electron–positron pair in the state $|b(l, n)\rangle$, the local energy density for the superposition state of the form (7) is always a positive constant. Therefore for a given particle content of the state, whether it is possible to detect negative energies is dependent upon the frame in which any measurement is to be carried out. This suggests that the sign of the energy density for a quantum state may well be a coordinate-dependent quantity. It is worth noting that the question of the observer dependence of negative energy for scalar fields was also discussed in two dimensions [25].

3. Negative energy and quantum inequalities

In the last section, we have found that under certain conditions, the energy density of the superposition state of two multi-particle states can be negative. Now, we want to demonstrate that the larger the magnitude of this negative energy, the shorter the duration that it persists. For simplicity, we will consider the ultra-relativistic limit with $\omega_{k_{\tau_z}} \gg \omega_{k_{\epsilon_y}}$ and $\omega_{k_{\tau_z}} \gg E_0$ for Case 1 and $\omega_{k_{\tau_z}} \gg E_a$ for Case 3, then both Eqs. (13) and (23) become

$$\langle \rho \rangle = \frac{\lambda \omega_{k_{\tau_z}}}{(1 + \lambda^2)V}\left[\lambda + \frac{1}{2}\sin \omega_{k_{\tau_z}}(t - x)\right] \quad (33)$$

and the condition for negative energy to arise is now $-1/2 < \lambda < 0$. Therefore, the energy density is that of a constant positive background plus propagating wave at the speed of light that alternates between negative and positive. At a fixed spatial point, the total energy can dip to negative for a certain period of time. The minimum value of $\langle \rho \rangle$ at a fixed point x is given by

$$\langle \rho \rangle_{\min} = \frac{\lambda \omega_{k_{\tau_z}}}{(1 + \lambda^2)V}\left(\lambda + \frac{1}{2}\right). \quad (34)$$

At the same time, the length of time when the energy density is negative is

$$\Delta t = \frac{(\pi - 2\sin^{-1} 2|\lambda|)}{\omega_{k_{\tau_z}}} = \frac{2}{\omega_{k_{\tau_z}}}\cos^{-1}\left(2|\lambda|\right)$$
$$= \frac{2\phi}{\omega_{k_{\tau_z}}}, \quad (35)$$

where $\phi \in (0, \pi)$. One can see that the larger the magnitude of the negative energy $-\langle \rho \rangle_{\min} V$ (or equivalently the larger $\omega_{k_{\tau_z}}$), the shorter its duration. In fact, we have

$$V|\langle \rho \rangle_{\min}|\Delta t = -\frac{\lambda(2\lambda + 1)}{(1 + \lambda^2)}\phi \leqslant -\frac{\lambda(2\lambda + 1)\pi}{(1 + \lambda^2)}$$
$$= \pi g(\lambda). \quad (36)$$

The function $g(\lambda)$ attains a maximum value of $\sqrt{5}/2 - 1$, leading to that $\pi g(\lambda) \approx 0.37$. Therefore, the negative energy satisfies the following quantum inequality

$$E \Delta t \leqslant 1, \quad (37)$$

where we have defined that $E = V|\langle \rho \rangle_{\min}|$. This implies that the amount of negative energy that passes by a fixed point in time Δt is less than the quantum energy uncertainty on that time scale, Δt^{-1}. It prevents attempts of using quantum matter to produce bizarre macroscopic effects. Finally, let us note that we can show, in essentially the same way as in Ref. [21], that the sampled energy density for the superposition states

$$\hat{\rho} = \frac{t_0}{\pi}\int_{-\infty}^{\infty}\frac{\langle \rho \rangle}{t^2 + t_0^2}\, dt \quad (38)$$

in the limits we considered above satisfies the quantum inequality which was originally proven for scalar and electromagnetic fields.

4. Conclusion

We have examined the energy densities of quantum states that are the superposition of two multi-electron–positron states. We have found that the energy densities can be negative only when these two states have the same number of electrons and positrons or when one state has one more electron–positron pair than the other and they are just positive constants for all the other possible cases. In the cases in which negative energy could arise, we have shown that the energy is that of a positive constant plus a propagating part which oscillates between positive and negative, and if the quantum states are properly manipulated, the energy can dip to negative at some places for a certain period of time. It has been demonstrated that the negative energy densities satisfy the quantum inequality, which means

that the product of its magnitude and its duration is less than unity. Last but not the least, we would like to note that in the case in which one state has one more electron–positron pair, the energy density is a positive constant in the center-mass frame of the pair in the state even it can be negative in other frames. Therefore, for a given particle content, the detection of negative energy is an operation that depends on the frame where any measurement is to be performed. This suggests that the sign of energy density for a quantum state may be a coordinate-dependent quantity in quantum theory.

Acknowledgements

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