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Pomeron fan diagrams with an infrared cutoff and running coupling

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Abstract

By direct numerical calculations the influence of a physically relevant infrared cutoff and running coupling on the gluon density and structure function of a large nucleus is studied in the perturbative QCD approach. It is found that the infrared cutoff changes the solutions very little. Running of the coupling produces a bigger change, considerably lowering both the saturation momentum and values of the structure functions.

1. Introduction

In high-colour perturbative QCD the interaction of a probe with the nucleus at low x is described by a non-linear BFKL-like Balitsky–Kovchegov evolution equation, which sums pomeron fan diagrams [1–3]. There has been considerable activity to study the resulting nuclear structure functions and gluon distributions [4–8]. However one has to remember that the BFKL dynamics [9] put at the basis of this equation is only an approximation, whose validity is restricted to not too small values of transverse momenta, where the mere notion of gluons becomes meaningless. Also the BFKL dynamics uses the development in powers of the fixed coupling constant. In the leading order it does not take into account the running of the strong coupling constant at all. Studies of the next-to-leading order, which takes into account terms linear in $\ln(q^2/\Lambda^2_{\mathrm{QCD}})$, have indicated that the fixed coupling constant of the leading order indeed starts to run according to the standard QCD rules [10]. So a reasonable first approximation seems to be the leading order in the running coupling constant, rather than in the fixed one. In view of these limitations inherent in the perturbative approach to small x physics we consider it fruitless to study properties of the BK equation as it stands at very low momenta (or large spatial distances). In particular its generalization to include pomeron dimensions much greater than those of the target nucleus and derive consequences as to the behaviour of the resulting amplitudes in the limit $x \to 0$ [11] are certainly interesting from the mathematical point of view but have little physical relevance in our opinion. Rather one has to study the dependence of the

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equation and its solutions on the infrared region and running of the coupling. Should such dependence be very strong, the results obtained in the current approach, where the coupling is fixed and no infrared cutoff is introduced, would have little physical sense. Note that these problems has been extensively studied for the original BFKL equation ([12,13] and references therein).

The first numerical solutions of the non-linear evolution equation have shown that the resulting gluon density is concentrated at quite high values of momenta, around the so-called saturation momentum $Q_s(Y)$ which grows exponentially with the rapidity $Y = \ln(1/x)$. This gives some hope that the non-linear equation is not sensitive to the infrared region at all and in this way retains a full physical meaning in the realistic world with confinement, unlike the original BFKL equation, in which at sufficiently low x the well-known diffusion into the infrared region [14] inevitably involves unphysical gluons of extremely small momenta and thus the problem of confinement. However this point has not been fully proven, due to the fact that the equation itself does not involve the gluon density itself but some integral of it, which is not at all negligible in the infrared region and, in fact, grows logarithmically towards small momenta. In this Letter we intend to study the infrared dependence of the non-linear evolution equation by direct numerical calculations. Our results confirm that the solutions at small enough x indeed depend on the infrared region only weakly.

Another point which we study is inclusion of the running coupling. Clearly this cannot be done in any rigorous way. We employ a simple intuitive model for the running of the coupling, taking it dependent on the smallest momentum in a given 3-gluon vertex. Our calculations show that with the running coupling the solutions do not change qualitatively, but the quantitative change is quite noticeable. In particular the slope of the dependence of the saturation momentum on Y drops by a factor 2–3, so that its values go down by 4 orders of magnitude at largest Y studied. The resulting structure functions also drop by an order of magnitude. We consider these results quite promising, since the values of Q_s obtained from the original non-linear equation, without infrared cutoff nor running of the coupling, are very large and grow unreasonably fast with Y.

2. The non-linear evolution equation with an infrared cutoff and running coupling

The non-linear evolution equation derived in [2,3] for an extended target reads

$$\left(\frac{\partial}{\partial y} + H\right)\phi(y,k) = -\phi^2(y,k). \tag{1}$$

Here $y = (\alpha_s N_c/\pi)Y$, H is the BFKL Hamiltonian and $\phi(y,k)$ is a Fourier transform of $\Phi(Y,r)/(2\pi r^2)$ where Φ has a meaning of the cross-section for the scattering of a colour dipole on a target at a given impact parameter b. In fact both ϕ and Φ also depend on b through the initial condition at $y = 0$. Eq. (1) is infrared stable, that is, preserves its meaning when k varies over the whole positive axis. Of course in numerical calculations one has to limit these values at both small and large k. Typically in our calculations [3,4] we chose $k_{\min} \sim 10^{-15}$ GeV/c and $k_{\max} \sim 10^{+40}$ GeV/c. With these values the solution does not change when the interval of k is taken still larger. Obviously these cutoffs served to a purely calculational purpose and the obtained solutions in fact correspond to the completely uncut equation.

The physical infrared cutoff has to be of the order $\Lambda_{\rm QCD} \sim 0.3$ GeV/c. We may introduce it into the equation in two different ways. One is simply to cut the allowed values of k to $k > k_{\min}$ and choose $k_{\min}$ to be around $\Lambda_{\rm QCD}$ (a "hard cutoff" choice). With such a cutoff the momenta of the intermediate real gluons are not cut and may be arbitrary small. To also cut these latter, one may introduce an effective gluon mass m_g of the same order in all gluon propagators and leave the overall cutoffs on momenta the same as in the original equation (a "soft cutoff choice"). As we shall see, our numerical calculations show that the resulting solutions are rather similar for both choices. In calculations we varied both $k_{\min}$ and m_g in the interval 0.3–0.6 GeV. No big difference was observed inside this interval. Below we report on the results with $k_{\min} = m_g = \Lambda_{\rm QCD} = 0.3$ GeV.

Passing to the running coupling, we recall that this is an unsolved problem even for the linear BFKL equation. To see the qualitative features of the solution, all one can do is to introduce the running coupling in a purely intuitive way, based on scale arguments. The 3-gluon BFKL vertex inside the BFKL Hamiltonian depends on the three gluon momenta k_1, k_2, k_3, two of the virtual gluons and the third of the emitted real gluon. We choose to introduce the running coupling for the vertex at a scale which is the smallest of the squares of these three momenta $k_0^2 = \min\{k_1^2, k_2^2, k_3^2\}$. This choice can be understood as follows. Assume that $k_1^2 \sim k_2^2 \gg k_3^2$. Then obviously $k_1 \simeq k_2$ and k_3 is orthogonal to them both. Passing to the system where $k_1 \simeq k_2 = 0$ we find that the vertex depends only on k_3^2. So for two momenta of the same order and larger than the third, the coupling has to depend on the smaller momentum. Our choice is a generalization to a situation where all three momenta may have different orders of magnitude. Using the soft cutoff we have further to define the coupling for values of momenta below $\Lambda_{\rm QCD}$. Our choice is to freeze the coupling below some scale, for which we take the same scale $\Lambda_{\rm QCD}$. To do this we change in the denominator of the running coupling $\alpha_s(k)$

$$\ln \frac{k^2}{\Lambda_{\rm QCD}^2} \to \ln\left(\frac{k^2}{\Lambda_{\rm QCD}^2} + c\right)$$

with a constant c chosen to have the desired frozen value of α_s at $k^2 = \Lambda_{\rm QCD}^2$. In our numerical calculations we have taken the frozen value of the coupling constant $\alpha_s^{(0)} = 0.2$.

3. Numerical results

3.1. Gluon distribution

We define the gluon distribution as in [3]

$$\frac{dx\,G(x,k^2)}{d^2b\,dk^2} = \frac{N_c}{2\pi^2\alpha_s} h(y,k),$$
$$h(y,k) = k^2 \nabla_k^2 \phi(y,k). \qquad (2)$$

Our aim is to study the influence of the cutoff and running coupling on this distribution. For this aim the dependence on the impact parameter is irrelevant, so that we shall assume $\phi(y=0,k)$ and consequently $\phi(y,k^2)$ independent of b. Physically it corresponds to assuming a constant nuclear profile function (approximation of a “cylindrical nucleus”). For the initial distribution, following [4], we chose the Golec-Biernat–Wuesthoff [15] form

$$\phi(0,k) = -\frac{1}{2}\,\mathrm{Ei}\left(-\frac{k^2}{0.3657}\right), \qquad (3)$$

where k^2 is in $(\mathrm{GeV}/c)^2$. In our calculations we compare four cases: no cutoff, no running coupling (case A), hard cutoff, no running coupling (case B), soft cutoff, no running coupling (case C) and finally soft cutoff, running coupling (case D). In all cases the gluon distribution turns out to have a sharp maximum at a certain “saturation momentum” $Q_s(y)$, which grows with y. With a fixed coupling (cases A–C) to a good precision $Q_s(y) \propto \exp(\Delta y)$. The slope Δ results practically independent of the introduced infrared cutoff and its value lies between 2.2 and 2.3. However with a running coupling $Q_s(y)$ grows with y much slowlier and not as the exponential, the slope Δ diminishing from 1.0 at $y = 5$ to 0.63 at $y = 10$. As a result, values of the saturation momentum with a running coupling are much lower than with a fixed one. In Fig. 1 we show the saturation momentum $Q_s(y)$ for the described four cases. With the running coupling the scaled rapidity y has been defined as $y = (\alpha_s^{(0)} N_c/\pi)Y$, which implies that for comparison the fixed coupling has been taken equal to 0.2 for cases A–C. The form of the gluon distribution is shown in Figs. 2 and 3 where $h(y,k)$ is plotted against the scaling variable $z = k/Q_s(y)$. In Fig. 2 the distributions are presented for a relatively small rapidity $y = 3$, when the initial conditions are not completely forgotten yet. One then observes a small difference in the three curves for fixed coupling (A–C) especially noticeable at small values of z. The running coupling curve is considerably different: it is narrower and its peak is larger. This difference becomes still more pronounced at higher rapidities, which is illustrated in Fig. 3, where we show distributions for all four cases at $y = 6$ and $y = 10$ simultaneously. All fixed coupling curves practically coincide for both rapidities, showing a clear scaling behaviour, which has been discovered earlier for the original solution without cutoffs [3,4,8]. So at these y the influence of the physically relevant infrared cutoff is totally forgotten: the gluon density does not change and remains scale invariant in spite of the introduction of a scale of the order 0.3 GeV/c. This is exactly what was conjectured in [3]: at not too small y the internal scale Q_s generated by the non-linear dynamics is much larger than infrared cutoffs, be it 0.3 GeV/c or much smaller.

On other hand, with the running coupling, the form of the gluon distribution changes considerably. Peaks

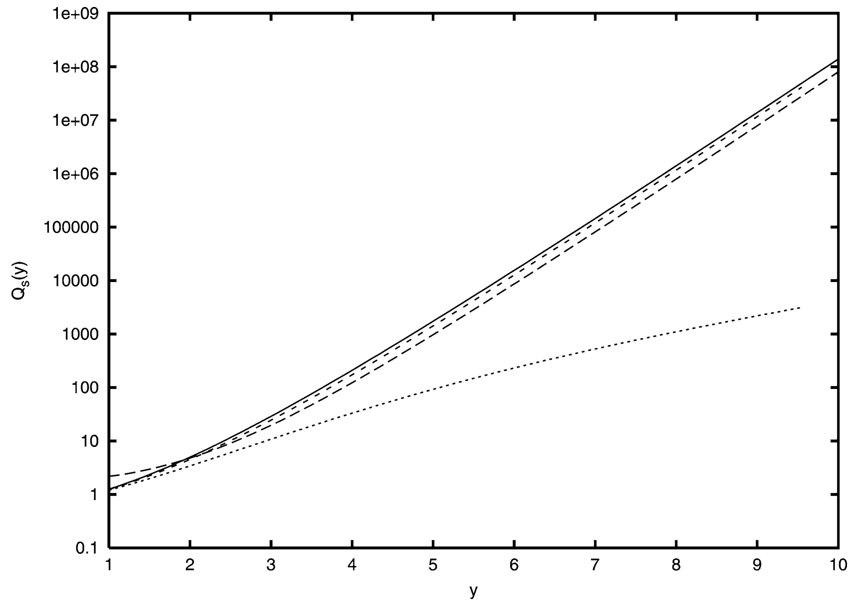

Fig. 1. The saturation momentum $Q_s(y)$ as a function of $y = (\alpha_s N_c/\pi)\ln(1/x)$. Curves from top to bottom correspond to cases A, C, B and D (see the text). The lower curve corresponds to the running coupling (case D).

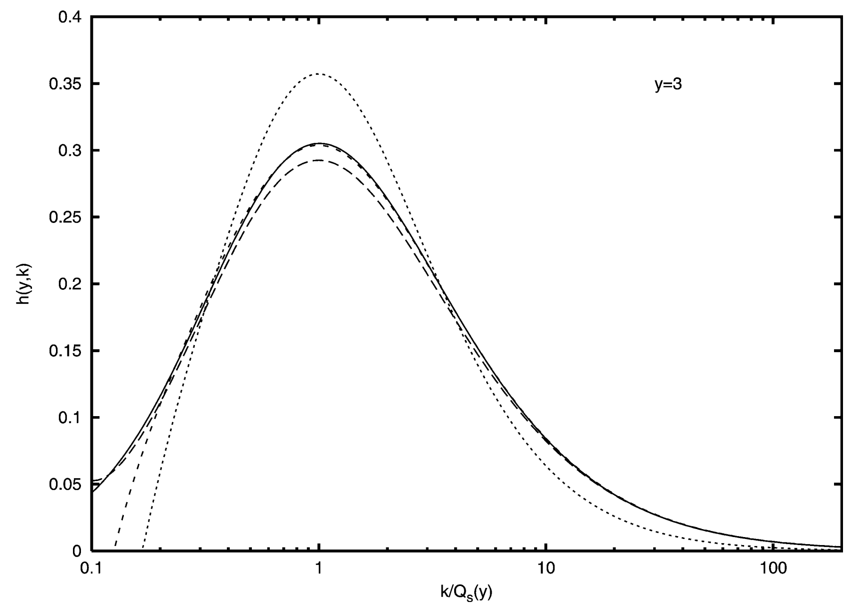

Fig. 2. The gluon distribution as a function of the scaling variable $k/Q_s(y)$ at $y = 3$. Curves with maxima from top to bottom correspond to cases D, A, C and B (the first referring to the running coupling).

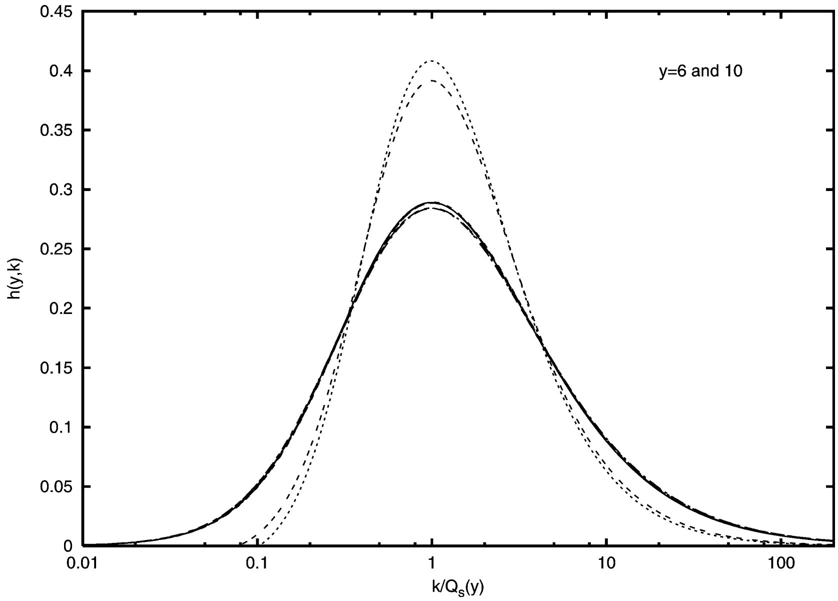

Fig. 3. Same as in Fig. 2 for $y = 6$ and 10. The two upper curves correspond to the running coupling (the uppermost for $y = 6$). All the rest correspond to fixed coupling, either with an infrared cutoff or without it.

of the running coupling curves become nearly twice larger than for a fixed coupling and the curves themselves become much narrower. However the difference between the running coupling curves at $y = 6$ and 10 is quite small, which indicates that even with a running coupling, to a good approximation, scaling in z is still observed.

3.2. Structure function

To see more physically noticeable consequences of the cutoffs and running coupling we also calculated the structure function of Pb for the mentioned 4 cases. For the initial function (now depending on b) we have chosen the same eikonalized Golec-Biernat–Wuesthoff distribution which was used in our earlier calculations according to the original equation, so that we could read the results for case A directly from [4]. Our results are presented in Figs. 4 and 5 for $Q^2 = 100$ and $10\,000$ $(\mathrm{GeV}/c)^2$, respectively. The change due to the physical infrared cutoff is now more pronounced: the structure functions with such a cutoff are somewhat smaller than without cutoff, the difference growing with Q^2. At $Q^2 = 10\,000$ $(\mathrm{GeV}/c)^2$ introduction of a infrared cutoff lowers the structure function by ~ 2 times at low x. Still the behaviour in x remains practically unchanged. A more significant change occurs with a running coupling. The structure function then grows with $1/x$ considerably slowlier. Its values at $Q^2 = 10\,000$ $(\mathrm{GeV}/c)^2$ and small x become ~ 4 times smaller than without cutoffs and running coupling and this difference seems to be growing at still smaller x.

4. Conclusions

By direct numerical calculations we studied the gluon density and structure function of a large nucleus at small x which follow from the non-linear evolution equation with a physically reasonable infrared cutoff and also with a running coupling. Our results show that the gluon density does not change qualitatively. In all cases it has a strong peak at a certain saturation momentum, $Q_s(y)$, which grows with $\ln(1/x)$. The introduction of an infrared cutoff of the order 0.3 GeV/c by itself does not practically change the value of Q_s nor the form of the gluon distribution around it. Running of the coupling, on the other hand, does change both: $Q_s(y)$ grows with $\ln(1/x)$ much slowlier and not as an exponential, the gluon distribution becomes narrower

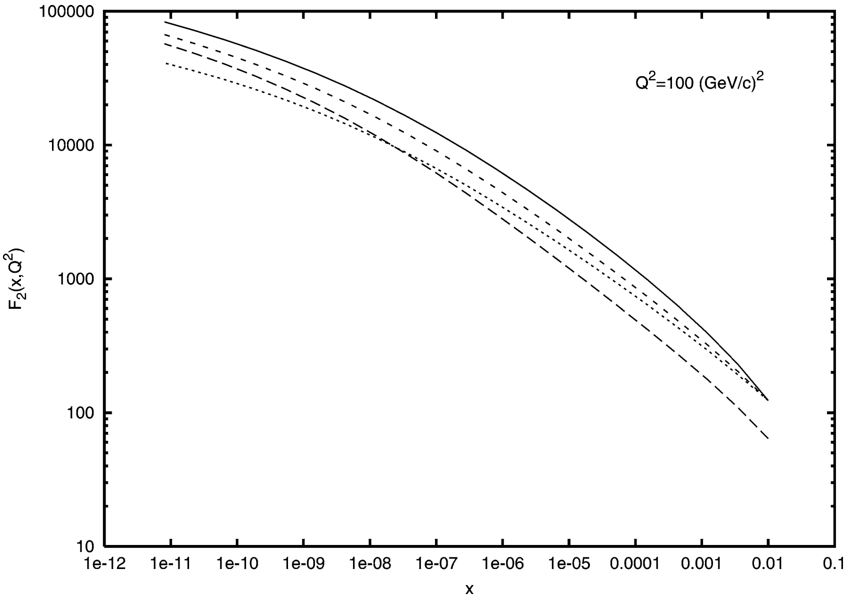

Fig. 4. The structure function of Pb at $Q^2 = 100$ $(\mathrm{GeV}/c)^2$. Curves from top to bottom on the left correspond to cases A, C, B and D (the last for the running coupling).

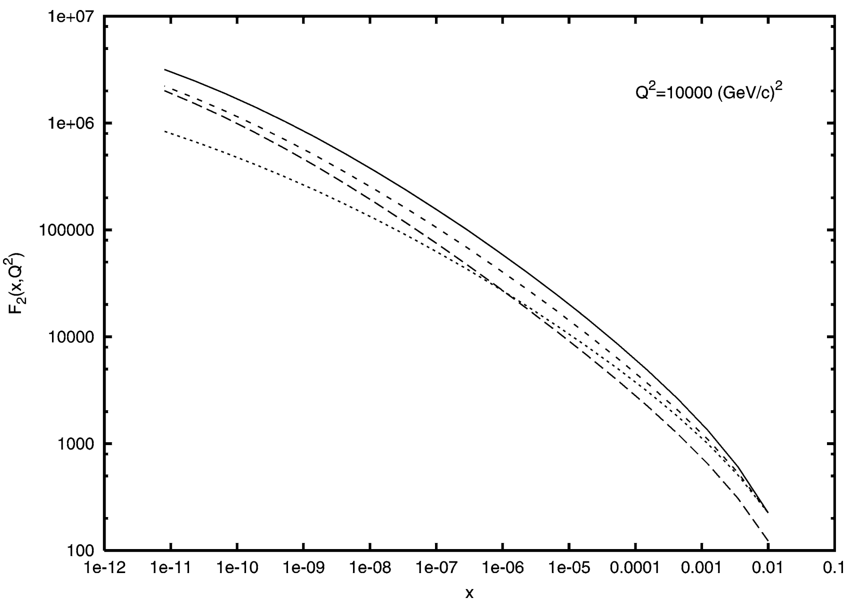

Fig. 5. Same as Fig. 4 for $Q^2 = 10\,000$ $(\mathrm{GeV}/c)^2$.

and its height greater. Still in all cases scaling of the distribution in k/Q_s is preserved.

The structure functions go down with the introduction of an infrared cutoff and especially with a running coupling. In the latter case the growth of the structure function with $1/x$ is found to be considerably slower.

Our results confirm that the non-linear evolution equation is more or less infrared stable, in contrast to the linear BFKL equation. Changes introduced by an infrared cutoff are of no qualitative nature and of minor quantitative influence. Running of the coupling produces a somewhat bigger change. It is important that this change is in the right direction: fixed coupling solutions lead to a very fast growth of $Q_s(y)$ with y and as a consequence to unreasonably large values for it. Introduction of the running coupling considerably improves the situation.

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