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Mathematical model for computing precise local tie vectors for CMONOC co-located GNSS/VLBI/SLR stations ☆

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ABSTRACT

The seven co-located sites of the Crustal Movement Observation Network of China (CMONOC) in Shanghai, Wuhan, Kunming, Beijing, Xi'an, Changchun, and Urumqi are equipped with Global Navigation Satellite System (GNSS), very long baseline interferometry (VLBI), and satellite laser ranging (SLR) equipment. Co-location surveying of these sites was performed in 2012 and the accuracies of the solved tie vectors are approximately 5 mm. This paper proposes a mathematical model that handles the least squares adjustment of the 3D control network and calculates the tie vectors in one step, using all the available constraints in the adjustment. Using the new mathematical model, local tie vectors can be more precisely determined and their covariance more reasonably estimated.

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1. Introduction

Sites equipped with two or more space geodesy instruments such as the Global Navigation Satellite System (GNSS), very long baseline interferometer (VLBI), and satellite laser ranging (SLR), are called co-located sites, which are essential for connecting diverse space geodetic techniques via local tie vectors for computing the International Terrestrial Reference Frame (ITRF) [1]. A local tie vector is a 3D baseline vector between the reference points (RPs) of two telescopes [2,3]. An RP is the perpendicular intersection point of the primary fixed axis and secondary axis of a telescope [4], which can be approximately regarded as the geometric rotation center of an SLR or VLBI telescope, or the antenna reference point (ARP) of a GNSS antenna [5]. To determine the RP of an SLR or VLBI telescope, targets mounted on the telescope need to be measured during rotation sequences around the primary and secondary axes. Methods for computing RPs using a target's observed coordinates are found in references [2,3,5–7].

The 7 co-located sites of the Crustal Movement Observation Network of China (CMONOC) are shown in Fig. 1. All the sites are equipped with GNSS instruments; the sites in Shanghai, Kunming, Beijing, Xi'an, Changchun, and Wuhan are also equipped with SLR instruments, and the sites in Shanghai, Kunming, and Urumqi are equipped with VLBI instruments. The tie vectors at Shanghai, Wuhan, and Beijing were measured several years ago [8–12]. To measure the targets fixed on the SLR and VLBI telescopes, at least two and four control points, respectively, are required and these were determined by first establishing a 3D control network. A precise terrestrial survey of CMONOC co-located sites was performed from September to November 2011 using both GNSS and conventional terrestrial measurements, and details of the field work can be referred from Gong [5].

In their tie vector solutions, Gong [5] and Ma [13] implicitly assume that the primary axis of a VLBI or SLR telescope intersects with and is perpendicular to its secondary axis, the deflection of the vertical can be neglected when computing the rotation center, and the rotation centers around the primary axis of different targets fixed on a telescope have the same horizontal coordinates, while those around the secondary axis have the same vertical coordinates. Besides, the least squares adjustment of the 3D control network and the determination of the RP were performed separately, and some conditions such as the distance between two targets not being changed during rotation of the telescopes were neglected. Moreover, Gong [5] also assumed that the coordinates of the rotation centers of different rotation circles were independent. Since these assumptions cause approximately 3 mm of error on the solved tie vector [3], this paper introduces a new mathematical model for obtaining precise local tie vector solutions.

2. Mathematical models for solving CMONOC local tie vectors

The GNSS vectors of the 3D control points were computed using GAMIT v10.35 and Bernese v5.0 Software, which use the absolute phase center variation models. Then, the 3D GNSS vectors, the terrestrial observations of the control network, and the target points were solved together in Gong [5] and Ma [13] using the 3D least squares adjustment by fixing the IGS stations as the initial values. Therefore, the 3D coordinates of all the target points were derived in the 3D adjustment. Since each target rotating around an axis forms a plane circle, it gives rise to two constraining conditions [14–16]:

$$a\overline{x}_i + b\overline{y}_i + c\overline{z}_i + d = 0 \tag{1}$$

and

$$(\overline{x}_i - u)^2 + (\overline{y}_i - v)^2 + (\overline{z}_i - w)^2 = r^2 \tag{2}$$

where a, b, c, d denote the parameters forming a plane, u, v, w are the coordinates of the rotation center and r is its radius,

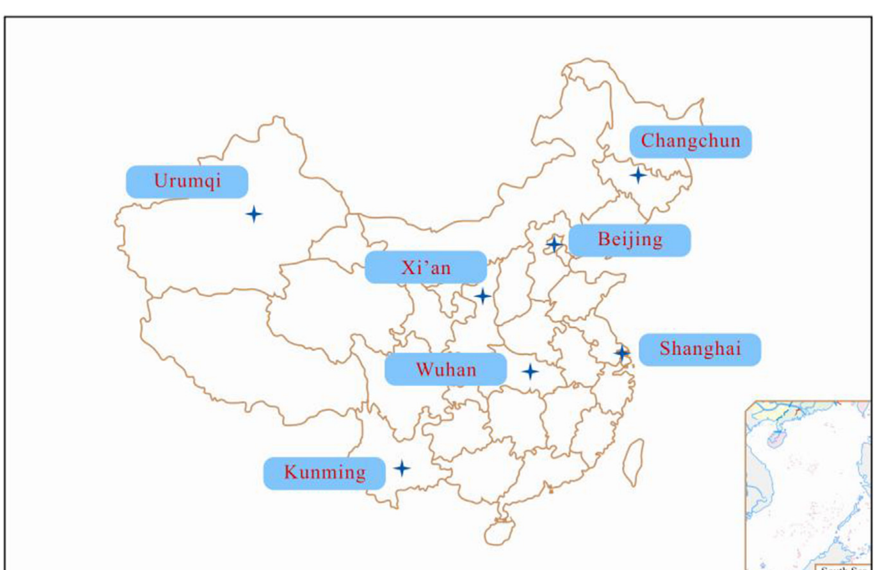

Fig. 1 – Locations of seven co-located sites in China.

$\bar{x}_i, \bar{y}_i, \bar{z}_i$ are the coordinates of target point i, which can be expressed as x_i, y_i, z_i derived by 3D least squares adjustment and corrections υ_{xi}, υ_{yi}, υ_{zi}. Thereby, equations (1) and (2) for all points can be linearized and expressed as the following pseudo observation equation,

$$\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{\upsilon} = \boldsymbol{w}_1 \tag{3}$$

where $\boldsymbol{x}$ is the vector of parameters, $\boldsymbol{A}$ is its design matrix, $\boldsymbol{\upsilon}$ denotes the correction vector of a target's coordinates, $\boldsymbol{B}$ is its design matrix, $\boldsymbol{w}_1$ is the misclosure vector of the constraining equations. The solution $\boldsymbol{x}$ of (3) can be derived using the least squares adjustment with its covariance matrix $\boldsymbol{\Sigma}_x$. If a total of m_1 and m_2 solutions are derived for the circles rotating around the primary and secondary axes, respectively, the coordinates of an RP computed by Gong [5] are,

$$\begin{pmatrix} N \\ E \end{pmatrix} = \frac{1}{m_1}\begin{pmatrix} -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ -\sin\lambda & \cos\lambda & 0 \end{pmatrix}\begin{pmatrix} \sum_{i=1}^{m_1} u_i^p & \sum_{i=1}^{m_1} v_i^p & \sum_{i=1}^{m_1} w_i^p \end{pmatrix}^{\mathrm{T}} \tag{4}$$

and $U = \dfrac{1}{m_2}(\cos\varphi\cos\lambda \;\; \cos\varphi\sin\lambda \;\; \sin\varphi)\begin{pmatrix} \sum_{i=1}^{m_2} u_i^s & \sum_{i=1}^{m_2} v_i^s & \sum_{i=1}^{m_2} w_i^s \end{pmatrix}^{\mathrm{T}}$ (5)

where N, E, and U are the coordinates of RPs in the terrestrial topocentric coordinate system, φ, λ are the geodetic latitude and longitude of the rotation center of the primary axis, and (u_i^p, v_i^p, w_i^p) and (u_i^s, v_i^s, w_i^s) are the coordinates of the rotation centers around the primary and secondary axes, respectively. Since the covariance matrices of the rotation centers have already been derived in the least squares adjustment, the covariance matrix of an RP can be easily derived from (4) and (5) using the law of error propagation. The simple average of the coordinates of the rotation centers in (4) and (5) imply that these coordinates are independent; however, they are actually correlated. Ma [13] introduced the following additional conditions to establish a direct relationship between the coordinates of RPs and the coordinates of rotation centers:

$$\begin{aligned} &\sin\varphi\cos\lambda u_i^p + \sin\varphi\sin\lambda v_i^p - \cos\varphi w_i^p - \sin\varphi\cos\lambda x_0 \\ &\qquad -\sin\varphi\sin\lambda y_0 + \cos\varphi z_0 = 0 \\ &\sin\lambda u_i^p - \cos\lambda v_i^p - \sin\lambda x_0 + \cos\lambda y_0 = 0 \\ &\cos\varphi\cos\lambda u_i^s + \cos\varphi\sin\lambda v_i^s + \sin\varphi w_i^s \\ &\qquad -\cos\varphi\cos\lambda x_0 - \cos\varphi\sin\lambda y_0 - \sin\varphi z_0 = 0 \end{aligned} \tag{6}$$

where x_0, y_0, z_0 denotes the coordinate of an RP. The conditions in (6) can be expressed as,

$$\boldsymbol{C}\boldsymbol{x} + \boldsymbol{C}_0\boldsymbol{x}_0 = \boldsymbol{w}_2 \tag{7}$$

where $\boldsymbol{x}_0 = (x_0, y_0, z_0)^{\mathrm{T}}$ is the coordinate of an RP, $\boldsymbol{C}$ and $\boldsymbol{C}_0$ are the coefficient matrices, and $\boldsymbol{w}_2$ is the misclosure vector. Solving the pseudo observation equation (3) and constraining it with (7) can directly provide the coordinate of an RP as well as its covariance. Therefore, the mathematical model of Ma [13] is an improved version of Gong [5]. However, the impacts of the deflections of the vertical are all neglected in equations (4), (5), and (7).

3. Results analysis

The local tie vectors (ΔX, ΔY, ΔZ) and their formal errors ($M_{\Delta X}$, $M_{\Delta Y}$, $M_{\Delta Z}$) in the ITRF2008 frame solved by Gong [5] are presented in Table 1, where BJFS, CHAN KUNM, SHAO, GUAO, WUHN, and XIAA denote the GNSS stations at the Beijing, Changchun, Kunming, Shanghai, Urumqi and Xi'an sites, respectively; the SLR and VLBI denote SLR and VLBI stations at those sites in addition to GNSS stations. The formal errors of all the coordinate components in Table 1 are less than 5 mm.

Ma [13] only solves the tie vectors for the Shanghai and Changchun sites, and these results and their discrepancies from Gong [5] are presented in Tables 2 and 3, respectively. The formal errors of Ma [13] are slightly smaller than those of Gong [5], and the largest discrepancy of the coordinate components is up to 0.6 mm.

The tie vectors at the Shanghai and Wuhan sites were also surveyed by the Institute Géographique National (IGN) in 2003 [9,10], and the discrepancies between Gong [5] and IGN are shown in Table 4. The discrepancy in the three coordinate components at the Wuhan site in Table 4 are all less than 10 mm, although the distance of the tie vector is up to 13 km and the surveying data of Gong [5] is approximately 10 years

Table 1 – Tie vectors and formal errors by Gong [5].

Tie vector	ΔX (m)	ΔY (m)	ΔZ (m)	$M_{\Delta X}$ (mm)	$M_{\Delta Y}$ (mm)	$M_{\Delta Z}$ (mm)
BJFS-SLR	−16.5166	118.3174	−146.2835	1.25	1.90	0.27
CHAN-SLR	40.2996	46.0158	−13.3399	0.35	1.01	0.17
KUNM-VLBI	103.1364	118.3366	−226.3731	2.37	0.62	2.90
KUNM-SLR	−20.2160	−18.8560	45.7754	0.39	0.72	1.24
SHAO-SLR	989.0580	914.3549	−296.5724	0.42	0.96	0.77
SHAO-VLBI	46.3460	67.6428	−41.8153	0.71	1.43	1.12
GUAO-VLBI	−68.5363	−24.1483	35.5471	0.66	4.90	0.54
WUHN-SLR	−11964.9994	−4386.8925	−1496.7445	4.68	2.02	1.37
XIAA-SLR	−14.8656	14.6918	−28.0790	2.03	1.01	0.87

Table 2 – Tie vectors and formal errors by Ma [13].

Tie vector	ΔX (m)	ΔY (m)	ΔZ (m)	$M_{\Delta X}$ (mm)	$M_{\Delta Y}$ (mm)	$M_{\Delta Z}$ (mm)
CHAN-SLR	40.3000	46.0155	−13.3405	0.2	0.2	0.2
SHAO-SLR	989.0582	914.3543	−296.5728	1.4	0.6	0.5
SHAO-VLBI	46.3462	67.6422	−41.8154	0.8	0.8	0.9

Table 3 – Discrepancies in tie vectors between Gong [5] and Ma [13] (unit: mm).

Tie vector	ΔX	ΔY	ΔZ
CHAN-SLR	0.4	−0.3	−0.6
SHAO-SLR	0.2	−0.6	0.4
SHAO-VLBI	0.2	−0.6	−0.1

Table 4 – Discrepancies of the tie vectors between Gong [5] and IGN (unit: mm).

Vector	ΔX	ΔY	ΔZ	ΔN	ΔE	ΔU
SHAO-VLBI	−10.0	17.4	10.2	−1.6	−0.5	22.4
WUHN-SLR	−2.5	−8.9	−5.6	−1.2	5.9	−8.9

late. At the Shanghai site, the discrepancies of the north and east (*N*, *E*) coordinate components are less than 2 mm, and the discrepancy of the vertical (*U*) coordinate component is as large as 22.4 mm.

4. Mathematical model for precise local tie vector solutions

As pointed out in Gong [5], the formal errors of their tie vectors are approximately 5 mm, because their field surveying and data processing methods only guarantee the precision of the tie vectors to approximately 5 mm. However, the ITRF innovation requires local tie vectors to achieve a precision of 2 mm or less. In addition, the field surveying work and data processing should be more rigorous. In this paper, we only discuss the mathematical model for obtaining precise local tie vector solutions. According to Dawson [3], neglecting the inter-axis and inter-circle geometrical conditions causes an error of 1.2–3.4 mm. Therefore, besides the conditions of (1) and (2), the following inter-axis and inter-circle conditions must be taken into account when computing precise local tie vectors.

First, all the rotation circles around the primary axis have the same normal vector denoted by $\boldsymbol{n}_p = (a_p, b_p, c_p)^T$. If the secondary axis doesn't change when it rotates for different circles, all circles also have the same normal vector expressed as $\boldsymbol{n}_s = (a_s, b_s, c_s)^T$. Furthermore, the same target also has the same intercepts for different circles, denoted by d_p and d_s for the circles around the primary and secondary axes, respectively. These conditions should be taken into account when solving the conditional equation (1).

Second, the same target must have the same rotation center and radius, denoted by (u_p, v_p, w_p) and r_p for circles around the primary axis, and (u_s, v_s, w_s) and r_s for those around the secondary axis. These conditions should also be taken into account when solving the conditional equation (2).

Third, when a telescope rotates, the distance between two targets is fixed. The following condition for targets *i* and *j* should be further introduced:

$$\left(\overline{x}_k^i-\overline{x}_k^j\right)^2+\left(\overline{y}_k^i-\overline{y}_k^j\right)^2+\left(\overline{z}_k^i-\overline{z}_k^j\right)^2=\left(\overline{x}_l^i-\overline{x}_l^j\right)^2+\left(\overline{y}_l^i-\overline{y}_l^j\right)^2+\left(\overline{z}_l^i-\overline{z}_l^j\right)^2 \tag{8}$$

where *k* and *l* denote the two positions of the telescope.

Fourth, the deflections of the vertical must be considered. If the deflection of the vertical is 10 s, its impact will be approximately 0.5 mm when computing the coordinates of an RP for a telescope with a radius of 10 m. Therefore, the conditional equations (4) and (5) or (6) cannot be used to obtain precise tie vector solutions. To determine the coordinates of an RP, the conditions of a linear line formed by the rotation center are first introduced for the primary axis:

$$\begin{pmatrix} u_p^i \\ v_p^i \\ w_p^i \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda_p^i \begin{pmatrix} a_p \\ b_p \\ c_p \end{pmatrix} \tag{9}$$

where λ_p^i is the scale parameter of the line, x_0, y_0, z_0 denote the coordinate of the RP, since RP is on the linear line. Similarly, the expression for the secondary axis is

$$\begin{pmatrix} u_s^i \\ v_s^i \\ w_s^i \end{pmatrix} = \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} + \lambda_s^i \begin{pmatrix} a_s \\ b_s \\ c_s \end{pmatrix} \tag{10}$$

where λ_s^i is the scale parameter of the line of the secondary axis, x_s, y_s, z_s represents the coordinate of a point on the secondary axis with the least distance to the primary axis. On the other hand, the secondary axis does not intersect the primary axis, and neglecting the offset between the two axes introduces an error of approximately 0.5 mm [3].

Fifth, the line with the minimum the distance from the secondary axis to the primary axis must be perpendicular to both axes. Therefore, its normal vector $\boldsymbol{n}$ is orthogonal to both $\boldsymbol{n}_p$ and $\boldsymbol{n}_s$, and can be computed as

$$\boldsymbol{n} = \boldsymbol{n}_p \times \boldsymbol{n}_s = \begin{pmatrix} b_p c_s - b_s c_p \\ a_s c_p - a_p c_s \\ a_p b_s - a_s b_p \end{pmatrix} \tag{11}$$

Because x_0, y_0, z_0 and x_s, y_s, z_s are all on the line, we have the following conditions:

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} + \lambda \begin{pmatrix} b_p c_s - b_s c_p \\ a_s c_p - a_p c_s \\ a_p b_s - a_s b_p \end{pmatrix} \tag{12}$$

where λ is the scale parameter to be estimated. Obviously, in equations (9), (10) and (12), nine unknowns have been introduced, i.e., x_0, y_0, z_0 and x_s, y_s, z_s as well as λ_p^i, λ_s^i, and λ. Even observing one circle around the primary axis and one circle around the secondary axis give nine conditions in equations (9), (10) and (12). Therefore, these unknowns can also be solved with the conditions.

All the above conditions can be briefly expressed as,

$$Ax + By = w \tag{13}$$

where x is the vector of parameters introduced by the conditional equations, A is its design matrix; y denotes the vector of parameters of the 3D control network, B is its design matrix; w is a misclosure vector. The observational equation of the 3D control network is denoted as,

$$l = Hy + \epsilon \tag{14}$$

where l is the observation vector with the observation error ϵ, and H is the design matrix. Because terrestrial observations, such as horizontal and vertical angles and leveling height differences, are determined based on the plumb line, the parameters of the deflections of the vertical must be introduced in the vector y. The observational equation (14) constrained by the conditional equation (13) is solved by least squares adjustment. We emphasize here that the coordinates of RPs and their covariance are directly computed in the adjustment, since the coordinates of RPs are introduced in the equations (9) and (12).

5. Conclusions

This study analyzed the solutions of Gong [5] and Ma [13] for determining the tie vectors of the seven co-located sites of the CMONOC. The accuracies of the solved tie vectors by Gong [5] and Ma [13] are approximately 5 mm, and the discrepancies of the tie vectors between them are up to 0.6 mm, which does not fulfill the requirement of obtaining precise local tie vector solutions. A mathematical model was presented for obtaining precise tie vector solutions in this study. The key points of our mathematical model are summarized as follows: 1) The 3D control network adjustment and the RP determination are processed in one step; 2)Except for the constraints of equations (1) and (2), the distances between different targets are fixed when the telescope rotates, and normal vectors as well as the distances between the rotation centers are also fixed in different rotation circles; 3)The offsets between the primary and secondary axes must be taken into account; 4)The deflections of the vertical cannot be neglected. With the new mathematical model, local tie vectors can be more precisely determined and the covariance matrix more reasonably estimated.

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