ELSEVIER
Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 645 (2007) 1–5
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

If Gauss–Bonnet interaction plays the role of dark energy

Abhik Kumar Sanyal [1]

Department of Physics, Jangipur College, Murshidabad, West Bengal 742213, India

Received 3 October 2006; received in revised form 20 November 2006; accepted 27 November 2006

Available online 15 December 2006

Editor: M. Cvetič

Abstract

A cosmological model has been constructed with Gauss–Bonnet-scalar interaction, where the Universe starts with exponential expansion but encounters infinite deceleration, $q \to \infty$ and infinite equation of state parameter, $w \to \infty$. During evolution it subsequently passes through the stiff fluid era, $q = 2$, $w = 1$, the radiation dominated era, $q = 1$, $w = 1/3$ and the matter dominated era, $q = 1/2$, $w = 0$. Finally, deceleration halts, $q = 0$, $w = -1/3$, and it then encounters a transition to the accelerating phase. Asymptotically the Universe reaches yet another inflationary phase $q \to -1$, $w \to -1$. Such evolution is independent of the form of the potential and the sign of the kinetic energy term, i.e., even a non-canonical kinetic energy is unable to phantomize ($w < -1$) the model.

1. Introduction

In the recent years lot of observations have been carried out that lead to a precise knowledge of the cosmological evolution. Important cosmological observations like abundance of galaxy clusters [1], statistics of large scale redshift surveys [1], angular power spectrum of cosmic microwave background radiation (CMBR) [2] and baryon oscillations [3] suggest that the Universe is nearly flat and almost 73% of the matter density is in the form of dark energy [4]. Further, the magnitude-redshift relation from standard candles such as type Ia supernovae (SnIa) [5] indicates that the Universe has recently entered a phase of accelerating expansion. Reconciling all these astronomical observations it is now quite clear that the so-called dark energy is slowly varying with negative pressure [6] having repulsive properties that can encounter the attractive gravitational force and the corresponding equation of state parameter ($w = p/\rho$) is presently pretty close to minus one (-1) [7]. So we can now brief the knowledge of the cosmological evolution that we have gathered so far from all the astronomical observations. Soon after the beginning, the Universe passed through a phase of exponential expansion (inflation) and during evolution it went through the stiff fluid era—when pressure balanced the energy density ($p = \rho$, i.e., $w = 1$. Thereafter, it encountered the radiation dominated era ($p = \rho/3$, i.e., $w = 1/3$) and finally the Universe entered the pressureless dust era ($p = 0$, i.e., $w = 0$). Presently, as already mentioned, it is accelerating with the equation state parameter w pretty close to minus one (-1) [7]. So far, all the attempts made to construct different dark energy models of the Universe encompassing all the observable phenomena went in vain [6,8]. An alternative approach to accommodate dark energy is to modify General Theory of Relativity by including additional curvature invariant terms such as Gauss–Bonnet (GB) term in the gravitational action. GB term arises naturally as the leading order of the α' expansion of heterotic superstring theory, where, α' is the inverse string tension [9]. Some interesting results appear in the literature with GB interaction. Avoidance of naked singularities in dilatonic brane world scenarios and the problem of fine tuning with scalar fields and GB interaction have been discussed in [10]. Further in string induced gravity near initial singularity, GB coupling with scalar field has been found [11] important for the occurrence of nonsingular cosmology. In addition, there are also some recent investigations [12] in the context of dark energy models. In these works, issues like phantom cos-

E-mail addresses: abhikkumar@gmail.com, sanyal_ak@yahoo.com (A.K. Sanyal).

[1] Also at Relativity and Cosmology Research Centre, Department of Physics, Jadavpur University, Calcutta 700032, India.

0370-2693
doi:10.1016/j.physletb.2006.11.070

mology with GB correction, interplay between GB term and quintessence scalar, experimental constraints on astronomical and cosmological observations and cosmological models with scalar dependent GB interaction have been addressed.

Gauss–Bonnet term is a topological invariant one and so to get some contribution in the four-dimensional space–time it requires dynamic dilatonic scalar coupling. In this work we shall consider such coupling and investigate cosmological consequence.

The Letter has been organized as follows. In the following section we write the action and the field equations. In Section 3, we explore a set of solutions with negative GB interaction. It is found that such solutions require non-canonical kinetic energy. In Section 4, we present another set of solutions for both positive and negative GB interacting terms. It has been observed that even a non-canonical kinetic energy might evolve through to a canonical one, without affecting the cosmic evolution.

2. Action and the field equations

We start with the following action containing Gauss–Bonnet interaction

$$S=\int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2}+\frac{\Lambda(\phi)}{8}G(R)-g(\phi)\phi_{,\mu}\phi'^{\mu}-V(\phi)\right], \tag{1}$$

where, $G(R)=R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss–Bonnet term which appears in the action with a coupling parameter $\Lambda(\phi)$. In the action there is yet another coupling parameter viz., $g(\phi)$. For the spatially flat Robertson–Walker space–time

$$ds^2=-dt^2+a^2(t)\left[dr^2+r^2\,d\theta^2+r^2\sin^2\theta\,d\phi^2\right],$$

the field equations are

$$2\frac{\ddot a}{a}+\frac{\dot a^2}{a^2}=-\kappa^2\left[g\dot\phi^2-V(\phi)+2\Lambda'\dot\phi\frac{\dot a\ddot a}{a^2}+\left(\Lambda'\ddot\phi+\Lambda''\dot\phi^2\right)\frac{\dot a^2}{a^2}\right]=-8\pi Gp, \tag{2}$$

$$3\frac{\dot a^2}{a^2}=\kappa^2\left[g\dot\phi^2+V(\phi)-3\Lambda'\dot\phi\frac{\dot a^3}{a^3}\right]=8\pi G\rho, \tag{3}$$

where, p and ρ are respectively the effective pressure and the energy density generated by the scalar field and the Gauss–Bonnet interaction. In addition we have got the ϕ variation equation

$$2g\left(\ddot\phi+3\frac{\dot a}{a}\dot\phi+\frac{1}{2}\frac{g'}{g}\dot\phi^2+\frac{V'}{2g}\right)=3\Lambda'\frac{\dot a^2\ddot a}{a^3}, \tag{4}$$

which may not be considered to be an independent equation, since it is derivable from the above two equations (2) and (3). In the above, over-dot and prime ($'$) stand for differentiations with respect to the proper time t and ϕ respectively. Now, we are to solve for a, ϕ, $g(\phi)$, $V(\phi)$ and $\Lambda(\phi)$ in view of the above two field equations (2) and (3), which requires three additional assumptions. The first assumption that we make is

$$\Lambda'\dot\phi=\lambda, \tag{5}$$

where λ is a constant, which is physically reasonable, since it implies that the Gauss–Bonnet coupling parameter $\Lambda(\phi(t))=\lambda t$, grows in time, and as a result it might contribute at the later epoch of cosmological evolution. Such an assumption also mathematically simplifies the field equations (2) and (3) considerably, which can be expressed as,

$$2\dot H+3H^2=-\kappa^2\left[g\dot\phi^2-V(\phi)+2\lambda H\dot H+2\lambda H^3\right]=-\kappa^2p, \tag{6}$$

$$3H^2=\kappa^2\left[g\dot\phi^2+V(\phi)-3\lambda H^3\right]=\kappa^2\rho, \tag{7}$$

where, $H=\dot a/a$ is the Hubble parameter. Now eliminating $V(\phi)$ between equations (6) and (7) we find,

$$\dot H+\kappa^2\left[g\dot\phi^2+\lambda H\dot H-\frac{\lambda}{2}H^3\right]=0. \tag{8}$$

One can also eliminate $g\dot\phi^2$ between the same pair of equations to express the potential as,

$$V=\frac{3}{\kappa^2}H^2+\frac{5\lambda-n^2}{2}H^3-\frac{\kappa^2n^2\lambda}{2}H^4. \tag{9}$$

Hence, we shall now deal with any three of the equations (5), (7), (8) and (9) to solve the field variables a, ϕ, the potential $V(\phi)$ along with the parameters g and λ of the theory, for which we shall require two suitable assumptions in addition. Two important parameters that we deal with, in the cosmological context, are the equation of state ($w=\frac{p}{\rho}$) and the deceleration ($q=-\frac{a\ddot a}{\dot a^2}$) parameters, which are expressed as,

$$w=-1-\frac{2\dot H}{3H^2}\quad\text{and}\quad q=-1-\frac{\dot H}{H^2}=\frac{1+3w}{2}. \tag{10}$$

3. Solution with negative Gauss–Bonnet interaction

The Gauss–Bonnet term interacts with the dilatonic scalar field through $\Lambda(\phi)$ which may appear with either the signs—positive or negative. To find explicit solutions, we impose the following condition,

$$g\dot\phi^2+\lambda H\dot H=0. \tag{11}$$

Under the above choice, Eq. (8) gets solved immediately to yield

$$H=\frac{1}{\kappa}\sqrt{\frac{2}{\lambda(t_s-t)}},$$

where, t_s is a constant. The Hubble parameter H in the above solution increases with time and finally faces the big-rip singularity [13]. However, the higher order correction to the Einstein–Hilbert action considered here, in the form of Gauss–Bonnet interaction eventually avoids such cataclysm through negative interaction, under a simple requirement, $\lambda=-n^2$. Thus the above solution is now given by,

$$H=\frac{1}{\kappa n\sqrt{t}}\quad\text{and}\quad a=a_0e^{\frac{2\sqrt{t}}{\kappa n}}, \tag{12}$$

where we have considered positive sign only, for both κ and n or the product of the two, to ensure expanding model and t_s has been set equal to zero without loss of generality. Now in view of Eq. (10) we can find the state and the deceleration parameters as

$$w = -1 + \frac{\kappa n}{3\sqrt{t}} \quad \text{and} \quad q = -1 + \frac{\kappa n}{2\sqrt{t}}. \tag{13}$$

It is to be noted that the above set of solutions (12) and (13) depends on the constant n, which determines the GB coupling parameter $\Lambda(\phi)$. Thus at the very early stage of the evolution of the Universe, i.e., at $t \to 0$, $q \to \infty$ and $w \to \infty$. However, at $w = 1$ velocity of sound equals that of light, and $w > 1$ does not carry any sense classically. It therefore suggests the need of some quantum treatment at this epoch. Nevertheless, in order to correlate such an exponential expansion with an era of inflation, it is required to see if the Universe is pushed towards a flat geometry and that if it generates approximately a scale invariant spectrum. However, at $t = \frac{\kappa^2 n^2}{36}$, the Universe enters stiff fluid era with $q = 2$ and $w = 1$. Now, if quantum cosmology is invoked at $t < \frac{\kappa^2 n^2}{36}$, then the constant n is related with the Planck's time scale and so the above set of solutions is free from undetermined parameters. Radiation dominated era starts at $t = \frac{\kappa^2 n^2}{16}$, when $q = 1$ and $w = \frac{1}{3}$. At $t = \frac{\kappa^2 n^2}{9}$, the Universe becomes dust filled, with $q = \frac{1}{2}$ and $w = 0$. There is a transition from the decelerating to the accelerating phase at $t = \frac{\kappa^2 n^2}{4}$, when $q = 0$ and $w = -\frac{1}{3}$. Finally with the increase of the proper time, $q \to -1$ and $w \to -1$ asymptotically. It is most important to notice that such evolution from the early to the late time of the Universe, encompassing all the experimental observations, is independent of the form of the potential $V(\phi)$. Gauss–Bonnet term has been introduced to account for the dark energy and so it is expected to modify the late time behavior of the cosmological evolution. However, interestingly enough, it has been found to play a dominant role even at the early stage of the cosmic evolution, when other kinds of matter, viz., radiation and cold dark matter would have been important. On the contrary, we have observed that the scalar field that enters into the coupling of Gauss–Bonnet term behaves as different kinds of matter at different epoch.

Now in view of Eq. (11) we get

$$g\dot{\phi}^2 = -\frac{\kappa^2 n^2}{2} H^4 = -\frac{1}{2\kappa^2 t^2}. \tag{14}$$

Thus the above solutions demand $g(\phi)$ has to be negative, so the kinetic term appears with a wrong sign. It is rather interesting to note that even a wrong sign of kinetic energy does not phantomize [13] the cosmological model under consideration, i.e., the state parameter w never goes beyond -1. However, to find explicit solutions of the model under consideration, we have to further fix up either the field variable ϕ, or one of the parameters among $g(\phi)$, $\Lambda(\phi)$ and the potential $V(\phi)$. As an example we consider the most natural choice, viz., $g = -\frac{1}{2}$. Thus Eq. (14) gets solved for ϕ as

$$\phi = \frac{\ln t}{\kappa}. \tag{15}$$

Eq. (5) now solves $\Lambda(\phi)$ as

$$\Lambda = -n^2 e^{\kappa\phi}, \tag{16}$$

and Eq. (7) solves the potential as

$$V = \frac{1}{2\kappa^4 n^2}\left[6e^{-\kappa\phi} + \kappa^2 n^2 e^{-2\kappa\phi} - 6\kappa n e^{-\frac{3\kappa\phi}{2}}\right]. \tag{17}$$

It is to be mentioned that the GB coupling parameter Λ is expressed (e.g., Mavromatos and Rizos in [10]) in the form,

$$\Lambda = -\lambda_0 e^{l\phi}, \tag{18}$$

where, $l = -4/\sqrt{6}$, in four dimensions. So, instead of fixing g, if one chooses Λ as in (18), then the solutions are found as

$$\phi = \frac{1}{l}\ln\left(\frac{n^2 t}{\lambda_0}\right) \quad \text{and} \quad g = -\frac{l^2}{2\kappa^2}, \tag{19}$$

while,

$$V = \frac{1}{\kappa^4\lambda_0}\left[3e^{-l\phi} + \frac{\kappa^2 n^4}{2\lambda_0}e^{-2l\phi} - \frac{3\kappa n^2}{\sqrt{\lambda_0}}e^{-\frac{3l\phi}{2}}\right]. \tag{20}$$

It is to be noted that the GB coupling parameter $\Lambda(\phi)$ and the potential $V(\phi)$ carry exponents with opposite signs automatically, and asymptotically the potential becomes a constant. As mentioned, one can also choose different forms of the potential to find explicit solutions which we shall not consider in this section. What we have observed is that the negative interaction with Gauss–Bonnet term leads to a non-canonical kinetic energy, which has got some interest in the context of phantom cosmology [13], but not in general. In the following section we study both positive and negative interactions and also the transition from non-canonical to canonical kinetic energy.

4. Solution with both signs of Gauss–Bonnet interaction

In the previous section we have observed that the condition (11) led to negative GB interaction which ultimately made the kinetic energy non-canonical. In this section we make a different assumption so that the GB interaction $\Lambda(\phi)$ may be positive as well. Let us consider,

$$g\dot{\phi}^2 + \lambda H\dot{H} - \frac{\lambda}{2}H^3 = \frac{n_1^2}{2}H^3, \tag{21}$$

where, n_1 is a constant. As a result, Eq. (8) is solved to yield

$$H = \frac{1}{\kappa n_1\sqrt{t}} \quad \text{and} \quad a = a_0 e^{\frac{2\sqrt{t}}{\kappa n_1}}. \tag{22}$$

Further, Eq. (10) can be expressed as,

$$w = -1 + \frac{\kappa n_1}{3\sqrt{t}} \quad \text{and} \quad q = -1 + \frac{\kappa n_1}{2\sqrt{t}}. \tag{23}$$

Thus we obtain the same set of solutions as in the previous section with the difference that the constant n that determined the solutions (12) and (13) in the previous Section 3, determined the GB coupling parameter $\Lambda(\phi)$ too, while in the above solutions (22) and (23) n_1 has nothing to do with $\Lambda(\phi)$. Thus λ here may be positive as well as negative and so is the GB interaction parameter $\Lambda(\phi)$. The other difference is that, we do not encounter the big-rip catastrophe here. Once again the product of κ and n_1 has been considered positive to ensure expanding model. The Universe evolves in the same manner as discussed in the previous section, starting from an exponential expansion with infinite deceleration and subsequently passing through the phases of stiff fluid, radiation dominated and the matter dominated era. It then finally encounters a transition to the accelerating phase when $w=-1/3$ and asymptotically reaches $q=-1$, $w=-1$. We would like to mention once again that such cosmological evolution remains independent of the form of potential $V(\phi)$. Eq. (21) now reduces to

$$g\dot{\phi}^2=\frac{1}{\kappa^2 n_1^2}\left[\frac{\lambda}{t^2}+\frac{\lambda+n_1^2}{\kappa n_1 t^{3/2}}\right]. \tag{24}$$

To find explicit solutions, we can explore different situations fixing up either $g(\phi)$, $\Lambda(\phi)$ or the potential $V(\phi)$. In the following we cite a few examples.

Case I. The most natural choice, $g=1/2$ leads to a well behaved solution, with

$$\phi=\frac{4}{(\kappa n_1)^{3/2}}\Bigg[\left\{\lambda\kappa n_1+\left(\lambda+n_1^2\right)\sqrt{t}\right\}^{1/2} + \frac{\sqrt{\lambda\kappa n_1}}{2}\ln\left\{\frac{\sqrt{\lambda\kappa n_1+(\lambda+n_1^2)\sqrt{t}}-\sqrt{\lambda\kappa n_1}}{\sqrt{\lambda\kappa n_1+(\lambda+n_1^2)\sqrt{t}}+\sqrt{\lambda\kappa n_1}}\right\}\Bigg]. \tag{25}$$

The potential can be expressed as a function of time as,

$$V=\frac{1}{\kappa^2 n_1^2}\left[\frac{3}{\kappa^2 t}+\frac{5\lambda-n_1^2}{2\kappa n_1 t\sqrt{t}}-\frac{\lambda}{2t^2}\right]. \tag{26}$$

However, it is extremely difficult to express the potential V and the GB interaction parameter Λ as functions of ϕ.

Case II. Next let us choose Λ as,

$$\Lambda=\frac{\beta}{\phi}. \tag{27}$$

Thus ϕ can be solved in view of Eq. (5) as,

$$\phi=\frac{\beta}{\lambda t}, \tag{28}$$

which decreases while Λ increases linearly with time. $g(\phi)$ can be found in view of Eq. (24) as,

$$g=\sqrt{\frac{\beta}{\lambda}}\left(\frac{n_1^2+\lambda}{\kappa^3 n_1^3}\right)\phi^{-5/2}+\frac{\lambda}{2\kappa^2 n_1^2}\phi^{-2}. \tag{29}$$

Thus, $g(\phi)\to\infty$ asymptotically. Now the potential takes the following form,

$$V=\frac{\lambda}{\kappa^2 n_1^2\beta}\left[\frac{3}{\kappa^2}\phi+\sqrt{\frac{\lambda}{\beta}}\left(\frac{5\lambda-n_1^2}{2\kappa n_1}\right)\phi^{3/2}-\frac{\lambda^2}{2\beta}\phi^2\right]. \tag{30}$$

It is interesting to observe that if one sets $\lambda=-m^2$, then β has to be negative making the Gauss–Bonnet interaction parameter negative. For such a situation, with $\beta=-c^2$, we have

$$g=\frac{c}{m}\left(\frac{n_1^2-m^2}{2\kappa^3 n_1^3}\right)\phi^{-5/2}-\frac{m^2}{2\kappa^2 n_1^2}\phi^{-2}. \tag{31}$$

Hence, for $n_1^2>m^2$, $g(\phi)<0$ at the beginning but becomes positive at a later stage of the cosmic evolution. So a non-canonical kinetic energy turns canonical at

$$\phi<\frac{c^2(n_1^2-m^2)^2}{\kappa^2 n_1^2 m^6},\quad \text{i.e., at}\quad t>\frac{\kappa^2 n_1^2 m^4}{(n_1^2-m^2)^2}. \tag{32}$$

This proper time has nothing to do with the transitions of the state parameter or the deceleration parameter in general. So even though a non-canonical kinetic energy evolves through to a canonical one, it does not play any significant role in the evolution of the Universe and the state parameter w always remains over the phantom divide line. However, by properly choosing m in terms of n_1, e.g., choosing $m=n_1/\sqrt{3}$, one can relate the time of flipping of the sign of the kinetic energy term to the time of transition of a decelerating Universe to an accelerating one.

Case III. Let us now choose Λ in the form

$$\Lambda=\beta e^{l\phi}, \tag{33}$$

where, β is a constant. So, in view of Eqs. (5) and (24)

$$\phi=\frac{1}{l}\ln\left(\frac{\lambda}{\beta}t\right)\quad\text{and}\quad g=\frac{l^2}{2\kappa^2 n_1^2}\left[\lambda+\sqrt{\frac{\beta}{\lambda}}\left(\frac{\lambda+n_1^2}{2\kappa n_1}\right)e^{\frac{l}{2}\phi}\right], \tag{34}$$

while the potential can be found from Eq. (9) as,

$$V=\frac{\lambda}{2\kappa^4 n_1^4\beta^2}\Big[6\beta n_1^2 e^{-l\phi}-\kappa^2 n_1^2\lambda^2 e^{-2l\phi} + \kappa n_1\sqrt{\beta\lambda}\left(5\lambda-n_1^2\right)e^{-\frac{3l\phi}{2}}\Big]. \tag{35}$$

For, $l=-4/\sqrt{6}$, ϕ becomes negative but that does not create any problem whatsoever, and the potential carries positive exponents. As before, here again we can choose $\lambda=-m^2$, $\beta=-c^2$ and $l=-s^2$ and find

$$g=\frac{s^4}{2\kappa^2 n_1^2}\left[-m^2+\frac{n_1^2-m^2}{2\kappa n_1}\left(\frac{c}{m}\right)e^{-\frac{s^2\phi}{2}}\right]. \tag{36}$$

Now since $\phi=\frac{1}{s^2}[\ln(\frac{c^2}{m^2 t})]$, so at $t\to 0$, $\phi\to\infty$ and $g<0$. It has a turning point at a later epoch $t=\frac{4\kappa^2 n_1^2 m^4}{(n_1^2-m^2)^2}$ after which g becomes positive provided $n_1>m$. So, here again we encounter the same situation as discussed in Case II, that the solutions remain unaffected even though a non-canonical kinetic energy evolves through to a canonical one.

Case IV. Here we choose a simple quadratic form of the potential, viz.,

$$V(\phi)=\lambda_0\phi^2=\frac{V_0^2}{2\kappa^2}\phi^2. \tag{37}$$

As a result,

$$\phi = \frac{1}{\kappa n_1 V_0}\left[\frac{6}{\sqrt{t}} - \frac{\kappa(n_1^2 - 5\lambda)}{t} - \frac{\kappa^2\lambda}{t\sqrt{t}}\right]^{1/2}. \tag{38}$$

Here, for a positive GB interaction $\lambda > 0$, $\Lambda > 0$ even an imaginary scalar field evolves to a real one, without affecting the nature of cosmic evolution. In view of Eqs. (5) and (24) it is now in principle possible to find the forms of $\Lambda(\phi)$ and $g(\phi)$.

5. Concluding remarks

A viable cosmological model has been presented in four dimensions considering Gauss–Bonnet-scalar coupling. To explain recent accelerating expansion of the Universe the Gauss–Bonnet term should dominate at the later epoch of cosmological evolution. Therefore under the physically reasonable assumption that the coupling parameter (Λ) grows linearly in time a set of solutions, independent of the signature of the coupling parameter (Λ) has been presented. The solutions demonstrate that the Universe starts with an exponential expansion, but with infinite deceleration ($q \to \infty$) and the equation of state ($w \to \infty$) parameters. In the process of evolution it subsequently passes through the stiff fluid era ($w = 1$), the radiation dominated era ($w = 1/3$) and the matter dominated (pressureless dust) era ($w = 0$). It then encounters a turning point at $w = -1/3$, after which the Universe starts accelerating. Asymptotically, both the deceleration and the equation of state parameters go over to -1. Hence the observations suggest that we are now living at the final stage of cosmological evolution and the dark energy is presently evolving rapidly from $w = -1/3$ to $w = -1$. For a negative coupling parameter ($\Lambda < 0$), the kinetic energy is non-canonical. It has been observed that a non-canonical kinetic energy might evolve to a canonical one without influencing the nature of the solutions in any way, and the equation of state parameter (w) never goes beyond the phantom divide line. Even an imaginary scalar field might evolve to a real one without affecting the cosmological evolution. Einstein–Hilbert action has been modified by the introduction of Gauss–Bonnet term with dynamic dilatonic-scalar coupling to investigate it's role as the dark energy at the later epoch of cosmological evolution. However, it has been found to play a crucial role throughout the cosmological evolution. The dilatonic-scalar plays the role of different kinds of matter at different epoch. The exponential expansion at the early stage of cosmic evolution with $w \to \infty$ and $q \to \infty$ can be treated as inflation only if it can be shown that such solution pushes the Universe towards a flat geometry and eventually if it generates an approximately scale invariant spectrum. This has not been shown in the present work. However, if it does not, then some quantum treatment should be accounted for at this epoch. It will be interesting to see how the introduction of cold dark matter modifies cosmological evolution. Whatsoever, dark energy in the form of Gauss–Bonnet-scalar interaction has been found to play an important role in the cosmological evolution.

References

[1] N.A. Bahcall, et al., Science 284 (1999) 1481;
W.J. Percival, et al., Mon. Not. R. Astron. Soc. 327 (2001) 1297;
M. Tegmark, et al., Phys. Rev. D 69 (2004) 103501;
L. Verde, et al., Mon. Not. R. Astron. Soc. 335 (2002) 432.
[2] D.N. Spergel, et al., Astrophys. J. Suppl. Ser. 148 (2003) 175.
[3] D.J. Eisenstein, et al., Astrophys. J. 633 (2005) 560.
[4] M.S. Turner, M. White, Phys. Rev. D 56 (1997) R4439;
P.H. Frampton, astro-ph/0409166;
T. Padmanabhan, Curr. Sci. 88 (2005) 1057.
[5] A.G. Riess, et al., Astron. J. 116 (1998) 1009;
A.G. Riess, et al., Astrophys. J. 607 (2004) 665;
S.J. Perlmutter, et al., Astrophys. J. 517 (1999) 565;
J.L. Tonry, et al., Astrophys. J. 594 (2003) 1;
R. Knop, et al., Astrophys. J. 598 (2003) 102;
B.J. Barris, et al., Astrophys. J. 602 (2004) 571;
P. Astier, et al., Astron. Astrophys. 447 (2006) 31.
[6] E.J. Copeland, M. Sami, S. Tsujikawa, hep-th/0603057;
L. Perivolaropoulos, astro-ph/0601014.
[7] A. Melchiorri, et al., Phys. Rev. D 68 (2003) 043509.
[8] N. Straumann, hep-ph/0604231.
[9] J. Callan, et al., Nucl. Phys. B 262 (1985) 593;
D.J. Gross, J.H. Sloan, Nucl. Phys. B 291 (1987) 41;
R.R. Metsaev, A.A. Tseytlin, Phys. Lett. B 191 (1987) 354;
M.C. Bento, O. Bertolami, Phys. Lett. B 368 (1995) 198.
[10] N.E. Mavromatos, J. Rizos, Phys. Rev. D 62 (2000) 124004;
N.E. Mavromatos, J. Rizos, Int. J. Mod. Phys. A 18 (2003) 57;
P. Binetruy, et al., Phys. Lett. B 544 (2002) 183;
A. Jakóbek, K.A. Meissner, M. Olechowski, Nucl. Phys. B 645 (2002) 217.
[11] I. Antoniadis, J. Rizos, K. Tamvakis, Nucl. Phys. B 415 (1994) 497;
P. Kanti, J. Rizos, K. Tamvakis, Phys. Rev. D 59 (1999) 083512.
[12] S. Nojiri, S.D. Odintsov, M. Sasaki, Phys. Rev. D 71 (2005) 123509;
S. Nojiri, S.D. Odintsov, Phys. Lett. B 631 (2005) 1;
S. Nojiri, S.D. Odintsov, M. Sami, hep-th/0605039;
G. Cognola, et al., Phys. Rev. D 73 (2006) 084007;
G. Calcagni, S. Tsujikawa, M. Sami, Class. Quantum Grav. 22 (2005) 3977;
M. Sami, et al., Phys. Lett. B 619 (2005) 193;
L. Amendola, C. Charmousis, S.C. Davis, hep-th/0506137;
G. Esposito-Farese, gr-qc/0306018;
T. Koivisto, D.F. Mota, astro-ph/0606078;
B.M.N. Carter, I.P. Neupane, hep-th/0512262;
B.M.N. Carter, I.P. Neupane, hep-th/0510109;
N. Deruelle, C. Germani, Nuovo Cimento B 118 (2003) 977;
I.P. Neupane, hep-th/0602097;
I.P. Neupane, hep-th/0605265.
[13] R.R. Caldwell, Phys. Lett. B 545 (2002) 23;
R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, Phys. Rev. Lett. 91 (2003) 071301;
S. Nesseris, L. Perivolaropoulos, Phys. Rev. D 70 (2004) 123529.