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Fuzzy Logic based Modelling and Simulation Approach for The estimation of Tire Forces

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Abstract

Tire modeling is an important aspect of vehicle dynamics as the forces and moments required to control the vehicle's motion are eventually transmitted through the tire and the tire road interface is also an important source for the dynamic excitation of the vehicle. This paper presents a fuzzy logic based approach for estimating tire forces, aligning moment of tire for the different slip ratio and slip angles. Proposed fuzzy logic approach requires slip angle and slip ratio, as the input variables, and estimates the longitudinal force, lateral force, aligning moment as the output variables. Membership functions of input, output variables and fuzzy rules are formulated based on the values obtained using the widely adopted Magic formula for tire model. Simulation values for longitudinal, lateral forces and aligning moment of the tire using the proposed fuzzy model is found to provide good correlation with the magic model. Proposed fuzzy logic frame work does not require the estimation of model parameters used in the Magic formula and it will be useful in developing vehicle control system.

Keywords: Tire modelling;Fuzzy Logic;Simulink

1. Introduction

Tire is a simple, however, an important element of a vehicle, as the ground interaction of the vehicle occurs through the contact patches of tires. Generally, when the tire makes contact with the road surface, the forces and moments will be generated due to the friction. These are the critical parameters for vehicle handling.

Nomenclature

F_x	longitudinal force (kN)
F_y	lateral force (kN)
M_z	aligning moment (Nm)
α	slip angle (deg)
k	slip ratio

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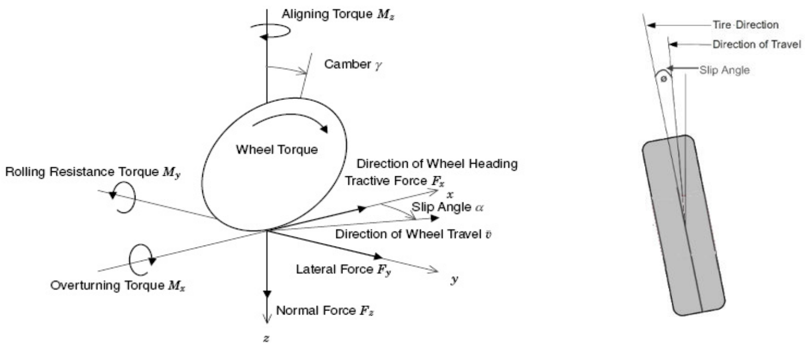
Hence, tire modeling is one of the most important aspects of vehicle dynamics and the knowledge of tire forces is essential for developing a vehicle control system [1]. Forces and torques acting on a tire are shown in Fig. 1 (a). In the aspect of tire modeling and vehicle handling, longitudinal force (F_x), lateral force (F_y) and self-aligning torque (M_z), slip ratio and slip angle are the important parameters [2]. Slip angle (α) is defined as the angle between its direction of motion and the wheel plane as shown in Fig. 1(b). The ratio between the slip velocity and vehicle velocity is defined as slip ratio (k) [3].



(a) Forces and moments acting on a tire [Jacob Svendenius, 2007] (b) Slip angle of a tire
Fig.1. General terminologies used in tire modelling

Various researchers have attempted different methodologies for developing a tire model. James Lacombe explained the factors such as friction/sliding and elastic deformation/slipping of the tire for the generation of tire forces. It is also given that the friction force is decreasing continuously with increasing velocities and also explained about the other aspects of tires such as, tread deflection, carcass/belt deflection, distributive of contact pressure and tire-road properties [4]. Peng et al. explained the tire characteristics and properties such as mechanical strength and adhesion on dry, wet conditions during acceleration and deceleration, service life, dimension and weights [5]. Van der Steer has developed a tire modeling technique based on finite element method which includes the tire/road interaction with friction [6]. Markus Schmid developed a single contact point transient tire model that can handle transient driving situations based on multi body dynamics [7]. Efstathios Velenis et al., derived the LuGre dynamic friction model from longitudinal to longitudinal/lateral motion [8]. In order to reduce the modeling complexity and computation time, Van Roji has restricted the tire modelling to only the contact region using a multi-scale approach for analyzing the tire-road interaction [9]. These models are not expected to give a very accurate correspondence with the measured values of tire forces; however it will predict the qualitative trends of the tire behavior.

New approaches have been developed for tire modeling based on the experimental data and empirical models. Dakhlallah et al., [10] presented a model based estimation approach in which tire-road friction forces are estimated from the vehicle dynamics and the measurements available from the vehicle. Apart from these modelling techniques, Magic Model which is derived by Pacejka [11], which is used to predict and simulate the forces exerted by a tire based on the experimental data. This method requires accurate estimation for a set of model coefficients involved in the empirical model. Mohammad Safwan Burhaumudin et al., analysed the magic formula for both combined slip and pure slip conditions and compared results with the simulation software carsim [12]. Chen Long et al., has done the comparative study between the magic formula and the neural network tire model based on genetic algorithm [13]. Cabrera et al. has given a new comparative method of magic formula with genetic algorithm [14].

It can be seen that the vehicle dynamics is an active area of research. Magic formula is widely used for modelling the tire forces, which requires estimation of model parameters. In order to simplify this task, a fuzzy logic based approach has been proposed in this work. The input parameters of the model are slip angle, slip ratio and the output parameters are longitudinal force, lateral force and aligning moment. The simulation results of the proposed fuzzy model are compared with the Matlab/Simulink model of magic formula.

2. Tire modeling using magic formula

The Magic Formula is a semi-empirical tire model that was introduced by Pacejka [18]. It is called the magic Formula because there is no true analytical methodology behind the form of the equations, the Magic Formula attempts to describe tire behavior via a formula that captures the shape of empirical data. Magic formula requires a set of model coefficients such as, stiffness (B), shape (C), curvature (E), peak (D) and vertical load (F_z) to describe the tyre properties. Fig. 2 shows the curve produced by the magic model.

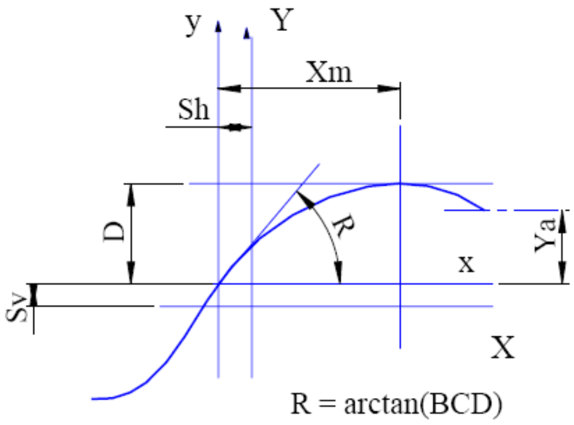


Fig. 2 Curve produced by the general form of the magic formula [18]

In this section, the detailed modelling of magic formula for both pure and combined slip are discussed. The camber angle has been considered as zero for easy calculation.

2.1 Pure slip

It is important to understand the fundamental property that a rubber tire generates forces only when deformed, as often occurs under slip conditions. Due to this property, tires are usually characterized in terms of the force or moment generated per unit of either longitudinal or lateral slip [18]. A condition in which a tire is exerting only a lateral or longitudinal force is known as pure slip. Longitudinal force for pure slip, F_{xo}, consists of coefficients *B*, *C*, *D*, *E* and S_{vx}. The subscript *x* represents condition along *x*-axis. Slip ratio, *κ,* is the input of F_{xo} as given by,

$$F_{x0} = D_x Sin(C_x \arctan\{B_x k_x - E_x(B_x k_x - \arctan(B_x k_x))\}) + S_{vx} \tag{1}$$

Lateral force for pure slip, F_{yo} consists of coefficients *B, C, D, E* and S_{vy}. The subscript *y* represents condition along *y*-axis. Slip angle, *α,* is the input to F_{yo} as given by,

$$F_{y0} = D_y Sin(C_y \arctan\{B_y \alpha_y - E_y(B_y \alpha_y - \arctan(B_y \alpha_y))\}) + S_{vy} \tag{2}$$

The aligning moment for pure slip is denoted as M_{z0} which consists of coefficients *D, C, B,* and *E*.

$$M_{z0} = -(t_0 . F_{y0}) + M_{zr0} \tag{3}$$

$$t_0 = D_t Cos(C_t \arctan\{B_t \alpha_t - E_t(B_t \alpha_t - \arctan(B_t \alpha_t))\}) \tag{4}$$

$$M_{zr0} = D_r Cos(C_r \arctan(B_r \alpha_r)) \tag{5}$$

Where, t_0=Pneumatic trail.

2.2 Combined slip

While pure slip is useful for analysis, in reality tires usually operate in a condition where both forces are produced, known as combined slip [10].Longitudinal force for combined slip, F_x, is the product of factor $G_{x\alpha}$ with pure longitudinal force, F_{xo}, as given by

$$F_x = G_{x\alpha} . F_{xo} \tag{6}$$

$$G_{x\alpha} = Cos[C_{x\alpha} \arctan\{B_{x\alpha}.\alpha_s - E_{x\alpha}(B_{x\alpha}.\alpha_s - \arctan(B_{x\alpha}.\alpha_s))\}]/G_{x\alpha 0} \quad (7)$$

$$G_{x\alpha 0} = Cos[C_{x\alpha} \arctan\{B_{x\alpha}.S_{Hx\alpha} - E_{x\alpha}(B_{x\alpha}.S_{Hx\alpha} - \arctan(B_{x\alpha}.S_{Hx\alpha}))\}] \quad (8)$$

The lateral force for pure slip is denoted as F_{x0} which consists of coefficients *D, C, B, E* and S_{Hy}.

$$F_y = G_{yk}.F_{yo} + S_{Vyk} \quad (9)$$

$$G_{yk} = Cos[C_{yk} \arctan\{B_{yk}.k_s - E_{yk}(B_{yk}.k_s - \arctan(B_{yk}.k_s))\}]/G_{yk0} \quad (10)$$

$$G_{yk0} = Cos[C_{yk} \arctan\{B_{yk}.S_{Hyk} - E_{yk}(B_{yk}.S_{Hyk} - \arctan(B_{yk}.S_{Hyk}))\}] \quad (11)$$

The longitudinal force for pure slip is denoted as F_{x0} which consists of coefficients *D, C, B* and *E*.

$$M_z = -(t.F_y) + (D_r Cos(C_r \arctan(B_r \alpha_{re}))) + s.F_x \quad (12)$$

$$t = D_t Cos(C_t \arctan(B_t \alpha_{te} - E_t(B_t \alpha_{te} - \arctan(B_t \alpha_{te})))) \quad (13)$$

$$s = 0.1(R_0) \quad (14)$$

Above mentioned mathematical equations are useful for calculating the longitudinal force, lateral force and aligning moment for the given values of model coefficients, *B, C, D* and *E*.

2.3 Simulation of lateral, longitudinal forces and aligning moment

In order to simulate the lateral, longitudinal forces and aligning moment, the mathematical equations which are given in the last section are modelled to Matlab/Simulink block diagram as shown in Fig.3. The inputs of the block diagram are slip ratio and slip angle.

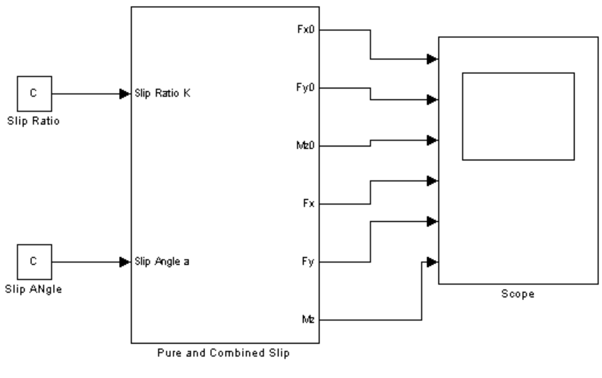


Fig. 3.Magic formula as simulink model

The parameters of Magic Formula have a physical meaning, so a good approximation of starting values can be made In the present work, the values for Lateral, Longitudinal Forces and Aligning Moment are simulated for the combined slip with the model coefficients such as B=0.08, C=1.65, D=1100, E= -4 and F_z=8000 N [18].The simulation results for the different values of slip angle and slip ratio are shown in Table 1.

Table 1.Simulation results of magic formula

Slip Ratio (k)	Slip Angle (α)														
	Longitudinal Force (F_x)(kN)					Lateral Force (F_y)(kN)					Aligning Moment (M_z)(Nm)				
	1	2	4	8	10	1	2	4	8	10	1	2	4	8	10
0.05	0.22	0.21	0.20	0.18	0.19	3.28	2.98	2.83	2.76	2.75	2.24	4.60	5.12	5.27	5.30
0.10	0.44	0.43	0.40	0.37	0.39	3.21	2.92	2.77	2.71	2.6	2.19	4.5	5.01	5.16	5.18
0.25	1.10	1.08	1.00	0.94	0.99	2.93	2.66	2.53	2.47	2.46	1.82	3.73	4.15	4.28	4.30
0.50	2.21	2.16	2.01	1.88	1.98	2.82	2.56	2.43	2.38	2.37	5.14	1.05	1.17	1.20	1.21
1.00	4.49	4.35	3.58	3.82	4.01	2.68	2.43	2.31	2.25	2.25	-1.12	-2.30	-2.58	-2.64	-2.65

It can be observed that the longitudinal force attains a maximum value of 4.49 kN when the slip ratio is maximum and slip angle is minimum, this is due to the fact that longitudinal force is a function of slip ratio [18]. Hence, as the slip ratio increases, the longitudinal force rises rapidly and provides an increasing trend. The lateral force attains a maximum value of 3.25 kN, when the slip ratio is minimum and slip angle is maximum, this is due to the lateral force is proportional to the slip angle and not to friction coefficient [18]. Thus when the slip angle is high then the resultant lateral force is also high. It can be seen that the magic formula is useful for the modelling of tire forces. However, it requires accurately estimated model coefficients. In order to simplify the tire modelling, a fuzzy logic approach has been proposed in the present work and it is explained in the next section.

3. Proposed tire model using fuzzy logic approach

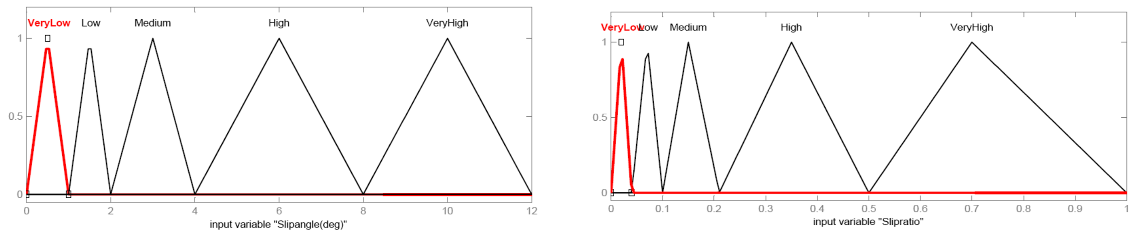
In the present work, fuzzy logic inference system is developed for modelling tire forces and aligning moment based on the slip angle and slip ratio. The input variables for the fuzzy inference system is slip and slip ratio. Output variables are considered as longitudinal, lateral forces and aligning moment. The typical steps in developing the fuzzy inference system involve fuzzification, rule formation, defuzzification [19] and it is explained in the subsequent sections.

3.1 Fuzzification of variables

The fuzzification is the process of transforming crisp values of the variables into of fuzzy sets describing with linguistic terms [17]. The input variables, such as slip angle, slip ratio are suitably partitioned and converted into linguistic variables such as: Very Low, Low, Medium, High, and Very High as shown in Table 2. Similarly, the output variables are partitioned and represented as fuzzy sets with linguistic terms as: Very Low, Low, Medium, and High. The selection of range for the input and output variables are obtained from the simulated values using magic formula as shown in Table 2.Triangular membership functions are used for graphical inference of the input and out variables as shown in Fig. 4 and Fig. 5.

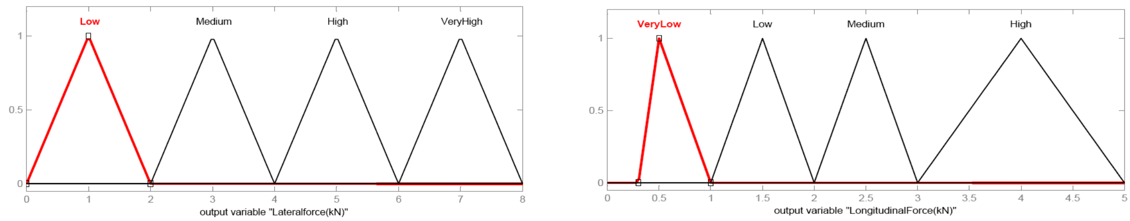
Table 2. Linguistic variables of fuzzy method

Linguistic variables	Fuzzy variables and ranges				
	Input Variables		Output variables		
	Slip Ratio	Slip Angle	Longitudinal Force	Lateral Force	Aligning Moment
Very Low (VL)	0 – 0.05	0 – 1	0.3 – 1	0 – 1	-3 – -1
Low (L)	0.05 – 0.10	1 – 2	1 – 2	1 – 2	-1 – 1
Medium (M)	0.1 – 0.25	2 – 4	2 – 3	2 – 3	1 – 2
High (H)	0.25 – 0.50	4 – 8	3 – 5	3 – 4	2 – 4
Very High (VH)	0.5 – 1.0	8 – 10	-	-	4 – 6

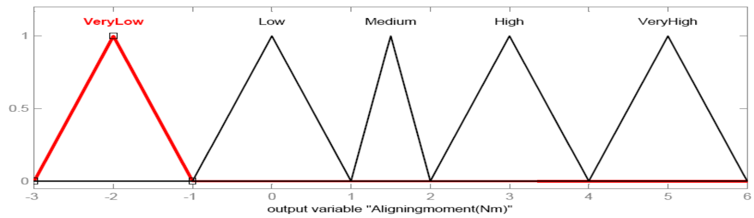


(a) Slip angle (deg) (b) Slip ratio

Fig 4.Input variables of fuzzy logic model



(a) Lateral force (kN) (b) Longitudinal force (kN)



(c) Aligning moment (Nm)

Fig 5.Output variables of fuzzy logic model

3.2 Fuzzy Rules

Fuzzy rule is the standard form of expressing knowledge and If/Then rules offer a convenient format for representing it [17]. A set of rules have been constructed based on the input variables: slip angle and slip ratio and the output variables: longitudinal, lateral forces, aligning moment as given in the Table 3, 4 and 5 respectively.

Table 3. Fuzzy rules for longitudinal force

Slip Ratio (k)	Slip Angle (α)				
	VL	L	M	H	VH
VL	L	L	L	VL	VL
L	H	H	H	L	L
M	M	M	L	VL	L
H	H	H	M	VL	L
VH	H	H	H	L	M

Table 4. Fuzzy rules for lateral force

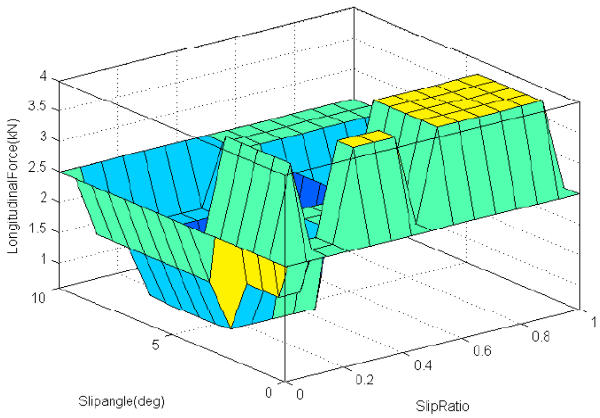
Slip Ratio (k)	Slip Angle (α)				
	VL	L	M	H	VH
VL	H	M	M	M	M
L	H	M	M	M	M
M	M	M	M	M	M
H	VL	VL	VL	VL	VL
VH	L	L	L	L	L

Table 5. Fuzzy rules for aligning moment

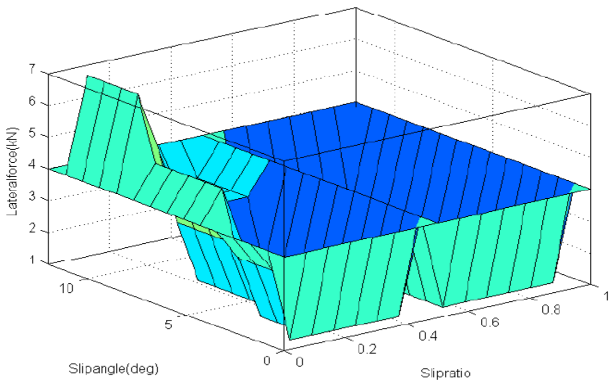
Slip Ratio (k)	Slip Angle (α) VL	L	M	H	VH
VL	H	VH	VH	VH	VH
L	H	VH	VH	VH	VH
M	M	H	VH	VH	VH
H	VH	M	M	M	M
VH	L	VL	VL	VL	VL

3.3 *Defuzzification*

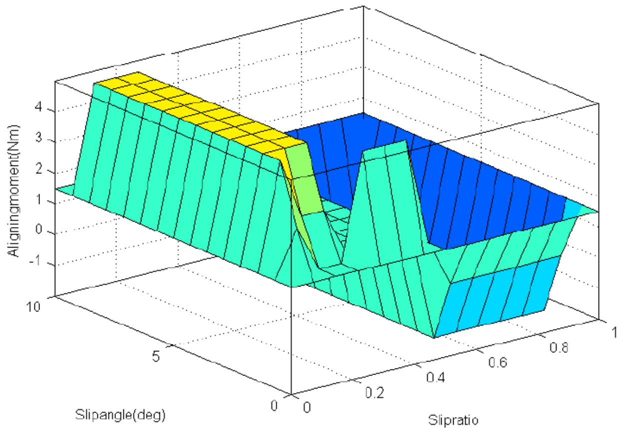
It is the conversion of a fuzzy quantity to a crisp value. In this work, centroid method is applied for defuzzification which has been widely used by the researchers [16]. Fig. 6 shows the relationship between slip angle, slip ratio, longitudinal force and lateral force, aligning moment. The defuzzified values for the longitudinal, lateral and aligning moment of fuzzy logic model are given in the Table 6.



(a) Longitudinal force (kN)



(b) Lateral force (kN)



(c) Aligning moment (Nm)

Fig 6.Graphical representations for the output variables of fuzzy model

Table 6.Simulation results using fuzzy model

Slip Ratio (k)	Slip Angle (α)														
	Longitudinal Force (F_x)(kN)					Lateral Force (F_y)(kN)					Aligning Moment (M_z)(Nm)				
	1	2	4	8	10	1	2	4	8	10	1	2	4	8	10
0.05	0.2	0.2	0.19	0.16	0.18	3.25	3.2	2.91	2.8	2.63	2.2	4.55	5	5.18	5
0.1	0.43	0.41	0.4	0.35	0.37	2.95	2.9	2.63	2.54	2.41	2	4.3	5	5.11	5.08
0.25	1.02	1.01	1.01	0.91	0.87	2.81	2.75	2.5	2.41	2.3	1.67	3.56	4.01	4.17	4.22
0.5	2.2	2.2	2	1.53	1.89	2.74	2.69	2.45	2.35	2.23	5.09	1	1.11	1.13	1.08
1	4.39	4.2	3.4	3.75	4	2.72	2.55	2.43	2.36	2.23	-1.1	-2.04	-2.39	-2.46	-2.57

It can be inferred that the maximum value for longitudinal force is achieved when the slip ratio is maximum and slip angle is minimum, which is similar to the results from the Magic model as listed in Table 1. From the above results, it is shown that the proposed fuzzy logic method provides an alternative approach for estimating the tire forces and aligning moment, without further estimation of model coefficients as used by Magic formula in Section.2.

4. Validation of the proposed fuzzy logic model

In order to validate the proposed fuzzy model, the simulation results for the tire forces and aligning moment were compared with the Magic formula. Fig. 7 shows the comparison of longitudinal force, lateral force and aligning moment of both magic model and fuzzy logic. The correlation coefficient is calculated for the longitudinal, lateral and aligning moment values obtained using the proposed fuzzy logic model and the Magic formula.

Fig.7 (a) shows the simulated values of longitudinal force, lateral force, aligning moment using Magic and fuzzy model for the different values of slip angle and slip ratio. It can be seen that the maximum correlation coefficient is found to be 0.99 and the minimum correlation coefficient is found to be 0.78. From Fig. 6(b) and 6(c), it can also be observed that the estimated values using the fuzzy model qualitatively follow the trend of the simulated values using magic model. These results prove that proposed fuzzy logic model provides an approximate estimation of tire forces and aligning moment for different values of slip ratio and slip angle as compared to magic model. The deviations between the simulated values using magic model and the fuzzy model can be minimized with suitable correction factors for the different cases

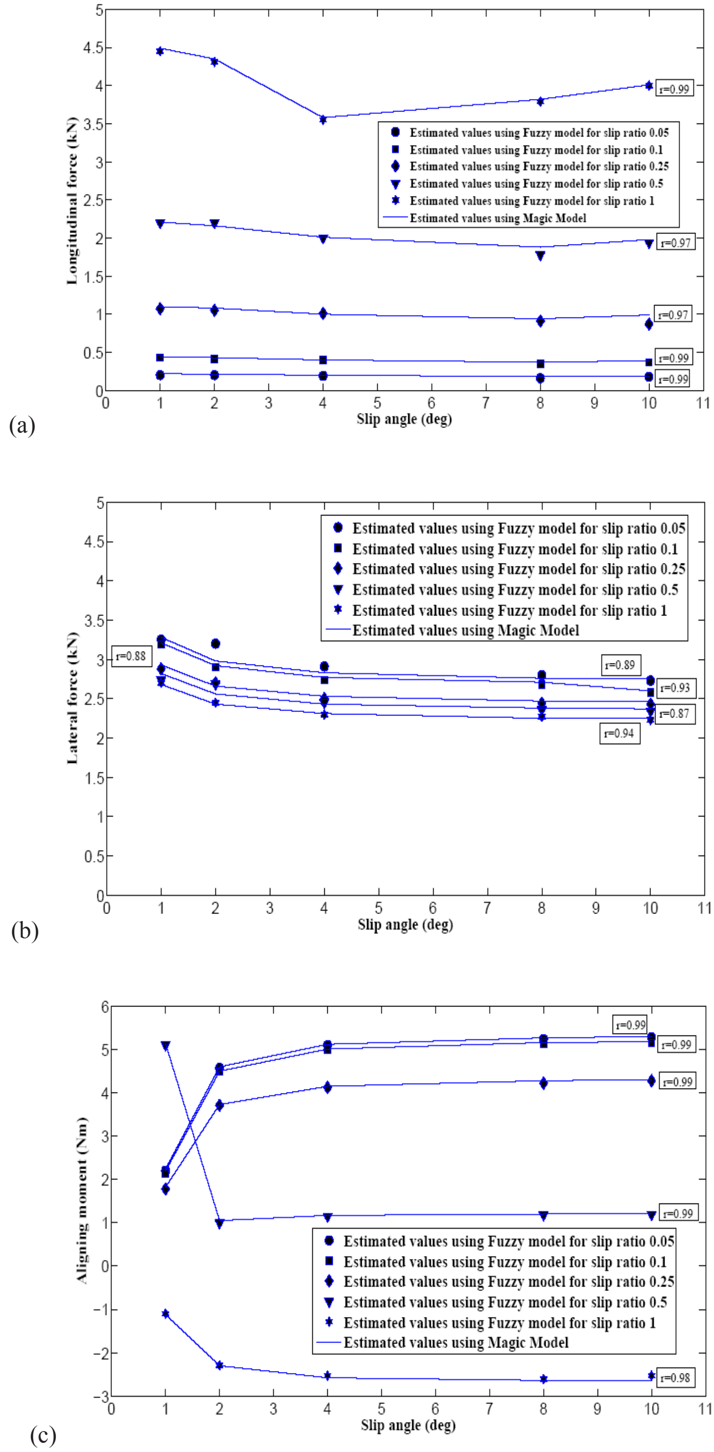


Fig 7.Comparison of simulated values using magic and fuzzy models

5. Conclusion

Tire behaviour is very important consideration in vehicle dynamics. This paper presents a new approach for modelling and simulating the tire forces using fuzzy logic principle. This method is implemented to provide three different parameters of tire model; such as longitudinal force, lateral force and aligning moment at combined slip conditions successfully for the different values of slip ratio and slip angle. The simulated values from the proposed fuzzy logic model are found to provide a higher correlation of 0.99 with widely used magic formula. Proposed modelling tool does not require high level knowledge and experience about the model coefficient, which is required by Magic formula. Proposed method provides a model free approach, which can be useful for developing feedback control system for tire forces in an automotive system.

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