Arbitrage and Volatility in Chinese Stock’s Markets

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Abstract

From the point of view of no-arbitrage pricing, what matters is how much volatility the stock has, for volatility measures the amount of profit that can be made from shorting stocks and purchasing options. With the short-sales constraints or in the absence of options, however, high volatility is likely to mean arbitrage from stock market. As emerging stock markets for China, investors are increasingly concerned about volatilities of Chinese two stock markets. We estimate volatility’s models for Chinese stock markets’ indexes using Markov chain Monte Carlo (MCMC) method and GARCH. We find that estimated values of volatility parameters are very high for all data frequencies. It suggests that stock returns are extremely volatile even at long term intervals in Chinese markets. Furthermore, this result could be considered that there seems to be arbitrage opportunities in Chinese stock markets.

1. Introduction

Under the assumption of geometric Brownian motions for stock prices, volatility is the key parameter. From the point of view of no-arbitrage pricing, what matters is how much volatility the stock has, for volatility measures the amount of profit that can be made from shorting stocks and purchasing options. Hence, studying and exploring volatility has very important financial applications.

Cont (2001) gives a survey of these features, which sometime are called stylized facts. These stylized facts in fact can be regarded as being related to volatility clustering of stock returns. Indeed, the strong time dependence in volatility is an important feature in stock market.
Volatility may be regarded as the exact function of a given set of variables, including the autoregressive conditional heteroskedasticity (ARCH) model of Engle (1982), the generalized ARCH (GARCH) model of Bollerslev (1986), and so on. And, a stochastic function is used to characterize volatility, so Stochastic Volatility (SV) models have been applied to study the behavior of financial variables.

SV models introduce the additional innovation to the conditional variance equation so that they are much more flexible than ARCH or GARCH models. It has been found to fit asset returns better and have residuals closer to standard normal (Jondeau, Poon and Rockinger, 2007). SV models also arise as discrete approximations to various diffusion processes of interest in the continuous-time asset-pricing literature (Hull and White, 1987; Melino and Turnbull, 1990).

Tsay (2005) indicates that the excess kurtosis of the ARCH (1) series is positive and the tail distribution is heavier than of a normal distribution. Jondeau, Poon and Rockinger (2007) report that, on average, a negative return shock has twice as much impact on volatility as a positive return shock for the log-returns on four stock market indices. Black (1976) calls this leverage effect, relating to the fact that when equity value decreases, leverage and, hence, risk and volatility increase. Campbell and Hentschel (1992) interpret the different responses as a stronger impact of bad news than good news.

In this paper, we primarily apply SV models to describe the behavior of volatility for different frequencies data for Chinese stock markets’ index returns. In order to compare the results form SV models with other models, we also estimate stock index returns’ volatility with GARCH models. In addition, we explain the relation between volatility and arbitrage, and then further analyze features of Chinese stock markets.

The first contribution is that our analyses almost cover all historical data of Chinese stock markets, providing a full description about volatility. The second contribution of this paper is that we set and estimate SV models for different data frequencies, so we give very comprehensive analyses for the behaviors of volatilities of stock returns. The third contribution is that we develop the relationship between volatility and arbitrage with the short-sales constraints or the incomplete option trading.

The rest of the paper is outlined as follows. Section 2 presents the financial theory about the relationship between volatility and arbitrage. In section 3, we introduce the basic principle and framework of SV models. Section 4 applies SV models, together with GARCH models, to analysis of Chinese stock markets’ index returns, and then provides and discusses the estimated results. A final section concludes.

2. Theory: arbitrage and volatility

Shreve (2004) makes use of option theory to illustrate that a long gamma portfolio is profitable in times of high stock volatility. Here we briefly restate this example to understand relationship between volatility and arbitrage.

Suppose at time $t$ the stock price is $x_t$. We purchase the option for $c(t, x_t)$, short $c_d(t, x_t)$ shares of stock, and invest the difference,

$$M = x_t c_d(t, x_t) - c(t, x_t),$$

in the money market account in order to take a long position in the option and hedge it. The initial portfolio value is zero at the moment $t$ when we set up these positions.

If the stock price were to instantaneously fall to $x_0$ as shown in Figure 1 and we do not change our positions in the stock or money market account, then our portfolio can obtain the gain equal to $c(t, x_0) - c_d(t, x_t)(x_0 - x_t) - c(t, x_t)$, in other words, our portfolio benefits from an instantaneous drop in the stock price.
price. On the other hand, if the stock price were to instantaneously rise to $x_2$ and we do not change our positions in the stock or money market account. We derive that our portfolio benefits from an instantaneous rise in the stock price with analogous mathematical tricks with the case of instantaneously fall in the stock price. The central factor of both gains from our portfolio is the convexity of $c(t, x)$ as described in Figure 1.

The portfolio we have set up is said to be delta-neutral and long gamma. The portfolio described above may at first appear to offer an arbitrage opportunity. Figure 1 is misleading because it is drawn with $t$ fixed. In fact, the derivative of $c(t, x)$ with respect to time is negative so that when moving forward in time, the curve $c(t, x)$ is shifting downward. The essence of the hedging is that if the stock really is a geometric Brownian motion with the right value of volatility, then rebalancing our portfolio, all these effects exactly cancel.

Of course, asset returns are not really geometric Brownian motion with constant volatility, so we would like to ask what happens if volatility is time-varying. Shreve (2004) points out that as volatility increases, so do option prices in the Black-Scholes-Merton model.

Note that the above discussed is based on the situation without the short-sales constraints and in the presence of continuous option trading. With the short-sales constraints or in the absence of option trading, things become more complicated and substantial high volatility could offer a number of arbitrage in stock markets.

3. SV Models and Estimating Technique

SV models cannot be estimated directly by Maximum Likelihood method because the process $\sigma_t^2$ is an unobservable variable. To estimate SV models, a quasi-likelihood method via Kalman filtering or Monte Carlo method is required. Jacquier, Polson, and Rossi (1994) provide some comparison of estimation results between quasi-likelihood and Markov chain Monte Carlo (MCMC) methods. In this paper, we model stochastic volatility using MCMC method

MCMC method is closely related with Bayesian inference. Bayesians represent uncertainty about unknown parameter values by probability distributions and proceed as if parameters were random

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1 Figure 1 is retrieved from Fig. 4.5.1 of Chapter 4 of Shreve (2004).
quantities (Gilks, Richardson, and Spiegelhalter, 1996). If we let $X$ represent the data that are observed and $\omega$ represent the model parameters, we can use Bayes’ Theorem to get the posterior distribution as follows

$$p(\omega \mid X) = \frac{p(\omega)p(X \mid \omega)}{\int p(\omega)p(X \mid \omega) d\omega}.$$

MCMC method is based on the idea that rather than compute $p(\omega \mid X)$, we may have a large random sample from $p(\omega \mid X)$. Intuitively, if the sample were large enough, we could approximate the form of the probability density using kernel density estimator or histograms and compute the mean and standard deviation of the large sample. This insight motivates the question of how to efficiently simulate a large number of random samples from $p(\omega \mid X)$. Metropolis, et al. (1953) shows that one could construct a Markov chain stochastic process for $(\omega_t, t \geq 0)$ that unfolds over time such that the stationary distribution which we use to draw samples is $p(\omega \mid X)$ after the Markov chain has been run for a long enough time. Therefore, we can run a Markov chain to produce a sample of $(\omega_t, t = 1, \cdots)$ from the posterior distribution and use simple descriptive statistics to examine any features of the posterior. The most widely used approaches to MCMC are Metropolis-Hastings method and Gibbs sampling method.


As authoritative statistical indicators widely adopted by domestic and overseas investors in measuring the performance of Chinese stock markets, many indices are compiled and published by Shanghai and Shenzhen stock exchanges. We choose the composite index of Shanghai stock exchange spanning the period from Dec 31, 1991 through Sep 30, 2009, and the component index of Shenzhen exchange, spanning the period from Jan 3, 1994 through Sep 30, 20092.

For the observed sequence, $t = 1, \ldots, T$, we let $X_1$ to be the last composite index of Shanghai stock exchange and $X_2$ to be the last component index of Shenzhen stock exchange. Then, we let $Y_1$ to be the returns of the composite index and $Y_2$ to be the returns of the component index. $Y_1$ and $Y_2$ are defined as follows.

$$Y_{1t} = \ln(X_{1t} / X_{1t-1}), \quad Y_{2t} = \ln(X_{2t} / X_{2t-1})$$

In order to examine the features of stock indexes’ returns for different frequencies, we consider three frequencies: daily returns, monthly returns, and quarterly returns. Denote that $Y_{1d}$, $Y_{1m}$, and $Y_{1q}$ are Shanghai composite indexes’ daily log returns, monthly log returns, and quarterly log returns, respectively. Denote that $Y_{2d}$, $Y_{2m}$, and $Y_{2q}$ are Shenzhen component indexes’ daily log returns, monthly log returns, and quarterly log returns, respectively. Each log return is measured in percent.

Consider SV models, $x_t = \mu^{SV} + \varepsilon_t$, $\varepsilon_t = \sigma_t z_t$, and $\ln \sigma_t^2 = \alpha_0^{SV} + \alpha_1^{SV} \ln \sigma_{t-1}^2 + \sigma_v^{SV} \nu_t$. To implement the Gibbs sampling, we assume that the prior distributions for $\mu^{SV}$ follows a normal distribution, the prior distributions for $\alpha^{SV} = (\alpha_0^{SV}, \alpha_1^{SV})'$ follows a jointly normal distribution, and the prior distributions for $(\sigma_v^{SV})^2$ follows an inverted chi-squared distribution with 5 degrees of freedom.

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Shu Quan Lu et al. / Physics Procedia 25 (2012) 756 – 762

For initial parameter values, we used the fitted values of the GARCH model described above for $\{\sigma_i^2\}$ and set $\left(\sigma_i^{SV}\right)^2$ and $\mu^{SV}$ to be the sample mean. We run the Gibbs sampling for 10000 iterations for daily log returns, but for 2000 iterations for monthly log returns and quarterly log returns because of their small sample size relative to daily log returns. We discard results of the first 100 iterations in order to make remaining samples independent. The estimated values of $\alpha^{SV}$ for $Y_{1t}^d$, $Y_{1t}^m$, and $Y_{1t}^q$ are 0.903, 0.891, and 0.891, and for $Y_{2t}^d$, $Y_{2t}^m$, and $Y_{2t}^q$ are 0.889, 0.857, and 0.889, respectively. All estimated results are statistically significant at 1%.

The prime advantage of our empirical investigations for volatilities of Chinese stock markets is that our analyses almost cover all historical data of Chinese stock markets since the Shanghai Stock Exchange (SSE) was founded on Nov. 26th, 1990. These analyses provide a full description about volatility of two stock markets’ index returns in China. The other advantage of this paper is that we set and estimate SV models for different data frequencies, so we give very comprehensive analyses for the behaviors of volatilities of stock returns.

A number of authors, such as Tsay (2005) and Jondeau, Poon and Rockinger (2007), have investigated the features of volatilities for many countries’ stock market indexes. Tsay (2005) models the SV model for monthly log returns with sample period covering Jan 1962 to Dec 1999 for the Standard and Poor’s 500 (SP500) from the US. Jondeau, Poon and Rockinger (2007) model the GARCH models for daily log returns with sample period covering Jan 2, 1980 to Aug 31, 2004 for SP500, the DAX 30 (DAX) from Germany, the FTSE ALL Shares (FTSE) from the UK, and the Nikkei 225 (Nikkei) from Japan. Jondeau, Poon and Rockinger (2007) also give estimated values for the GARCH models for weekly log returns for SP500. A summary of the estimated values of volatility parameter $\alpha_i$ for different country’s stock market is provided in Table 1.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Stock Markets</th>
<th>Frequency</th>
<th>Models</th>
<th>Volatility Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tsay (2005)</td>
<td>SP500</td>
<td>monthly</td>
<td>SV</td>
<td>0.685</td>
</tr>
<tr>
<td>Jondeau, Poon and Rockinger (2007)</td>
<td>SP500</td>
<td>daily</td>
<td>GARCH</td>
<td>0.919</td>
</tr>
<tr>
<td></td>
<td>SP500</td>
<td>weekly</td>
<td>GARCH</td>
<td>0.7862</td>
</tr>
<tr>
<td></td>
<td>DAX</td>
<td>daily</td>
<td>GARCH</td>
<td>0.868</td>
</tr>
<tr>
<td></td>
<td>FTSE</td>
<td>daily</td>
<td>GARCH</td>
<td>0.872</td>
</tr>
<tr>
<td></td>
<td>Nikkei</td>
<td>daily</td>
<td>GARCH</td>
<td>0.864</td>
</tr>
</tbody>
</table>

In particular, the volatility parameter $\alpha_i$ seems to decline with the time interval for SP500 log returns in the Table 4, so this implies that the estimated results for SP500 log returns could be consistent with the general rule, which is that serial correlation of volatility would be likely to decline with the length of time interval. Even for the quarterly log returns, however, our empirically estimated results show that volatility parameter $\alpha_i$ could remain high, as for the daily log returns. In fact, estimated values of volatility parameters for Chinese stock markets in Table 3, regardless of data frequencies, are closed to estimated values volatility parameters of daily log returns for other four countries’ stock markets in Table 4. This comparison implies that log returns are extremely volatile for Chinese stock markets even with a long time interval.

In Section 2, we discuss the relationship between volatility and arbitrage based on the financial theory. We show that a long gamma portfolio is profitable in times of high stock volatility and we also
present that things become more complicated with short-sales constraints or in the absence of option trading. Short-sales constraints now work well, together with option markets’ incompleteness, in the current stock exchanges in China; hence it is possible for such high volatility of stock returns to offer arbitrage opportunities in Chinese stock markets.

Lu, Ito, and Voges (2008) indicate that stock returns of Shenzhen component index exhibit long memory processes. Lu and Ito (2009) find that there is a two-way feedback between Chinese two stock markets by using the expectational model to trace the response of a stock market to the other stock market. These studies could suggest that high volatilities of stock markets could give rise to long memory or strong two-way feedback for stock market. Moreover, Lu and Ito (2008) have showed Chinese many macroeconomic series seem to turn out to be unstable. Hence, this suggests that macroeconomic instability could cause the stock market to be extremely volatile.

5. Conclusion

Volatility is very important issue for studying the behavior of stock markets. There are two stock markets in China: Shanghai Stock exchange and Shenzhen Stock exchange. As emerging stock markets, investors are increasingly concerned about volatilities of the two stock markets. One interest is how to model these volatilities of stock returns. There are essentially two types of models for describing the dynamics of volatility: one is ARCH or GARCH models, and the other is SV models.

ARCH or GARCH models consider only one source of uncertainty, while SV models introduce the additional innovation to the conditional variance equation. SV models can be estimated by a quasi-likelihood method or Monte Carlo method rather than directly by Maximum Likelihood method. In this paper, we estimate SV models of Chinese stock markets’ returns using Markov chain Monte Carlo method.

The prime contribution of our empirical investigations is that our analyses almost cover all historical data, providing a full description about volatility for Chinese stock markets. The other contribution of this paper is that we set and estimate SV models for different data frequencies, so we give very comprehensive analyses for the behaviors of volatilities of stock returns.

According to our empirical analyses for Chinese stock markets, we show that SV models provide improvements in model fitting relative to GARCH models since most of parameters are statistically significant. Furthermore, for all data frequencies of Chinese stock markets, we find that estimated values of volatility parameters are such high as to be closed to estimated values volatility parameters of daily log returns for other four countries’ stock markets. This comparison implies that log returns are extremely volatile even with a long time interval in Chinese stock markets.

In terms of financial theory, a long gamma portfolio is profitable in times of high stock volatility. With short-sales constraints or the incomplete option trading, it is possible for such high volatility of stock returns to offer arbitrage opportunities in Chinese stock markets.

References


* Shu Quan Lu is also the student of PHD candidate of School of Economics, Fudan University now.