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$1/R$ gravity and scalar-tensor gravity

Takeshi Chiba

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

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Abstract

We point out that extended gravity theories, the Lagrangian of which is an arbitrary function of scalar curvature R, are equivalent to a class of the scalar-tensor theories of gravity. The corresponding gravity theory is $\omega = 0$ Brans–Dicke gravity with a potential for the Brans–Dicke scalar field, which is not compatible with solar system experiments if the field is very light: the case when such modifications become important recently.

The problem of dark energy is the problem of Ω: $\Omega = 8\pi G\rho_M/3H^2 < 1$. Since Ω can be regarded as the ratio of the right-hand side of the Einstein equation (matter) to the left-hand side of the Einstein equation (curvature = gravity), in order to make $\Omega = 1$ one requires either (i) introduction of new form of matter (energy): dark energy or (ii) modification of gravity in the large, so that the total energy density is equal to the critical density, which is required by theory (inflation) or by observation (WMAP).

Recent attempts to modify gravity by introducing R^{-1} term [1,2] fall in the latter possibility:[1]

$$S = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\left(R - \frac{\mu^4}{R}\right) + S_{\text{matter}}(g_{\mu\nu}), \tag{1}$$

where $\kappa^2 = 8\pi G$. The Newtonian limit of such modified gravity theories is studied in [3], and it is found that Newton gravity is reproduced (as it should be). In this note, we point out that modified gravity theories with the Lagrangian of an arbitrary function of R are equivalent to a special class of scalar-tensor theories of gravity. We also calculate the PPN (parameterized post-Newtonian) parameter of such gravity theories. To this end, we utilize the dynamically equivalent action by introducing a new field ϕ [4]:

$$S = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\left(\left(1 + \frac{\mu^4}{\phi^2}\right)R - \frac{2\mu^4}{\phi}\right) + S_{\text{matter}}(g_{\mu\nu}). \tag{2}$$

One can easily verify that the field equation for ϕ gives $\phi = R$, which reproduces the original action Eq. (1).[2]

E-mail address: chiba@tap.scphys.kyoto-u.ac.jp (T. Chiba).

[1] If such models are phenomenologically viable, R^{-1} gravity might be called "*c*-essence" (*c* for curvature).

[2] The field ϕ is not an auxiliary field since the field equations contain the second derivative of ϕ through the equation of motion of the metric.

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The equivalence is easily generalized to an arbitrary function of R:

$$S=\frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\,F(R)+S_{\text{matter}}(g_{\mu\nu}). \tag{3}$$

The equivalent action is [5,6]:

$$S=\frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\,\big(F(\phi)+F'(\phi)(R-\phi)\big)+S_{\text{matter}}(g_{\mu\nu}), \tag{4}$$

where $F'(\phi)=dF/d\phi$. One can easily verify that the field equation for ϕ gives $\phi=R$ if $F''(\phi)\neq 0$, which reproduces the original action. After the conformal transformation such that $F'(\phi)g_{\mu\nu}=g^E_{\mu\nu}$ along $\phi=R$, the action is reduced to that of the scalar field minimally coupled to the Einstein gravity [7–10]:

$$S=\frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g_E}\bigg(R_E-\frac{3}{2F'(\phi)^2}g_E^{\mu\nu}\nabla_{E\mu}F'(\phi)\nabla_{E\nu}F'(\phi)-\frac{1}{F'(\phi)^2}\big(\phi F'(\phi)-F(\phi)\big)\bigg)$$
$$+\,S_{\text{matter}}\big(g^E_{\mu\nu}/F'(\phi)\big). \tag{5}$$

Introducing a canonical scalar field φ such that $F'(\phi)=\exp(\sqrt{2/3}\,\kappa\varphi)$, Eq. (5) can be written as

$$S=\int d^4x\,\sqrt{-g_E}\bigg(\frac{1}{2\kappa^2}R_E-\frac{1}{2}(\nabla\varphi)^2-V(\varphi)\bigg)+S_{\text{matter}}\big(g^E_{\mu\nu}/F'\big(\phi(\varphi)\big)\big),$$
$$V(\varphi)=\big(\phi(\varphi)F'\big(\phi(\varphi)\big)-F\big(\phi(\varphi)\big)\big)/2\kappa^2F'\big(\phi(\varphi)\big)^2. \tag{6}$$

So the question arises: what is the gravity described by the original frame metric $g_{\mu\nu}$? Since the gravity described by $g^E_{\mu\nu}$ is the Einstein-scalar system and $g_{\mu\nu}$ $(=g^E_{\mu\nu}/F'(\phi))$ is admixture of spin 0 degree of freedom and spin 2 degree of freedom, the gravity by $g_{\mu\nu}$ should be a class of scalar-tensor theories of gravity which are subject to observational constraints coming from the solar system experiments of gravity [11]. Usually higher derivative modifications of gravity are thought to be important in the early universe, and hence the bounds on ω by the present time experiments are not important. However, if such modifications become important recently, there is the danger that such theories may be in conflict with experiments. In fact, the absence of the kinetic term in Eq. (4) implies that the Brans–Dicke parameter is vanishing, $\omega=0$ (or the PPN parameter $\gamma=(\omega+1)/(\omega+2)$ is $\gamma=1/2$). The current bound on ω is $\omega>3500$ (or $|\gamma-1|<2.8\times10^{-4}$) [11]. This bound applies to the Brans–Dicke type theory with the very light Brans–Dicke scalar field with mass $\lesssim$ (1 A.U.)$^{-1}\sim10^{-27}$ GeV (e.g., extended quintessence) [12].

We estimate the effective mass for two examples. The first example is the Starobinsky model [13]: $F(R)=R+R^2/M^2$ with $M\sim10^{12}$ GeV. In terms of the scalar field φ, the effective potential can be rewritten as

$$V(\varphi)=\frac{M^2e^{-2\sqrt{2/3}\kappa\varphi}}{8\kappa^2}\big(e^{\sqrt{2/3}\kappa\varphi}-1\big)^2, \tag{7}$$

where we have neglected the matter term for simplicity. Evaluating the second derivative of $V(\varphi)$[3] around the Minkowski vacuum ($\varphi=0$) gives the effective mass squared of the scalar field of order M^2, which is much larger than H_0^2. Hence the constraints by the solar system experiments do not apply here.

The second example is CDTT model [1,2]: $F(R)=R-\mu^4/R$ with $\mu\sim H_0\simeq10^{-42}$ GeV. Again, in terms of φ, the effective potential is given by

$$V(\varphi)=\frac{\mu^2e^{-2\sqrt{2/3}\kappa\varphi}}{\kappa^2}\sqrt{e^{\sqrt{2/3}\kappa\varphi}-1}. \tag{8}$$

[3] Note that $3\,d^2V/d\varphi^2=1/F''+\phi/F'-4F/F'^2$.

Evaluating V'' around $\phi = R \sim H_0^2$ ($\kappa\varphi \sim 1$) gives the effective mass squared of order μ^2 (and tachyonic for $8/9 < e^{\sqrt{2/3}\kappa\varphi} < 2$), which is very light. Together with $\omega = 0$, the solar system experiments exclude such a theory.[4]

To conclude, we have shown that extended gravity theories, the Lagrangian of which is an arbitrary function of scalar curvature R, are equivalent to a class of the Brans–Dicke type theories of gravity with a potential. The corresponding Brans–Dicke parameter is $\omega = 0$. If such modifications become important recently, the scalar field is generically very light and mediates a gravity force of long range. Hence such theories are not compatible with solar system experiments. Thus c-essence may cease to exist. It remains to be seen whether other modification of gravity (higher-dimensional origin [15], massive graviton [16], etc.) could be phenomenologically viable alternative to dark energy.

Note added

Ref. [17] addresses the stability issue of Eq. (1).

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[4] If we can create a dip in $V(\varphi)$ at $\varphi \sim \mu$ so that $V'' \gg \mu^2$ there (like Albrecht–Skordis model [14]), then we may evade the constraints. However, we are currently unable to construct such $F(R)$.