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[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb) $g_{B^*B\pi}$ -coupling in the static heavy quark limit

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## ABSTRACT

By means of QCD simulations on the lattice, we compute the coupling of the heavy–light mesons to a soft pion in the static heavy quark limit. The gauge field configurations used in this calculations include the effect of  $N_f = 2$  dynamical Wilson quarks, while for the static quark propagator we use its improved form (so-called HYP). On the basis of our results we obtain that the coupling  $\hat{g} = 0.44 \pm 0.03^{+0.07}_{-0.00}$ , where the second error is flat (not Gaussian).

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## 1. Introduction

The static quark limit of QCD offers a simplified framework to solving the non-perturbative dynamics of light degrees of freedom in the heavy–light systems. That dynamics is constrained by heavy quark symmetry (HQS): it is blind to the heavy quark flavor and its spin. As a result the total angular momentum of the light degrees of freedom becomes a good quantum number ( $j_\ell^P$ ), and therefore the physical heavy–light mesons come in mass-degenerate doublets. In phenomenological applications the most interesting information involves the lowest lying doublet, the one with  $j_\ell^P = (1/2)^-$ , consisting of a pseudoscalar and a vector meson, such as  $(B_q, B_q^*)$  or  $(D_q, D_q^*)$  states, where  $q \in \{u, d, s\}$ . When studying any phenomenologically interesting quantity from the QCD simulations on the lattice that includes heavy–light mesons (decay constants, various form factors, bag parameters and so on), one of the major sources of systematic uncertainty is related to the necessity to make chiral extrapolations. The reason is that the physical light quarks, which are expected to most significantly modify the structure of the QCD vacuum, are much lighter than the ones that are directly simulated on the lattice,  $m_q \gg m_{u,d}$ . Here by “ $q$ ” we label the light quark masses that are attainable from the lattice. Since the QCD dynamics with very light quarks is bound to be strongly affected by the effects of spontaneous chiral symmetry breaking, a more suitable (theoretically more controllable) way to guide such

extrapolations is by using the expressions derived in heavy meson chiral perturbation theory (HMChPT), which is an effective theory built on the combination of HQS and the spontaneous chiral symmetry breaking  $[SU(N_f)_L \otimes SU(N_f)_R \rightarrow SU(N_f)_V]$ . Its Lagrangian is given by [1]

$$\begin{aligned} \mathcal{L}_{\text{heavy}} &= -\text{tr}_a \text{Tr}[\bar{H}_a i v \cdot D_{ba} H_b] + \hat{g} \text{tr}_a \text{Tr}[\bar{H}_a H_b \gamma_\mu \mathbf{A}_{ba}^\mu \gamma_5], \\ D_{ba}^\mu H_b &= \partial^\mu H_a - H_b \frac{1}{2} [\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger]_{ba}, \\ \mathbf{A}_\mu^{ab} &= \frac{i}{2} [\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger]_{ab}, \end{aligned} \quad (1)$$

where

$$H_a(v) = \frac{1 + \not{v}}{2} [P_\mu^{*a}(v) \gamma_\mu - P^a(v) \gamma_5], \quad (2)$$

is the heavy meson doublet field containing the pseudoscalar,  $P^a(v)$ , and the vector meson field,  $P^{*a}(v)$ . In the above formulas, the indices  $a, b$  run over the light quark flavors,  $\xi = \exp(i\Phi/f)$ , with  $\Phi$  being the matrix of  $(N_f^2 - 1)$  pseudo-Goldstone bosons, and “ $f$ ” is the pion decay constant in the chiral limit. We see that the term connecting the Goldstone boson ( $\mathbf{A}_\mu$ ) with the heavy-meson doublet  $[H(v)]$  is proportional to the coupling  $\hat{g}$ , which will therefore enter into every expression related to physics of heavy–light mesons with  $j_\ell^P = (1/2)^-$  when the chiral loop corrections are included.<sup>1</sup> Being the parameter of effective theory, its value cannot be predicted but should be fixed in some other way. It can

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<sup>1</sup> A special attention should be given to the problem related to the presence of the nearby excited states as discussed in Ref. [2]. Any precision lattice calculation

be related to the measured decay width  $\Gamma(D^* \rightarrow D\pi)$  [3], with the resulting value  $\hat{g}_{\text{charm}} = 0.61(7)$ . That value turned out to be much larger than predicted by all of the QCD sum rule approaches [4], but consistent with some model predictions such as the one in Ref. [5], in which a more detailed list of predictions with their references can be found. The large value for  $g_{D^*D\pi}$ -coupling was confirmed by the quenched lattice QCD study in Ref. [6], and recently also in the unquenched case [7]. Since the charm quark is not very heavy, the use of  $g_{D^*D\pi}$  to fix the value of  $\hat{g}$ -coupling, via

$$\hat{g} = \frac{g_{D^*D\pi}}{2\sqrt{m_D m_{D^*}}} f_\pi, \quad (3)$$

and its use in chiral extrapolations of the quantities relevant to  $B$ -physics phenomenology may be dangerous mainly because of the potentially large  $\mathcal{O}(1/m_c^n)$ -corrections. Unfortunately the decay  $B^* \rightarrow B\pi$  is kinematically forbidden and therefore, to determine the size of  $\hat{g}$ , we have to resort to a non-perturbative approach to QCD. Unlike for the computation of the heavy-to-light form factors, QCD sum rules proved to be inadequate when computing  $g_{D^*D\pi}$ , most likely because of the use of double dispersion relations when the radial excitations should be explicitly included in the analysis, as claimed in Ref. [8]. In this Letter, instead, we compute the  $\hat{g}$ -coupling on the lattice by using the unquenched gauge field configurations, with  $N_f = 2$  dynamical light quarks, and in the static heavy quark limit. The attempts to compute this coupling in this limit were made in Ref. [9], and very recently in Ref. [10]. On the basis of the currently available information, the coupling  $\hat{g}$  in the static limit is indeed smaller than the one obtained in the charmed heavy quark case.

In the remainder of this Letter we will briefly describe the standard strategy to compute this coupling, list the correlation functions that are being computed to extract the bare coupling  $\hat{g}_q$ , as well as the axial vector renormalization constants. We then give details concerning the gauge field configurations used in this work, and present our results.

## 2. Definitions and correlation functions to be computed

In the limit in which the heavy quark is infinitely heavy and the light quarks massless, the axial coupling of the charged pion to the lowest lying doublet of heavy-light mesons,  $\hat{g}$ , is defined via [9]

$$\langle B | \vec{A} | B^* \rangle(\varepsilon) = \hat{g} \vec{\varepsilon}_\lambda, \quad (4)$$

where the non-relativistic normalisation of states  $|B^{(*)}\rangle$  is assumed,  $\langle B_a(v) | B_b(v') \rangle = \delta_{ab} \delta(v - v')$ . For the heavy-light hadrons at rest ( $\vec{v} = \vec{v}' = \vec{0}$ ), the soft pion that couples to the axial current,  $A_\mu = \bar{u} \gamma_\mu \gamma_5 d$ , is at rest too,  $|\vec{q}| = 0$ .  $\varepsilon_\mu^\lambda$  is the polarisation of the vector static-light meson. In the typical situation on the lattice we are away from the chiral limit ( $\hat{g} \rightarrow \hat{g}_q$ ), and the coupling  $\hat{g}_q$  becomes the axial form factor whose value should be extrapolated to the chiral limit, in which the soft pion theorem relating the matrix element of the axial current to the pionic coupling applies [9].

The standard strategy to compute the above matrix element on the lattice consists in evaluating the following correlation functions:

$$C_2(t) = \left\langle \sum_{\vec{x}} P(x) P^\dagger(0) \right\rangle_U \stackrel{\text{HQS}}{=} \frac{1}{3} \left\langle \sum_{i, \vec{x}} V_i(x) V_i^\dagger(0) \right\rangle_U$$

cannot be fully trusted if the chiral extrapolations are made without discussing the problem of discerning the mixing with the  $j_\ell^p = (1/2)^+$  states in the chiral loop diagrams.

$$= \left\langle \sum_{\vec{x}} \text{Tr} \left[ \frac{1 + \gamma_0}{2} W_x^0 \gamma_5 S_{u,d}(0, x) \gamma_5 \right] \right\rangle_U, \\ C_3(t_y, t_x) = \left\langle \sum_{i, \vec{x}, \vec{y}} V_i(y) A_i(x) P^\dagger(0) \right\rangle_U \\ = \left\langle \sum_{\vec{x}, \vec{y}} \text{Tr} \left[ \frac{1 + \gamma_0}{2} W_0^y \gamma_i S_u(y, x) \gamma_i \gamma_5 S_d(x, 0) \gamma_5 \right] \right\rangle_U, \quad (5)$$

where  $\langle \dots \rangle_U$  denotes the average over independent gauge field configurations, the interpolating fields are  $P = \bar{h} \gamma_5 q$ ,  $V_i = \bar{h} \gamma_i q$ , with  $h(x)$  and  $q(x)$  the static heavy and the light quark field, respectively. In what follows, we drop the dependence on  $t_y$ . In practice its value is fixed to one or several values as it will be specified in the text. In Eq. (5) we also expressed the correlation functions in terms of quark propagators: the light ones,  $S_q(x, y)$ , and the static heavy one, which becomes a Wilson line,

$$W_x^y = \delta(\vec{x} - \vec{y}) \prod_{\tau=t_y}^{t_x-1} U_0^{\text{impr.}}(\tau, \vec{x}). \quad (6)$$

The latter is merely obtained from the discretized static heavy quark action [11]

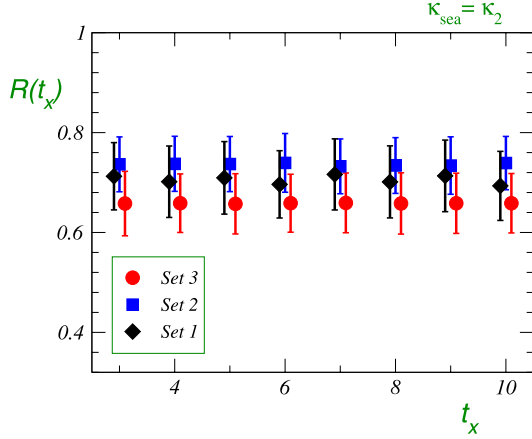
$$\mathcal{L}_{\text{HQET}} = \sum_x \bar{h}^\dagger(x) [h(x) - U_0^{\text{impr.}}(x - \hat{0})^\dagger h(x - \hat{0})], \quad (7)$$

where for  $U_0^{\text{impr.}}$ , the time component of the link variable, we use its improved form, obtained after applying the hyper-cubic blocking procedure on the original link variable, with the parameters optimized in a way described in Ref. [12], namely with  $\vec{\alpha} = (0.75, 0.6, 0.3)$ . That step is essential as it ensures the exponential improvement of the signal to noise ratio in the correlation functions with respect to what is obtained by using the simple product of link variables [13].

The spectral decomposition of the three point function, given in Eq. (5), reads

$$C_3(t_x) = \sum_{m,n} [\mathcal{Z}_n e^{-\mathcal{E}_n^q t_y} \langle B_n | A_i | B_m^* \rangle e^{-(\mathcal{E}_m^q - \mathcal{E}_n^q) t_x} \mathcal{Z}_m \varepsilon_i^{(m)}],$$

where the sum includes not only the ground states ( $m = n = 0$ ) but also their radial excitations ( $m, n > 0$ ), which are heavier and thus exponentially suppressed. Note a shorthand notation,  $\mathcal{Z}_n = |\langle 0 | \bar{h}^\dagger \gamma_5 q | B_n \rangle|$ , and the fact that we do not distinguish  $\mathcal{Z}_n$  from couplings to the vector interpolating operator because of the HQS. If the non-diagonal terms in the above sum were important ( $n \neq m$ ) the correlation function  $C_3(t_x)$  would exhibit some exponential dependence in  $t_x$ . In practice, it appears that the correlation functions  $C_3(t_x)$ , as defined in Eq. (5) are very flat ( $t_x$ -independent) for all the data sets that we use in this work and the details of which will be given in the next section (c.f. Fig. 1). This observation in fact agrees with what one can deduce from various quark models, and in particular from the one in Ref. [5]. We will therefore discard the non-diagonal terms in the spectral decomposition of  $C_3(t_x)$ . We are still left with the problem of contamination of the desired signal ( $n = 0$ ) by the axial transitions among radial excitations,  $n = m > 0$ . To solve that problem we should employ some smearing procedure and suppress the couplings of the source operators to the radial excitations. To that purpose we use the smearing technique proposed in Ref. [14], which essentially means that – in Eq. (5) – the interpolating fields are replaced by  $\bar{h}(x) \gamma_5 q(x) \rightarrow \bar{h}(x) \gamma_5 q^S(x)$ , and similarly for the source of the heavy-light vector mesons, where



**Fig. 1.** Ratio  $R(t_x)$  in Eq. (12) as obtained from our data for all three sets and for which  $\kappa_{\text{val}} = \kappa_{\text{sea}} = \kappa_2$ , with  $\kappa_2$  specified in Table 1. This plot also shows the flatness of the signal of  $C_3(t_x)$  defined in Eq. (5). For completeness, we also note that  $t_y = 13$ .

$$q^S(x) = \sum_{r=0}^{R_{\text{max}}} \varphi(r) \sum_{k=x,y,z} \left[ q(x+r\hat{k}) \prod_{i=1}^r U_k(x+(i-1)\hat{k}) + q(x-r\hat{k}) \prod_{i=1}^r U_k^\dagger(x-i\hat{k}) \right], \quad (8)$$

and  $\varphi(r) = e^{-r/R}(r+1/2)^2$ . The link variables on the right-hand side of Eq. (8) are fuzzed as discussed in Ref. [9]. After several trials we chose the smearing parameters to be  $R = 1.3$  and  $R_{\text{max}} = 4$ , to highly enhance the overlap with ground states. From the fits of our two-point functions computed with both the local (“loc.”) and smeared sources (“sm.”) to two exponentials on the large interval  $4 \leq t \leq 15$ , we obtain that  $Z_0^{\text{sm.}}/Z_0^{\text{loc.}} \gtrsim 45$ , while  $Z_1^{\text{sm.}}/Z_1^{\text{loc.}} < 0.05$ . More importantly,  $Z_1^{\text{sm.}}/Z_0^{\text{sm.}} < 0.04$ , or it cannot be fitted, when it is completely absent. We therefore deduce that our smearing is efficient and the contribution of the radial excitations is most probably negligible. To further check this point we reorganized the operators in  $C_3(t_x)$  and fixed the transition operator [ $A_i$  in Eq. (5)] at  $t = 0$ , one source operator at  $t_y \equiv t_{\text{fix}} = -5$ , and have let the other source operator free (c.f. also Ref. [16]). In that situation the spectral decomposition looks as follows,

$$C'_3(t_x) \simeq \sum_n Z_n^2 e^{-\mathcal{E}_n^q(t_x - t_{\text{fix}})} \langle B_n | A_i | B_n^* \rangle \varepsilon_i^{(n)}, \quad (9)$$

which allows us to check whether or not its effective binding energy, with the smeared source operators,

$$\mathcal{E}_{\text{eff}}^q(t_x) = \log \left( \frac{C'_3(t_x)}{C'_3(t_x + 1)} \right), \quad (10)$$

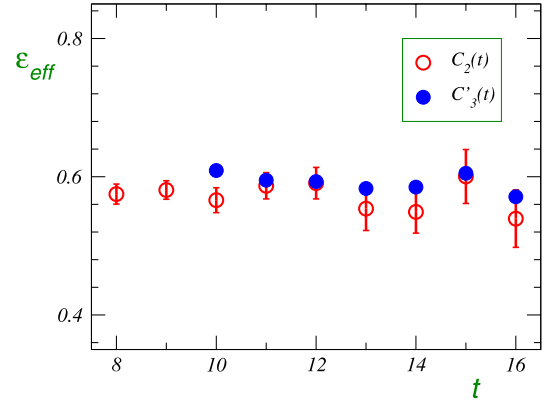
agrees with what is obtained from the two-point correlation functions,

$$\mathcal{E}_{\text{eff}}^q(t) = \log \left( \frac{C_2(t)}{C_2(t+1)} \right). \quad (11)$$

This is illustrated in Fig. 2, which we find satisfactory. After these checks, we extract  $\hat{g}_q$  from the fit to a constant of the ratio,<sup>2</sup>

$$R(t_x) = \frac{1}{3} \frac{C_3(t_x)}{(Z_0^{\text{sm.}})^2 e^{-\mathcal{E}_0^q t_y}} \longrightarrow \hat{g}_q. \quad (12)$$

<sup>2</sup> The use of index “q” in  $\hat{g}_q$  should not be confusing to the reader. Here it simply labels the light quark directly accessed from our lattices.



**Fig. 2.** Comparison of the effective binding energy extracted from  $C_2(t)$  and from  $C'_3(t_x)$  for the Set 2 and  $\kappa_1$  (cf. Table 1). Notice that the separation of the sources  $t = t_x - t_{\text{fix}}$  and in our case  $t_{\text{fix}} = -5$ .

All our fits are made on the common interval,  $5 \leq t_x \leq 8$ . On one ensemble of our gauge-field configurations we also checked that the value of  $\hat{g}_q$  extracted from Eq. (12) is fully consistent with what is obtained if the computation is organized as in Eq. (9).

### 2.1. Axial current renormalization constant

The final ingredient necessary to relate the results of our calculation to the continuum limit is the appropriate axial current renormalization. We prefer to apply the same procedure to all our data sets and determine non-perturbatively the axial renormalization constant. To avoid any notational ambiguity we stress that in this subsection we discuss only the light bilinear quark non-singlet operators,  $P(x) = \bar{q}(x)\gamma_5 q(x)$ ,  $V_\mu = \bar{q}(x)\gamma_\mu q(x)$ ,  $A_\mu = \bar{q}(x)\gamma_\mu \gamma_5 q(x)$ , i.e. no reference to the static heavy quark will be needed in this subsection. To evaluate  $Z_A(g_0^2)$  we use the hadronic Ward identity [15], which is readily derived by imposing the invariance under the axial chiral rotations of  $\langle \sum_{\vec{x}} V_i(x) A_i(0) \rangle$ , and  $\langle \sum_{\vec{x}} V_0(x) P(0) \rangle$ . One then obtains

$$\frac{Z_V^2}{Z_A^2} \left\langle \sum_{\vec{x}} V_i(x) V_i(0) \right\rangle = \left\langle \sum_{\vec{x}} A_i(x) A_i(0) \right\rangle - Z_V \int_{\mathcal{V}} d^4 z \left\langle \sum_{\vec{x}} 2m_{\text{AWI}}^{(0)} P(z) V_i(x) A_i(0) \right\rangle, \quad (13)$$

$$\left\langle \sum_{\vec{x}} A_0(x) P(0) \right\rangle = Z_V \int_{\mathcal{V}'} d^4 z \left\langle \sum_{\vec{x}} 2m_{\text{AWI}}^{(0)} P(z) V_0(x) P(0) \right\rangle, \quad (14)$$

where the integration volume  $\mathcal{V}$  ( $\mathcal{V}'$ ) does (does not) include zero. The bare quark mass defined via the axial Ward identity reads,  $2m_{\text{AWI}}^{(0)} = \langle \sum_{\vec{x}} A_0(x) P(0) \rangle / \langle \sum_{\vec{x}} P(x) P(0) \rangle$ . For notational simplicity in the above Ward identities we wrote  $Z_{V,A} \equiv Z_{V,A}(g_0^2, am_q)$ .

### 3. Lattice details and results

We use the publicly available gauge field configurations generated with  $N_f = 2$  dynamical light (“sea”) quarks which were produced by using the Wilson gauge and Wilson quark actions. In Table 1 we provide a basic information on the data sets used in this Letter. Concerning the discretized Yang-Mills part, the configurations explored in this Letter were generated by the standard Wilson plaquette action and (in one of the sets) by its improved form, known as the Iwasaki action. The effects of dynamical

quarks in the QCD vacuum fluctuations are simulated by using the Wilson quark action, both the ordinary one, and its non-perturbatively  $\mathcal{O}(a)$ -improved version, which is usually referred to as the “Clover”-action. From the publicly available configurations we chose those with small lattice spacings,  $a \lesssim 0.1$  fm. In Table 1 we also provide the references containing detailed information about the simulation parameters and the algorithms used in producing these configurations. The values of lattice spacings, given in Table 1, are obtained from  $r_0/a$ , computed on each of these lattices, extrapolated to the chiral limit and then by choosing  $r_0 = 0.467$  fm. Other popular choice is  $r_0 = 0.5$  fm, which would make the lattice spacing 7% larger. To our purpose that error on fixing the lattice spacing is, however, completely immaterial. We should emphasize that we do not work in the partially quenched situations. Instead, we fix the hopping parameter ( $\kappa_q$ ) of our valence light quark in correlation functions (5) and in those appearing in Eqs. (13), (14) to be equal to that of the corresponding dynamical (“sea”) quark, also listed in Table 1.

In this Letter we do not use the so-called “all-to-all” propagators. The feasibility study of using that technique in the computation of  $\hat{g}$ -coupling has been made recently in Ref. [10], showing the substantial reduction in statistical errors. We plan to adopt that technique in our future studies.

In Table 2 we provide the list of all results relevant to the subject of this Letter, that we directly extracted from the correlation functions computed on all lattices from Table 1. For an easier comparison, the values of the pseudoscalar light meson masses, as well as of the bare light quark masses inferred from the axial vector Ward identity, are given in lattice units. They are fully consistent with those reported in Refs. [17–19]. Concerning the

**Table 1**

Basic information on the sets of unquenched gauge field configurations with  $N_f = 2$  dynamical Wilson quarks, the hopping parameters of which are specified for each set. “WP” stands for the Wilson–Plaquette gauge action, and Clover is a standard distinction to indicate the non-perturbative  $\mathcal{O}(a)$ -improved Wilson quark action. More information on each set of configurations can be found in the quoted references. All lattice volumes are  $24^3 \times 48$ .

Action	$\beta$ [a (fm)]	# meas.	$\kappa_{\text{sea}}$	Ref.
WP/Wilson Set 1	5.8 [0.054(2)]	50	$\kappa_1 = 0.1535$	[17]
		50	$\kappa_2 = 0.1538$	
		50	$\kappa_3 = 0.1540$	
		50	$\kappa_4 = 0.1541$	
Iwasaki/Clover Set 2	2.1 [0.099(0)]	100	$\kappa_1 = 0.1357$	[18]
		100	$\kappa_2 = 0.1367$	
		100	$\kappa_3 = 0.1374$	
		100	$\kappa_4 = 0.1382$	
WP/Clover Set 3	5.29 [0.075(1)]	60	$\kappa_1 = 0.1355$	[19]
		80	$\kappa_2 = 0.1359$	
		100	$\kappa_3 = 0.1362$	

**Table 2**

Direct numerical results extracted from the correlation functions calculated on all of the ensembles of the lattices with parameters enumerated in Table 1.

$\beta$	$\kappa_q$	$am_\pi$	$am_{\text{AWI}}^{(0)}$	$Z_V(g_0^2, m_q)$	$Z_A(g_0^2, m_q)$	$\hat{g}_q$
5.8 Set 1	0.1535	0.262(4)	0.0333(6)	0.720(38)	0.908(50)	0.683(52)
	0.1538	0.236(4)	0.0260(3)	0.705(41)	0.858(51)	0.714(71)
	0.1540	0.221(3)	0.0215(4)	0.643(57)	0.795(70)	0.742(81)
	0.1541	0.182(7)	0.0180(4)	0.654(61)	0.827(81)	0.628(62)
2.1 Set 2	0.1357	0.631(2)	0.1078(5)	0.742(13)	0.822(13)	0.738(42)
	0.1367	0.519(2)	0.0743(3)	0.751(25)	0.839(18)	0.736(56)
	0.1374	0.422(2)	0.0513(4)	0.754(38)	0.847(26)	0.691(55)
	0.1382	0.298(2)	0.0252(4)	0.752(82)	0.829(50)	0.622(61)
5.29 Set 3	0.1355	0.327(2)	0.0357(3)	0.772(2)	0.793(5)	0.683(63)
	0.1359	0.245(2)	0.0206(2)	0.770(4)	0.786(9)	0.658(61)
	0.1362	0.155(2)	0.0086(3)	0.760(10)	0.779(14)	0.711(62)

renormalization constants  $Z_{V,A}$ , they are obtained from the “light–light” correlation functions which we computed on the lattice and then combined to verify the Ward identities in Eqs. (13), (14). After inspection, we found the common plateau-region for all our 11 data sets to be between  $10 \leq t \leq 14$ . Finally, in the three-point correlation function in Eq. (5) the fixed source operator is set at  $t_y = 13$ , and – as already mentioned – the results for  $\hat{g}_q$  are obtained from the fit to a constant in Eq. (12), on the interval  $5 \leq t_x \leq 8$ . We checked that our results remain stable when  $t_y = 12$ . Directly extracted values for the bare couplings  $\hat{g}_q$ , from all of the lattice data sets considered in this Letter, i.e. before multiplying them by its corresponding  $Z_A$ , are listed in Table 2. In Fig. 3, instead, we plot the renormalized coupling  $\hat{g}_q$ , as a function of the squared light–light pseudoscalar meson (“pion”), mass now given in physical units (in  $\text{GeV}^2$ ). That conversion is made by computing  $r_0 m_\pi$  for each of our data sets and then use  $r_0 = 0.467$  fm (or,  $r_0 = 2.367 \text{ GeV}^{-1}$ ). We reiterate that opting for  $r_0 = 0.5$  fm ( $r_0 = 2.534 \text{ GeV}^{-1}$ ), does not alter our final results in any significant way.

The last step to reach the coupling  $\hat{g}$ , which is our final goal, is to make the extrapolation to the chiral limit. To that end we attempt either a simple linear fit or a fit guided by the expression derived in HMChPT [20], i.e.,

$$\hat{g}^q = \hat{g}_{\text{lin}}(1 + c_{\text{lin}} m_\pi^2), \quad (15)$$

$$\hat{g}^q = \hat{g}_0 \left[ 1 - \frac{4\hat{g}_0^2}{(4\pi f)^2} m_\pi^2 \log(m_\pi^2) + c_0 m_\pi^2 \right], \quad (16)$$

where  $\hat{g}_0$  is then the soft pion coupling that is to be used in applying the HMChPT formulas when extrapolating the phenomenologically interesting quantities computed on the lattice to the physical light quark mass limit. From Fig. 3 it is obvious that this task is quite difficult if one is doing it separately for each  $\beta$ . More specifically, applying the linear fit (15) to each of our data sets we obtain

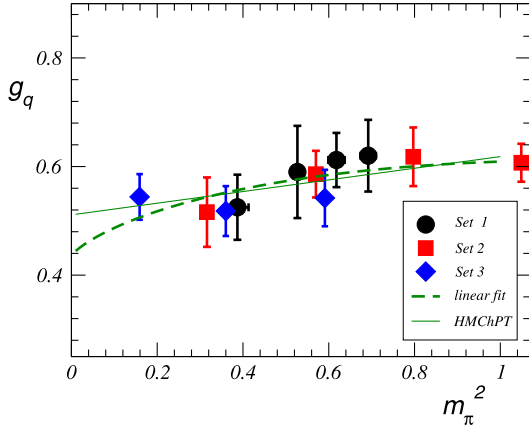
$$\hat{g}_{\text{lin}} = \{(0.40 \pm 0.15)_1, (0.52 \pm 0.07)_2, (0.54 \pm 0.06)_3\}, \quad (17)$$

while from the fit to HMChPT (16) we get

$$\hat{g}_0 = \{(0.36 \pm 0.11)_1, (0.43 \pm 0.04)_2, (0.48 \pm 0.04)_3\}, \quad (18)$$

where the index on the right-hand side labels the data sets like in Tables 1 and 2. As it could have been anticipated from Eq. (16), the results of the HMChPT fit ( $\hat{g}_0$ ) are lower than the results of linear extrapolation ( $\hat{g}_{\text{lin}}$ ). The values obtained from different sets are consistent within the errors. It is obvious that we cannot make a precision determination of this coupling yet, but it is clear that the unquenched lattice data also point to the fact that the  $\hat{g}$  coupling is considerably smaller in the static-heavy quark limit than in the case of the heavy charm quark. Since the heavy quark is





**Fig. 3.**  $\hat{g}_q$  computed from the ratio in Eq. (12) for all of our lattice data sets listed in Table 1, after accounting for the axial current renormalization constants computed on the same ensembles of gauge field configurations. They are plotted as a function of the light pseudoscalar meson (“pion”) mass squared (in  $\text{GeV}^2$ ).

only a spectator, this information – that the  $1/m_h^n$ -corrections are large – is significant, and somewhat surprising. If one simply feeds the difference by a linear  $1/m_c$ -term, it is quite interesting to notice that from the light cone QCD sum rules one get a similar size is such a correction in spite of the fact that the absolute value for the pionic couplings were considerably underestimated [4].

Since we are not aiming at a percent-level precision determination of this coupling we can try and see what happens if all the data are combined and fit them together to Eqs. (15), (16). We are, of course, aware that our three sets suffer from different discretization errors but since the lattice spacing is small ( $a < 0.1$  fm) and the common renormalization procedure has been applied to all of them, it is reasonable to assume that the remaining discretization errors are not likely to matter, in view of our statistical error ( $\sim 10\%$ ). If we combine all of our data, we then obtain

$$\hat{g}_{\text{lin}} = 0.51 \pm 0.04, \quad c_{\text{lin}} = (0.21 \pm 0.12) \text{ GeV}^{-1}, \quad (19)$$

while with the HMChPT formula (15) we have

$$\hat{g}_0 = 0.44 \pm 0.03, \quad c_0 = (0.40 \pm 0.12) \text{ GeV}^{-1}. \quad (20)$$

Another possibility is to exclude the data with  $m_\pi^2 \geq 0.6 \text{ GeV}^2$ , which gives  $\hat{g}_0 = 0.46 \pm 0.04$ . We also checked that our resulting  $\hat{g}_0$  is insensitive to the variation of  $f \in (120, 132) \text{ MeV}$ , latter being  $f_\pi^{\text{phys}}$ .

Before concluding we should compare our result to the existing unquenched value for  $\hat{g}$  reported in Ref. [10]. The main advantage of the calculation presented in Ref. [10] with respect to ours is that they used the so-called all-to-all light quark propagators so that their resulting statistical errors are much smaller. However, the lattices we used here are finer and the associated discretization errors should be smaller. In addition, here we also use various gauge and quark actions, to show that our results are robust in that respect too (of course within our error bars). A reasonable comparison with Ref. [10] can be made by using our results from Set-2 because these data correspond to the same gauge and quark actions as those used in Ref. [10], although the lattice spacing we use here is smaller. Comparing the bare quantities, we see that – for example – when the pion mass is  $m_\pi \approx 0.75 \text{ GeV}$ , from Ref. [10] we read  $g_q^{\beta=1.8} = 0.68(1)$ ,  $g_q^{\beta=1.95} = 0.69(1)$ , while our  $g_q^{\beta=2.1} = 0.69(6)$ . Therefore they fully agree although our statistical errors are much larger. Using the perturbative (boosted) 1-loop expression (bpt), the result for the overall renormalization constant in all three cases are equal among themselves within less

than 1%, so that the renormalization constants computed in bpt would not spoil this comparison, which seems to indicate that the discretization errors are indeed small. The step in which we go beyond Ref. [10] is that we evaluate the axial renormalization constant non-perturbatively,  $Z_A^{\text{npr}}$ . This is particularly important when using the data obtained with Iwasaki gauge action because in that case the strong coupling is very large and the use of perturbation theory is far from being justified. Various boosting procedures can lead to various estimates of  $Z_A$ . We show that the boosting procedure used in Refs. [10,18] leads to the values very close to our non-perturbative estimate. More precisely, at our  $\beta = 2.1$ , we have  $Z_A^{\text{npr}}/Z_A^{\text{bpt}} = 0.90(1), 0.94(2), 0.97(3), 0.97(6)$ , when going from the heaviest to the lightest quark mass.

#### 4. Conclusions

In this Letter we report on the results of our calculations of the soft pion coupling to the lowest lying doublet of static heavy-light mesons. From our computations, in which we use the fully unquenched set-up and three different sets of gauge field configurations, all produced with Wilson gauge and fermion actions, we obtain that  $\hat{g}_0 = 0.44 \pm 0.03_{-0.00}^{+0.07}$ . The second error reflects the uncertainty due to chiral extrapolation and it is the difference between the results of linear fit and the fit in which HMChPT is used. If our result is to be used in the chiral extrapolations of the phenomenologically relevant quantities in  $B$ -physics the second error should be considered as flat. The reason is that our central value is obtained via HMChPT fit, but since the domain of applicability of HMChPT is still unclear [2] – as of now – both results (extrapolated linearly or by using the chiral loop correction) are equally valid.

On the more qualitative level, our results show/confirm that this coupling is smaller in the static limit than what is obtained when the heavy quark is propagating and is of the mass equal to that of the physical charm quark,  $\hat{g}_{\text{charm}} = 0.68 \pm 0.07$  [7]. It is intriguing that the  $\mathcal{O}(1/m_c^2)$  corrections are quite large for the quantity in which the heavy quark contributes only as a spectator. That feature can be safely studied on the lattice by means of the relativistic heavy quark action of Ref. [21], and we plan to do such a study. An obvious perspective concerning the determination of  $\hat{g}_0$  is to further reduce the errors, both statistical (by using the “all-to-all” propagator technique, like in Ref. [10]), and the systematic ones (in particular, those associated with chiral extrapolations). Once a per-cent accuracy is reached, it will be important to study carefully the effects of mixing with the lowest heavy-light excited states [those with  $j_\ell^P = (1/2)^+$ ]. The expression derived in HMChPT which accounts for those effects in  $\hat{g}_q$  already exist (first paper in Ref. [20]), but their use requires the knowledge of another pionic coupling, the one that parametrizes the  $S$ -wave pion emitted in the transition from a  $(1/2)^+ \rightarrow (1/2)^-$  states. Finally, the numerical tests concerning the impact of inclusion of heavier quarks in the vacuum fluctuations ( $s$  and  $c$ ) on the size of  $\hat{g}_0$ , would be highly welcome too.

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