



Multi-objective Optimisation of Marine Propellers

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Abstract

Real world problems have usually multiple objectives. These objective functions are often in conflict, making them highly challenging in terms of determining optimal solutions and analysing solutions obtained. In this work Multi-objective Particle Swarm Optimisation (MOPSO) is employed to optimise the shape of marine propellers for the first time. The two objectives identified are maximising efficiency and minimising cavitation. Several experiments are undertaken to observe and analyse the impacts of structural parameters (shape and number of blades) and operating conditions (RPM) on both objective. The paper also investigates the negative effects of uncertainties in parameters and operating conditions on efficiency and cavitation. Firstly, the results showed that MOPSO is able to find a very accurate and uniformly distributed approximation of the true Pareto optimal front. The analysis of the results also shows that a propeller with 5 or 6 blades operating between 180 and 190 RPM results in the best trade-offs for efficiency and cavitation. Secondly, the simulation results show the significant negative impacts of uncertainties on both objectives.

Keywords: Multi-objective Particle Swarm Optimisation, MOPSO, Marine Propeller Design, Cavitation, Efficiency

1 Introduction

Optimisation of real engineering problems is usually very challenging. There are many issues to address, such as multi-objectivity, feasibility, multi-modality, and uncertainty. Multi-objectivity refers to having multiple objectives for a problem. There is generally not a single, optimal solution: a solution might be better than others in one of the objectives but worse in others.. In this case a set of solutions, called the Pareto optimal set, is the answer to a problem with multiple objectives. The Pareto optimal set contains Pareto optimal solutions that represent the best possible trade-offs between the objectives of a particular problem.

Generally speaking, there are two methods for solving multi-objective problems: *a priori* versus *a posteriori* [1]. In the former method all objectives are aggregated into a single objective, and the importance of each of the objectives is defined by a set of weights (coefficients) according

to the decision maker's needs. In the latter approach the multi-objective formulation of the problem is maintained and the aim is to find all or some of the Pareto optimal solutions. The main drawbacks of the *a priori* methods is the difficulty of finding proper weights to satisfy the decision makers' preferences, and their inability to solve certain classes of problems. In contrast, maintaining the multi-objective formulation of the problem allows the exploration of its behaviour across a range of design parameters and operating conditions. Despite this substantial advantage, *a posteriori* methods require specific operators and algorithms to handle conflicting objectives, which are generally computationally expensive.

Some *a posteriori* techniques for handling multi-objective are Multi-Objective Evolutionary Algorithms (MOEA) [3]. MOEAs are considered as stochastic methods which start the optimisation process with a random population. The random population is then evolved over a pre-defined number of steps called generations. The individuals are compared with respect to their objective values during optimisation. Finally, the optimisation process is terminated by satisfaction of an end criterion. MOEAs have been applied to many real problems. This is due to such features as their derivative-free mechanism, local Pareto optimal solutions avoidance, and obtaining the Pareto optimal set in one run.

Some of the well-regarded MOEAs are Non-dominated Sorting Genetic Algorithm second version (NSGA-II) [5], Multi-Objective Particle Swarm Optimisation (MOPSO) [4], and MOEA/D. These techniques have been widely applied in science and industry [3]. This work concentrates on multi-objective optimisation of marine propellers using MOPSO. Propeller designers consider two key objectives: efficiency and cavitation. The ultimate goal is to design a propeller with maximum efficiency and minimum cavitation. However, the process of optimising the shape of a propeller is very challenging due to the nature of Computational Fluid Dynamics (CFD) problems. In general, CFD problems have many constraints. Such constraints provide very narrow feasible regions for the search space, making CFD problems very complex. The rest of the paper is organised as follows:

For reasons of space, details of the MOPSO algorithm have been omitted. The form of the algorithm used was that of Coello Coello and Lechuga [4]. The search process was terminated by satisfaction of an end condition as outlined in Mirjalili *et al.* [6]. Section 2 discusses the problem of propeller design, relevant preliminaries, and problem formulation. The results and discussion are then provided in Section 3. Finally, Section 4 concludes the work and suggests some directions for future research.

2 Marine Propeller design and related work

2.1 Propeller design

Due to the relatively high density of water, the efficiency of propellers for marine vehicles is very important in comparison with aircraft. The efficiency of propellers refers to the amount of the power of the motor(s) that is converted to thrust. In addition to the efficiency, this conversion should be done with a minimum level of vibration and noise. The third characteristic of a good propeller is low surface erosion which is caused by cavitation. Finding a balance between these three features is a challenging task which should be considered during the design process of a propeller. The main part of a propeller is its blades. The geometric shape of these blades should satisfy all the above-mentioned requirements.

The propeller adds velocity (Δv) to an incoming velocity (v) of the surrounding fluid. This acceleration is created in two places: the first half in front of the propeller and the second half behind the propeller. A propeller is rotated, which swirls the outflow. The amount of this swirl

is based on the rotation speed of the motor and energy loss. Efficient propellers lose 1% to 5% of their power because of swirl. The thrust of propellers is calculated as follows [2]:

$$T = \frac{\pi}{4} D^2 (v + \frac{\Delta v}{2}) p \Delta v \quad (1)$$

where T is thrust, D is the propeller diameter, v is the velocity of the incoming flow, Δv is the additional velocity which is created by the propeller, and p is the density of the fluid.

It may be seen in equation 1 that the final thrust depends on the the volume of the incoming stream which has been accelerated per unit of time, the amount of this acceleration, and the density of the medium.

Power is defined as force times distance per time. The required power to drive a vehicle with a velocity of v using the available thrust is calculated as follows:

$$P_a = Tv \quad (2)$$

One of the objectives of optimisation in propellers is to create as much thrust as possible with the smallest amount of power. This is the efficiency of propellers which can be expressed as follows:

$$\eta = \frac{P_a}{P_{engine}} = \frac{Tv}{P_{engine}} \quad (3)$$

The efficiency of a propeller can be calculated as follows:

$$\eta(x) = \frac{JK_T(x)}{2\pi K_Q(x)} \quad (4)$$

where J is the advance number, K_T is the propeller thrust coefficient, and K_Q is the propeller torque coefficient. J is defined as follows:

$$J = \frac{V_a}{nD} \quad (5)$$

where V_a is the axial velocity, n is rotational velocity, and D is the diameter of the propeller.

By substitution of terms the efficiency can also be presented as follows:

$$\eta(x) = \frac{V_a}{2\pi nD} \frac{K_T(x)}{K_Q(x)} \quad (6)$$

The thrust coefficient (K_T) and torque coefficient (K_Q) are calculated as follows:

$$K_T = \sum_{n=1}^{39} C_{T_n}(J)^{s_n} \left(\frac{P}{D}\right)^{t_n} \left(\frac{A_e}{A_o}\right)^{u_n} (Z)^{v_n} \quad (7)$$

$$K_Q = \sum_{n=1}^{47} C_{Q_n}(J)^{s_n} \left(\frac{P}{D}\right)^{t_n} \left(\frac{A_e}{A_o}\right)^{u_n} (Z)^{v_n} \quad (8)$$

where P/D is the pitch ratio, A_e/A_o is the disk ratio of the propeller, Z is the number of blades, and C_{T_n} , C_{Q_n} , s_n , t_n , u_n , v_n are corresponding regression coefficients.

There is another issue in propellers called cavitation. When the blades of a propeller move through water at high speed, low pressure regions form as the water accelerates and moves past the blades. This can cause bubbles to form, which collapse and can cause strong local

shockwaves which result in erosion of propellers. The sensitivity of the propeller to cavitation is calculated as follows:

$$\sigma_{n,0.8} = \frac{(p_a + pgh_{0.8} - p_v)}{0.5\rho(\pi nD)^2} \quad (9)$$

where p_a is the atmospheric pressure, p_v indicates the vapour pressure of water, g is the acceleration due to gravity, and $h_{0.8}$ shows immersion of 0.8 blade radius when the blade is at the position of 12:00.

The ultimate goal here is to design a propeller with the highest efficiency and the lowest cavitation sensitivity.

In order to find the final geometrical shape of the blade, standard NACA airfoils were selected as shown in Fig. 1. It may be seen in this figure that two parameters define the shape of the airfoil: maximum thickness and chord length. In this paper ten airfoils were considered along the blade, so the total number of parameters is 20.

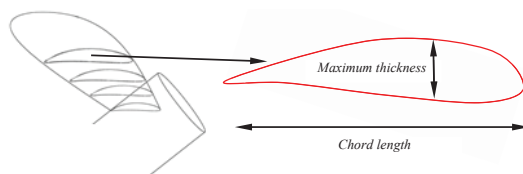


Figure 1: Airfoils along the blade define the shape of the propeller (NACA a=0.8 meanline and NACA 65A010 thickness)

The final parameter vector is as follows:

$$\vec{X}_i = (T_1, C_1, T_2, C_2, \dots, T_{10}, C_{10}) \quad (10)$$

where T_i and C_i indicates the thickness and chord length of the i -th airfoil along the blade.

Finally, the problem can be formulated as follows:

$$\text{Suppose : } \vec{X}_i = (T_i, C_i), i = 1, 2, \dots, 10 \quad (11)$$

$$\text{Maximise : } \eta(X) \quad (12)$$

$$\text{Minimise : } V_c(X) \quad (13)$$

$$\text{Subject to : } \text{Thrust} \geq 40000 \quad (14)$$

2.2 Related work

Work using an heuristic algorithm to optimise the shape of a B-series propeller has been reported in the literature [8]. The NSGA-II algorithm was employed to optimise the shape of a propeller with specific performance in given conditions. Designing a propeller for ships was considered as a multi-objective problem with two objectives: minimising propeller efficiency and maximising the thrust coefficient. The author considered two main constraints for this problem. These constraints were wake friction and thrust deduction. The author did not specify the exact number of variables. NGA II provided 15 Pareto optimal solutions. Finally, a decision making technique was used to select one of the Pareto optimal solutions as the best solution. One other study has investigated the application of the NSGA-II multi-objective optimisation algorithm to the design of marine propellers [7]. Difficulties were reported with the nature of the design space: constraints applied isolated feasible results into “small islands” and the optimisation algorithm failed to converge.

3 Results and discussion

A MOPSO algorithm was employed to find the Pareto optimal front. A population of 100 search agents and maximum number of 200 iterations were chosen for MOPSO. The main case study is a ship propeller with 2 metre diameter as shown in Fig. 2.

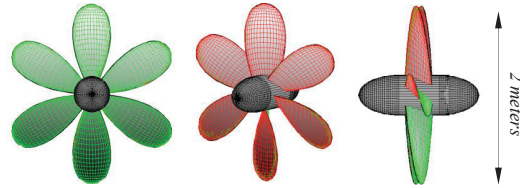


Figure 2: Propeller used as the case study

The experiments undertaken using MOPSO were as follow:

1. Observing the behaviour of MOPSO in finding an accurate approximation and well-spread Pareto optimal solutions
2. Observing the effect of the number of blades on the efficiency and cavitation of the propeller
3. Finding the optimal number of blades
4. Observing the effects of Revolutions Per Minute (RPM) on the efficiency and cavitation of the propeller
5. Finding the optimal values (range) for RPM
6. Observing the effects of uncertainties in operating conditions (RPM) on the the Pareto optimal fronts obtained by MOPSO
7. Observing the effects of uncertainties in structural parameters on the Pareto optimal fronts obtained by MOPSO
8. Post analysis of the result to extract the possible physical behaviour and impacts of the parameters on the efficiency and cavitation of the propeller

The following subsections present and discuss the results for each of these experiments.

3.1 Approximating the Pareto Front using the algorithm

The MOPSO algorithm was run 4 time on the problem and the best Pareto Front obtained is illustrated in Fig. 3. The blue points in the figure, at left, show that the MOPSO algorithm was able to find a set of uniformly distributed Pareto optimal solutions across both objectives. The search history of points sampled by particles during optimisation is illustrated by black points. The search history also shows that the MOPSO algorithm explored and exploited the search space efficiently, which results in obtaining this uniformly distributed and accurately converged Pareto optimal front. The accurate convergence of the solutions obtained is due to the intrinsic high exploitation of the MOPSO algorithm around the selected leaders, *gbest* and *pbests*, in each iteration. The uniform distribution originates from the selection mechanism of leaders in MOPSO. Since particle guides were selected from the less populated parts of the archive, there

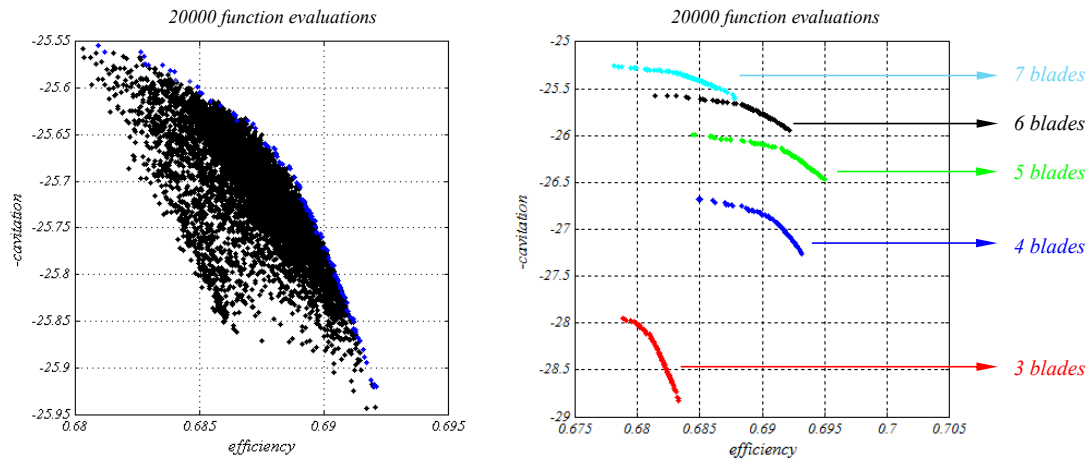


Figure 3: (left) Pareto optimal front obtained by the MOPSO algorithm, (right) Pareto optimal fronts for different numbers of blades

was always a high tendency toward finding Pareto optimal solutions along the regions of the Pareto Front with lower distribution.

3.2 Number of blades

The effects of the number of blades on efficiency and cavitation were investigated. Five problems were first formulated by altering the number of blades from 3 to 7. The MOPSO algorithm was then employed to approximate the Pareto optimal fronts. The MOPSO algorithm was run 4 times on each of the problems and the best Pareto optimal fronts obtained are illustrated in Fig. 3, at right.

This figure shows that the efficiency increases proportional to the number of blades, up to a limit of 5 blades. Beyond this number, efficiency decreases. The figure also shows that cavitation decreases in proportion to the number of blades. The reason why the majority of ship propellers are made of 5 or 6 blades is due to the shape of the fronts in Fig. 3. The highest efficiency, which is the main objective in ship propellers, is achieved by 5 or 6 blades. Therefore, 5 blades are chosen by default unless cavitation is a major issue.

3.3 Revolutions Per Minute (RPM)

RPM is one of the most important operating conditions for propellers. In order to observe the effects of this parameter on the efficiency and cavitation of the propeller, a 5-blade version of the propeller investigated in the previous section was selected. The RPM considered was limited to the range of 150 to 250. Since changing the RPM changes the operating conditions of the propeller significantly, two types of experiments were done, as follows:

1. Finding the Pareto optimal front for the propeller at RPM increments of 10.
2. Parametrising the RPM and finding the optimal front for it using MOPSO.

The MOPSO algorithm was employed to find the Pareto optimal front for the propeller at each of the 11 RPM varying from 150 to 250. The algorithm was run 4 times on each case and the

best Pareto optimal fronts obtained are illustrated in Fig. 4, at left. This figure first shows that there is no feasible Pareto optimal solution when $RPM = 150, 160$, or 250 . For the remaining RPMs, it may be observed that increasing RPM generally results in decreasing efficiency and increasing cavitation. Although increasing the RPM seems to increase the thrust, these results show that high RPM is not very effective and risks increased damage to the propeller in long term use due to the high cavitation. The peak of the high efficiency and low cavitation occurred between $RPM = 170$ and $RPM = 180$. Therefore, such RPM rates can be recommended when using a 5-blade version of the ship propeller investigated.

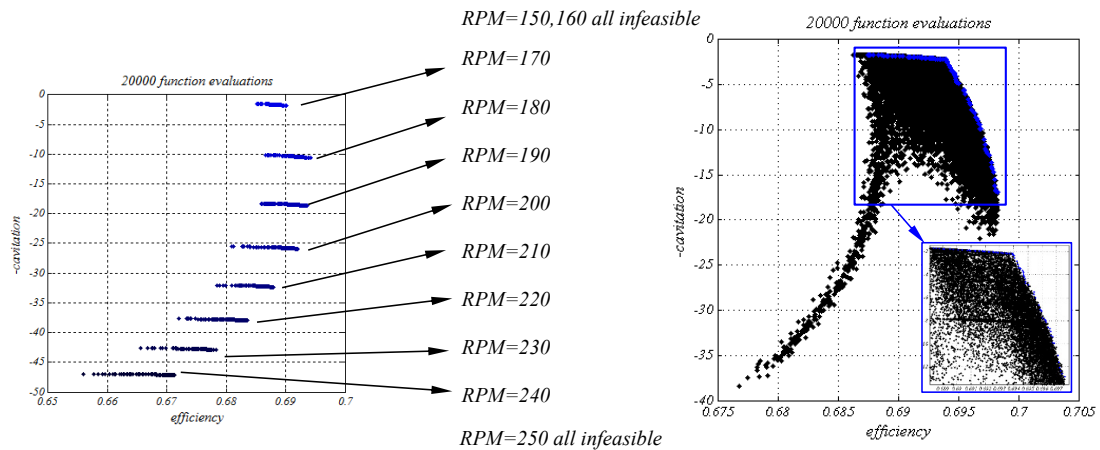


Figure 4: (left) Best Pareto Fronts obtained for different RPM (right) Optimal RPM

To find the optimal values for the RPM, this operating condition was parametrised and optimised by MOPSO as well. The number of parameters increases to 21 when considering RPM as a parameter, but the same number of particles and iteration were chosen to approximate the Pareto optimal front. The best Pareto optimal front is illustrated in Fig. 4, at right.

The Pareto optimal front obtained shows that the Pareto optimal solutions mostly tend to the best Pareto optimal front found for $RPM = 170$. Almost 20% of the solutions are distributed between the Pareto optimal fronts for $RPM = 170$ and $RPM = 180$. The search history of the MOPSO algorithm is also illustrated in Fig. 4 to make sure that all of the Pareto fronts obtained in the previous experiment have been explored. The search history clearly illustrates that the fronts have been found by MOPSO, but all of them are dominated by the Pareto optimal front for $RPM = 170$ and the solutions between $RPM = 170$ and $RPM = 180$ (blue points).

A parallel coordinates visualisation of the solutions from the Pareto optimal front in Fig. 4(right) is shown in Fig. 5, for RPM between 170 and 180.

It may be observed in this figure that the range of the RPM is between 170 to 180. However, the density of solutions is higher close to $RPM = 170$. Other features that can be seen in this representation are the diversity of solution values for the first three airfoils ($P1 - P6$) suggesting there are not specific values for these critical to performance. In contrast, values for $P7 - P10$ show clustering to particular values, indicating the significant influence of the shape of the fifth and sixth airfoils on performance.

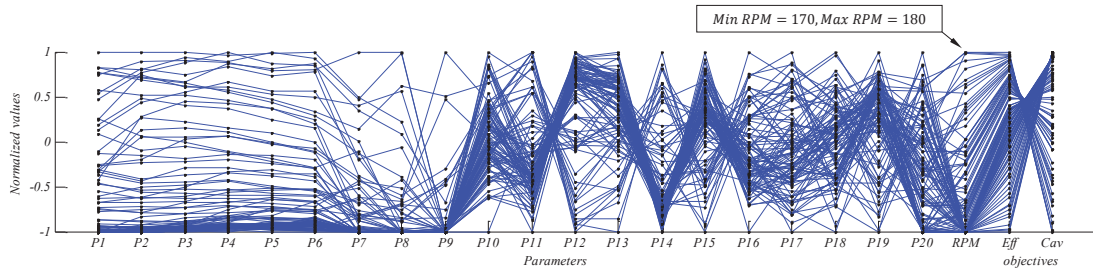


Figure 5: Optimal RPM coordinates

3.4 Effects of uncertainties in operating conditions on the objectives

This experiment was to investigate the effects of uncertainties in the RPM on the efficiency/-cavitation of the propeller. To do this, the best Pareto optimal front obtained for the 5-blade propeller in Fig. 3 was selected as the main front. The efficiency and cavitation of the Pareto optimal solutions in this front were then re-calculated by changing the RPM as the most important environmental condition. The projections of the solutions are illustrated in Fig. 6. Note that the perturbation considered is $\delta = \pm 1$, which has been recommended by an expert in the field of mechanical engineering.

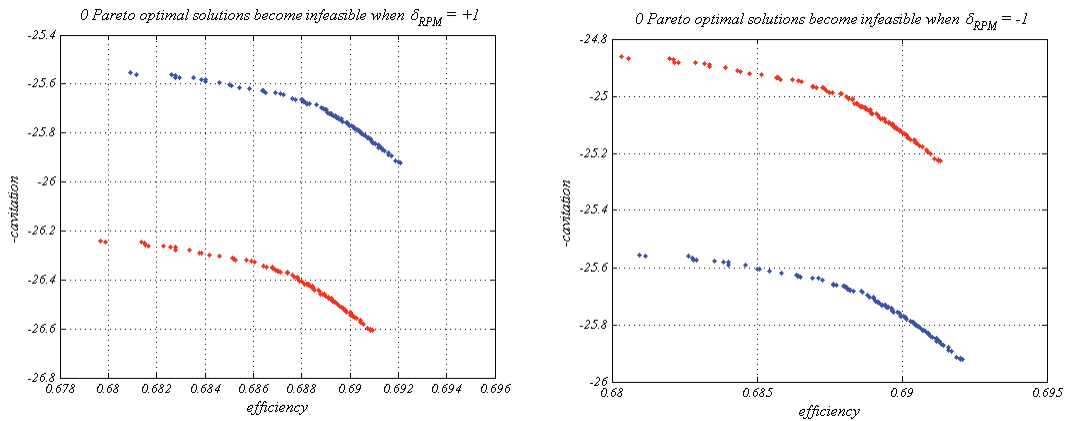


Figure 6: Pareto optimal solutions in case of (left) $\delta_{RPM} = +1$, (right) $\delta_{RPM} = -1$ fluctuations in RPM (right). Original values are shown in blue, perturbed results in red.

As Fig. 6 (left) shows, the efficiencies of all the Pareto optimal solutions obtained decrease when $\delta_{RPM} = +1$ perturbations occur. The cavitation of Pareto optimal solutions is also increased. A similar behaviour for the efficiency can be observed in Fig. 6 (right). This figures shows that the efficiencies of Pareto optimal solutions decrease when $\delta_{RPM} = -1$. However, the cavitation is decreased, which is obviously due to the lower rate of RPM. These results shows that perturbations in RPM can have significant negative impacts on the expected and desired efficiencies. The cavitation can also vary substantially with uncertainties in RPM.

3.5 Uncertainties in the structural parameters

Uncertainties may occur in the structural parameters as well. This type of uncertainty mostly originates from manufacturing errors. This subsection considers the maximum permitted errors, according to ISO 484/2-1981, that can alter the optimal values obtained by MOPSO. Note that the perturbation considered is $\delta = 1.5\%$.

The Pareto optimal solutions obtained in Fig. 3 are first selected. Maximum positive and negative perturbations are then applied to parameters. Finally, the objectives of the Pareto optimal solutions obtained are re-calculated. The results are illustrated in Fig. 7.

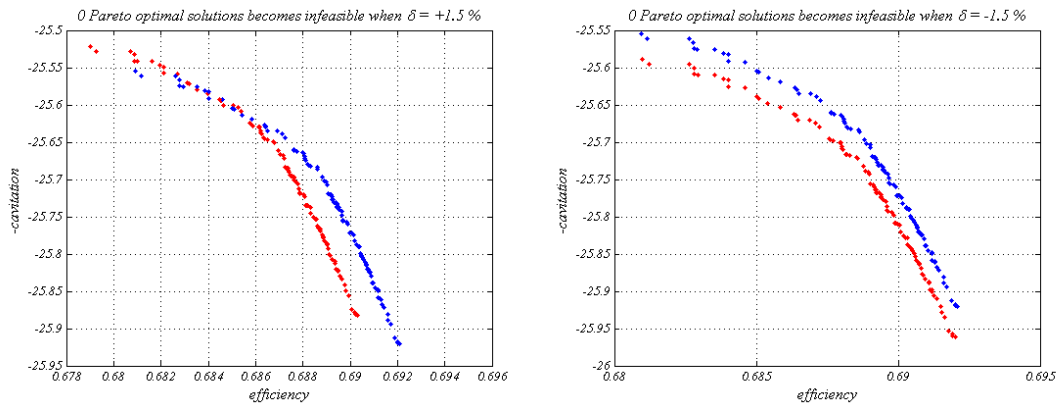


Figure 7: Pareto optimal solutions in case of (left) $\delta = +1.5\%$ (right) $\delta = -1.5\%$ perturbations in parameters. Original values are shown in blue, perturbed results in red.

The trend is similar to the results of the preceding subsection, in that the uncertainties in parameters also degrade the expected efficiency significantly. In addition, the results show that the cavitation can vary dramatically in case of uncertainties in parameters.

In summary, these results strongly show the remarkably negative impacts of perturbations on the performance of marine propellers and emphasise the importance of considering such undesirable inputs when designing propellers. As a further illustration of the effect on efficiency, it may be noted that the perturbations in structural parameters gave rise to reductions in efficiency of about 0.25%. This translates directly to increased fuel consumption, the biggest cost in marine shipping. For the vessels for which the propeller tested is suited, generally those up to 100 tonnes displacement, the difference may be an increase of 40 litres per day. Scaling the effect to typical container ships operating under normal conditions, the increased fuel usage could be over half a tonne of bunker oil a day, increasing not only costs but also environmental emissions.

4 Conclusion

In this paper, the shape of a ship propeller was optimised considering two objectives: efficiency versus cavitation. MOPSO was first employed to find the best approximation of the true Pareto optimal front for the propeller, then to undertake several experiments investigating the effect of the number of blades, RPM, and uncertainties in manufacturing and operating parameters. The results of MOPSO were also analysed to identify the possible physical behaviour of the propeller.

The results showed that the best efficiency and cavitation can be achieved by having five or six blades, since any other number of blades significantly degrades one of the objectives. It was observed that the best Pareto optimal front can be obtained when the propeller is operating at $\text{RPM} = 170$ to 180 . However, the results of the impact of uncertainties on RPM show that the optimal RPM is very sensitive to perturbation: efficiency and cavitation can be degraded significantly by a small amount of uncertainty. Simulation of manufacturing perturbations also revealed that both of the objectives for the Pareto optimal solutions obtained can vary dramatically.

For future work, we are planning to apply robust optimisation techniques to handle uncertainties in RPM and shape parameters as this appears critically necessary for propeller design, due to the severity of the impact of uncertainties. We also intend to explore more detailed variation of blade shape.

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