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Null Controllability of Semilinear Integrodifferential Systems in Banach Space

J. P. DAUER
Department of Mathematics
University of Tennessee at Chattanooga
Chattanooga, TN 37403, U.S.A.

P. BALASUBRAMANIAM
Department of Mathematics, Kongu Engineering College
Perundurai, Tamil Nadu, India

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Abstract—Sufficient conditions for null controllability of semilinear integrodifferential systems with unbounded linear operators in Banach space are established. The results are obtained using semigroup of linear operators, fractional powers of operators, and the Schauder fixed point theorem. An application to partial integrodifferential equations is given.

Keywords—Controllability, Semilinear systems, Integrodifferential systems.

1. INTRODUCTION

Controllability of linear and nonlinear systems represented by ordinary differential equations in finite dimensional space has been extensively studied. Several authors have extended the concept to infinite dimensional systems represented by the evolution equations with bounded operators in Banach spaces [1–4]. Recently, Balachandran *et al.* [5] established sufficient conditions for the local null controllability of nonlinear functional differential systems using the Schauder fixed point theorem with unbounded operators in Banach space. The purpose of this paper is to extend the use of fixed point theorems to integrodifferential systems with unbounded operators and infinite delay in Banach space. In particular, results are obtained for the null controllability of semilinear integrodifferential systems with unbounded linear operators in Banach space. The considered system is an abstract formulation of parabolic partial functional differential equations discussed in [6].

2. PRELIMINARIES

Consider the semilinear integrodifferential system

$$\begin{aligned} \dot{x}_t(\phi) + A(t)x_t(\phi) &= \int_{-\infty}^{t} f(t, s, x_s(\phi))\, ds + (Bu)(t), \qquad && t \in J = [0,T], \\ x_t(\phi) &= \phi(t), && t \in (-\infty, 0]. \end{aligned} \tag{1}$$

Here $\{A(t) : t \geq 0\}$ is a family of unbounded linear operators mapping a Banach space X to itself. The state $x(t)$ takes the values in the Banach space X and the control function u is given

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in $L^2(J,U)$, a Banach space of admissible control functions with U as a Banach space. B is a bounded linear operator from U into X. Let X_α denote the interpolation space defined in the α power of $A(0)$, that is,

$$X_\alpha = \{x : x \in D(A^\alpha(0))\},$$

with $\|x\|_\alpha = \|A^\alpha(0)x\|$. The space C_α is the space of bounded uniformly continuous functions from $(-\infty,0]$ to X_α endowed with the supremum norm

$$\|\phi\|_{C_\alpha} = \sup\{\|\phi(\theta)\|_\alpha : \theta \in (-\infty,0]\}.$$

Further, let $\phi \in C_\alpha$ for some α, $0<\alpha<1$, and f is a continuous nonlinear operator of $J \times J \times X_\alpha$ into X.

For the existence of a solution of (1), we need the following assumptions (see [6]).

(i) The domain $D(A)$ of $A(t)$, $t \in J$, is dense in the Banach space X and independent of t.

(ii) For each $t \in [0,\infty)$, the resolvent $R(\lambda, A(t))$ exists for all λ such that $\operatorname{Re}\lambda \geq 0$, and there exists $C>0$ such that

$$\|R(\lambda, A(t))\| \leq \frac{C}{(|\lambda|+1)}.$$

(iii) For any $t,s,\tau \in J$, there exists a $0<\delta<1$ and $K>0$ such that

$$\left\|(A(t)-A(\tau))A^{-1}(s)\right\| \leq K|t-\tau|^\delta.$$

And for each $t \in [0,T]$ and some $\lambda \in \rho(A(t))$, the resolvent $R(\lambda, A(t))$ set of $A(t)$ is a compact operator.

Conditions (i) and (ii) imply that for each $r \geq 0$, $-A(r)$ is the infinitesimal generator of an analytic semigroup $\{e^{-tA(r)} : t \geq 0\}$. The fact that $0 \in \rho(A(r))$ and that $-A(r)$ generates an analytic semigroup implies that fractional powers of $A(r)$ can be defined for $0<\alpha<1$. We set

$$A^{-\alpha}(r) = \frac{1}{\Gamma(\alpha)} \int_0^\infty s^{\alpha-1} e^{-sA(r)}\, ds,$$

where $\Gamma(\cdot)$ denotes the Eulerian gamma function. The operator $A^{-\alpha}(r)$ can be shown to be a bounded linear operator with a well-defined inverse [7]. Let $A^\alpha(r) = (A^{-\alpha}(r))^{-1}$.

If Conditions (i)–(iii) are satisfied, then there exists an operator valued function $X(t,\tau)$ which is defined on the triangle $0 \leq \tau \leq t < \infty$ and has values in $B(U)$ [8, p. 109]. $X(t,\tau)$ is strongly continuous jointly in t and in τ, and maps X into $D(A)$ if $t>\tau$. The family $\{X(t,\tau) : 0 \leq \tau \leq t < \infty\}$ satisfies the identity

$$X(t,\tau) = X(t,s)X(s,\tau), \qquad \text{for } 0 \leq \tau \leq s \leq t < \infty.$$

The derivative $\frac{\partial X(t,\tau)}{\partial t}$ exists in the strong operator topology and belongs to X whenever $0 \leq \tau < t$ [9, p. 129]. Finally, $X(t,\tau)$ satisfies the following initial value problem:

$$\begin{aligned} \frac{\partial X(t,\tau)}{\partial t} &= A(t)X(t,\tau), \qquad \text{for } t>\tau, \\ X(\tau,\tau) &= I, \end{aligned}$$

when I is the identity operator.

Further, if $0 \leq \nu \leq 1$, $0 \leq \beta \leq \delta < 1+\mu$, then for any $0 \leq \tau \leq t < t+\Delta t < T$ and $s \in J$, there exists a $K(\beta,\nu,\delta)$ such that (see [6])

$$\left\|A^\nu(s)\left[X(t+\Delta t,\tau) - X(t,\tau)\right]A^{-\beta}(\tau)\right\| \leq K(\beta,\nu,\delta)(\Delta t)^{\delta-\nu}|t-\tau|^{\beta-\delta}. \tag{2}$$

(iv) There exists a $\beta_0 > \alpha$ and $\omega > 0$, such that for all $0 \le \beta < \beta_0$, there exists a $K_\beta > 0$ satisfying

$$\left\|A^\beta(0)X(t,s)\right\| \le K_\beta(t-s)^{-\beta}e^{-\omega(t-s)}. \tag{3}$$

(v) The function $f : J \times J \times X_\alpha \to X$ is continuous, $f(t,s,0) = 0$ for all $s \le t$, and there exists an $L > 0$ and $\nu > 0$ such that

$$\|f(t,s,x) - f(t,s,y)\| \le e^{-\nu(t-s)}L\|x-y\|_\alpha. \tag{4}$$

(vi) The bounded linear operator $A^{-\alpha}(t)$ is compact for all $\alpha \in (0,1]$.

Note that (v) is satisfied by the function

$$f(t,s,x) = \begin{cases} 0, & \text{for } s \le t, \\ ce^{-\alpha(t-s)}\arctan x, & \alpha > 0 \text{ and } c > 0. \end{cases}$$

Suppose that (i)–(iii) and (v) are satisfied. If $\phi \in C_\alpha$ and $\phi(0) \in D(A^\beta(0))$ for some $\beta > \alpha$ and (iv) is satisfied for some $\beta_0 > \beta$, then there exists a unique function $x(\phi)(\cdot) : J \to X$ such that

$$\begin{aligned} x_t(\phi) &= X(t,0)\phi(0) + \int_0^t X(t,s)\int_{-\infty}^s f(s,\tau,x_\tau(\phi))\,d\tau\,ds, \\ &\quad + \int_0^t X(t,s)(Bu)(s)\,ds, \qquad \text{for } t \in J, \end{aligned} \tag{5a}$$

$$x_t(\phi) = \phi(t), \qquad t \in (-\infty, 0]. \tag{5b}$$

Moreover, $x_t(\phi)$ is continuously differentiable for $t > 0$ and satisfies (1).

DEFINITION. *The system (1) is said to be* **null controllable** *on the interval J if for every continuous initial function $\phi \in C_\alpha$, there exists a control $u \in L^2(J,U)$ such that the solution $x_t(\phi)$ of (1) satisfies $x_T(\phi) = 0$.*

3. MAIN RESULT

THEOREM 3.1. *Suppose Conditions (i)–(vi) hold and the linear operator W from U into X defined by*

$$Wu = \int_0^T X(T,s)Bu(s)\,ds$$

has an invertible operator W^{-1} defined on $L^2(J;U)/\ker W$. If there exist positive constants N_1, N_2 such that

$$\|B\| \le N_1 \quad \text{and} \quad \left\|W^{-1}\right\| \le N_2,$$

then the system (1) is null controllable on J.

PROOF. Using the hypothesis, define the control

$$u(t) = -W^{-1}\left[X(T,0)\phi(0) + \int_0^T X(T,s)\int_{-\infty}^s f(s,\tau,x_\tau(\phi))\,d\tau\,ds\right](t). \tag{6}$$

Now it is shown that, when using this control, the operator defined by

$$\begin{aligned} \Phi x_t(\phi) &= \phi(t), \qquad \text{for } t \in (-\infty,0], \\ \Phi x_t(\phi) &= X(t,0)\phi(0) - \int_0^t X(t,\mu)BW^{-1}\left[X(T,0)\phi(0)\right. \\ &\quad \left. + \int_0^T X(T,s)\int_{-\infty}^s f(s,\tau,x_\tau(\phi))\,d\tau\,ds\right](\mu)\,d\mu \\ &\quad + \int_0^t X(t,s)\int_{-\infty}^s f(s,\tau,x_\tau(\phi))\,d\tau\,ds \end{aligned} \tag{7}$$

has a fixed point. This fixed point is a solution of equation (1). Clearly $\Phi x_T(\phi) = 0$, which means that the control u steers the semilinear integrodifferential system from the initial function ϕ to 0 in time T provided the nonlinear operator Φ has a fixed point.

Define

$$S = \{x \in C((-\infty, T] : X_\alpha(0)) : \|x_t(\phi)\| = \|\phi(t)\| \text{ on } t \in (-\infty, 0] \text{ and } \|x_t(\phi)\| \leq N,\ t \in J\}.$$

Consider the transformation

$$\Phi : S \to C((-\infty, T] : X_\alpha(0))$$

defined by (7).

It is claimed that $\Phi : S \to S$. For that

$$\begin{aligned} \|\Phi x_t(\phi)\|_\alpha &\leq \|\phi(0)\| K_\alpha t^{-\alpha} e^{-\omega t} + K_\alpha L \int_0^t (t-s)^{-\alpha} e^{-\omega(t-s)} \int_{-\infty}^s e^{-\nu(s-\tau)} \|x_\tau(\phi)\|_\alpha \, d\tau \, ds \\ &+ K_\alpha \int_0^t (t-\mu)^{-\alpha} e^{-\omega(t-\mu)} N_1 N_2 \Big\{ \|\phi(0)\| K_\alpha t^{-\alpha} e^{-\omega t} \\ &+ K_\alpha L \int_0^T (T-s)^{-\alpha} e^{-\omega(T-s)} \int_{-\infty}^s e^{-\nu(s-\tau)} \|x_\tau(\phi)\|_\alpha \, d\tau \, ds \Big\} (\mu) \, d\mu. \end{aligned}$$

Thus, there exists a $K_1 > 0$ such that for $t \in [\epsilon, T_0)$ $(\epsilon > 0)$

$$\begin{aligned} \|\Phi x_t(\phi)\|_\alpha &\leq K_1 + K_\alpha L \int_0^t (t-s)^{-\alpha} e^{-\omega(t-s)} \int_{-\infty}^s e^{-\nu(s-\tau)} \|x_\tau(\phi)\|_\alpha \, d\tau \, ds \\ &+ K_\alpha N_1 N_2 \int_0^t (t-\mu)^{-\alpha} e^{-\omega(t-\mu)} \Big\{ K_1 \\ &+ K_\alpha L \int_0^T (T-s)^{-\alpha} e^{-\omega(T-s)} \int_{-\infty}^s e^{-\nu(s-\tau)} \|x_\tau(\phi)\|_\alpha \, d\tau \, ds \Big\} (\mu) \, d\mu. \end{aligned}$$

We choose $\delta > 0$, and observe that

$$\begin{aligned} e^{-\delta t} \|\Phi x_t(\phi)\|_\alpha &\leq K_1 + K_\alpha L \int_0^t (t-s)^{-\alpha} e^{-(\omega+\delta)(t-s)} \int_{-\infty}^s e^{-(\nu+\delta)(s-\tau)} e^{-\delta\tau} \|x_\tau(\phi)\|_\alpha \, d\tau \, ds \\ &+ K_\alpha N_1 N_2 \int_0^t (t-\mu)^{-\alpha} e^{-\omega(t-\mu)} \Big\{ K_1 + K_\alpha L \int_0^T (T-s)^{-\alpha} e^{-(\omega+\delta)(T-s)} \\ &\times \int_{-\infty}^s e^{-(\nu+\delta)(s-\tau)} e^{-\delta\tau} \|x_\tau(\phi)\|_\alpha \, d\tau \, ds \Big\} (\mu) \, d\mu, \end{aligned}$$

for $K_2 > \sup\{K_1, \|\phi\| C_\alpha\}$, and for $y(t) = e^{-\delta t} \|x_t(\phi)\|_\alpha$,

$$\begin{aligned} \|\Phi y(t)\|_\alpha &\leq K_2 + K_\alpha L \int_0^t (t-s)^{-\alpha} e^{-(\omega+\delta)(t-s)} \int_{-\infty}^s e^{-(\nu+\delta)(s-\tau)} y(\tau) \, d\tau \, ds \\ &+ N_1 N_2 \int_0^T K_\alpha (t-\mu)^{-\alpha} e^{-\omega(t-\mu)} \Big\{ K_2 \\ &+ K_\alpha L \int_0^T (T-s)^{-\alpha} e^{-(\omega+\delta)(T-s)} \int_{-\infty}^s e^{-(\nu+\delta)(s-\tau)} y(\tau) \, d\tau \, ds \Big\} (\mu) \, d\mu. \end{aligned}$$

If $K_3 > 0$, introduce a function $z(\cdot)$ by

$$z(s) = \begin{cases} K_3, & s \in [0, T_0], \\ K_3 e^{-\delta s}, & s < 0, \end{cases}$$

and note that for $t \geq 0$,

$$
\begin{aligned}
&K_2 + K_\alpha L \int_0^t (t-s)^{-\alpha} e^{-(\omega+\delta)(t-s)} \int_{-\infty}^s e^{-(\nu+\delta)(s-\tau)} y(\tau)\, d\tau\, ds \\
&\quad + N_1 N_2 K_\alpha \int_0^T (t-\mu)^{-\alpha} e^{-\omega(t-\mu)} \Big\{ K_2 \\
&\quad + K_\alpha L \int_0^T (T-s)^{-\alpha} e^{-(\omega+\delta)(T-s)} \int_{-\infty}^s e^{-(\nu+\delta)(s-\tau)} y(\tau)\, d\tau\, ds \Big\} (\mu)\, d\mu \\
&\leq K_2 + K_\alpha L \Gamma(1-\alpha) \left(\frac{1}{\nu}\right) (\omega+\delta)^{\alpha-1} K_3 + N_1 N_2 K_\alpha \int_0^T (t-\mu)^{\alpha} e^{-\omega(t-\mu)} \\
&\quad \times \left\{ K_2 + K_\alpha L \Gamma(1-\alpha) \left(\frac{1}{\nu}\right) (\omega+\delta)^{\alpha-1} K_3 \right\} (\mu)\, d\mu.
\end{aligned}
$$

Thus, if $\delta > 0$ is chosen large enough to ensure that

$$K_\alpha L \Gamma(1-\alpha) \left(\frac{1}{\nu}\right) (\omega+\delta)^{\alpha-1} < 1,$$

and K_3 is such that

$$K_3 > K_2 \left[1 - 3K_\alpha L \Gamma(1-\alpha) \left(\frac{1}{\nu}\right) (\omega+\delta)^{\alpha-1} \right]^{-1},$$

then for suitable constants N_1, N_2, it follows that:

$$\sup_{t \in [0,T_0)} \|\Phi x_t(\phi)\| \leq K_3\, e^{\delta T} = N.$$

To prove equicontinuous, if $0 < \beta < \theta$, apply $A^\alpha(0)$ to each side of (7) and use (4):

$$
\begin{aligned}
&\|\Phi x_{t_1}(\phi) - \Phi x_{t_2}(\phi)\|_\alpha \leq \|A^\alpha(0)\, (X(t_1,0) - X(t_2,0))\, \phi(0)\| \\
&\quad - \int_0^t \|A^\alpha(0)\, (X(t_1,\mu) - X(t_2,\mu))\| \, BW^{-1} \Big\{ \|A^\alpha(0) X(T,0) \phi(0)\| \\
&\quad + \int_0^T \|X(T,s)\| \int_{-\infty}^s \|f(s,\tau,x_\tau(\phi))\|\, d\tau\, ds \Big\} (\mu)\, d\mu \\
&\quad + \int_{t_1}^{t_2} \|A^\alpha(0) X(t_2,\mu)\| \, BW^{-1} \Big\{ \|A^\alpha(0) X(T,0) \phi(0)\| \\
&\quad + \int_0^T \|X(T,s)\| \int_{-\infty}^s \|f(s,\tau,x_\tau(\phi))\|\, d\tau\, ds \Big\} (\mu)\, d\mu \\
&\quad - \int_0^{t_1} \|A^\alpha(0)\, (X(t_1,s) - X(t_2,s))\| \int_{-\infty}^s \|f(s,\tau,x_\tau(\phi))\|\, d\tau\, ds \\
&\quad - \int_{t_1}^{t_2} \|A^\alpha(0) X(t_2,s)\| \int_{-\infty}^s \|f(s,\tau,x_\tau(\phi))\|\, d\tau\, ds \\
&\leq K(0,\alpha,\theta)(t_1-t_2)^{\theta-\alpha} t_2^{-\theta} + N_1 N_2 K(0,\alpha,\theta)(t_1-t_2)^{\theta-\alpha} \int_0^{t_1} (t_2-\mu)^{-\theta} \\
&\quad \times \left\{ K_\alpha T^{-\alpha} e^{-\omega T} \|\phi(0)\| + K_\alpha L \int_0^T (T-s)^{-\alpha} e^{-\omega(T-s)} \int_{-\infty}^s e^{-\nu(s-\tau)} N\, d\tau\, ds \right\} (\mu)\, d\mu
\end{aligned}
$$

$$+ K_\alpha \int_{t_1}^{t_2} (t_2-\mu)e^{-\omega(t_2-s)} \Bigg\{ K_\alpha T^{-\alpha} e^{-\omega T} \|\phi(0)\|$$

$$+ K_\alpha L \int_0^T (T-s)^{-\alpha} e^{-\omega(T-s)} \int_{-\infty}^{s} e^{-\nu(s-\tau)} N \, d\tau \, ds \Bigg\} (\mu) \, d\mu$$

$$+ LK(0,\alpha,\theta)(t_1-t_2)^{\theta-\alpha} \int_0^{t_1} (t_2-s)^{-\theta} \int_{-\infty}^{s} e^{-\nu(s-\tau)} N \, d\tau \, ds$$

$$+ K_\alpha L \int_{t_1}^{t_2} (t_2-s)e^{-\omega(t_2-s)} \int_{-\infty}^{s} e^{-\nu(s-\tau)} N \, d\tau \, ds.$$

Hence, the set $Y_1 = \{(\Phi x)(t) : x \in Y_0\}$ is equicontinuous. The set Y_1 is also precompact for each fixed $t \in (-\infty, T]$. $A^{-\alpha}(0)$ is compact for all $\alpha \in (0,1]$, since $A^{-1}(0)$ is compact by Condition (iv). Therefore, for any β such that $1 > \beta > \alpha \geq 0$ and $t \in (-\infty, T]$, the set $\{A^\beta(0)\Phi x_t(\phi)\}$ is bounded in X since

$$\left\|A^\beta(0)\Phi x_t(\phi)\right\|_\alpha \leq \left\|A^\beta(0)X(t,0)\phi(0)\right\|$$

$$+ N_1 N_2 \int_0^t \left\|A^\beta(0)X(t,\mu)\right\| \Bigg\{ \left\|A^\beta(0)X(T,0)\phi(0)\right\|$$

$$+ \int_0^T \left\|A^\beta(0)X(T,s)\right\| \int_{-\infty}^{s} \|f(s,\tau,x_\tau(\phi))\| \, d\tau \, ds \Bigg\} (\mu) \, d\mu$$

$$+ \int_0^t \left\|A^\beta(0)X(t,s)\right\| \int_{-\infty}^{s} \|f(s,\tau,x_\tau(\phi))\| \, d\tau \, ds$$

$$\leq K_\alpha T^{-\beta} e^{-\omega t} \|\phi(0)\| + K_\alpha N_1 N_2 \int_0^t (t-\mu)^{-\beta} e^{-\omega(t-\mu)} \Bigg\{ K_\alpha T^{-\beta} e^{-\omega T} \|\phi(0)\|$$

$$+ K_\alpha L \int_0^T (T-s)^{-\beta} e^{-\omega(T-s)} \int_{-\infty}^{s} e^{-\nu(s-\tau)} N \, d\tau \, ds \Bigg\} (\mu) \, d\mu$$

$$+ K_\alpha L \int_0^t (t-s)^{-\beta} e^{-\omega(t-s)} \int_{-\infty}^{s} e^{-\nu(s-\tau)} N \, d\tau \, ds.$$

Now, since the mapping $A^{-\beta} : X \to X_\alpha(0)$ is compact for each $\beta > \alpha$, it follows that the set Y_1 is a precompact set of C_α. Hence, the result follows by applying Schauder's fixed point theorem to the mapping Φ.

EXAMPLE. Consider the partial integrodifferential equations

$$\begin{aligned} \frac{\partial u}{\partial t} + A(t,u,D)x &= \int_{-\infty}^{t} g(t,s)f(\nabla u) \, ds + (Bv)(t), \\ u(x,t) &= 0, \qquad (x,t) \in \partial\Omega \times R, \\ u(x,\theta) &= \phi(x,\theta), \qquad \theta \in (-\infty, 0], \end{aligned} \tag{8}$$

where

$$A(t,u,D) = \sum_{|x|\leq 2} a_n(x,t) D^n.$$

The operators $A(t,x,D)$ are assumed to be uniformly elliptic. Thus, there exists a constant $C_0 > 0$ such that

$$-\operatorname{Re} \sum_{|n|\leq 2} a_n(x,t)\xi^n \geq C_0|\xi|^2,$$

where Ω is a bounded domain in R^3 with smooth boundary. The coefficients $a_n(x,t)$ are smooth functions of $(x,t) \in \Omega \times R^+$, and there exist constants $C_2 > 0$ and $\mu \in (0,1)$ such that

$$|a_n(x,t) - a_n(x,\tau)| \leq C_2 |t-\tau|^\mu, \qquad \text{for } t,\tau > 0, \quad x \in \Omega.$$

Finally, stipulate that the coefficients $a_n(\cdot) : \overline{\Omega} \to R$ are such that

$$\lim_{t\to 0} \sup_x |a_n(x,t) - a_n(x)| = 0.$$

The nonlinearity $f(\cdot) : R^3 \to R$ vanishes at zero and has the property that there exists a $C_3 > 0$ satisfying

$$|f(u) - f(v)| \le C_3 \sum_{i=1}^{3} |u_i - v_i|.$$

The function $g(\cdot) : R^+ \to R$ is Lipschitz continuous. Moreover, there exist $C_4 > 0$ and $\nu > 0$ such that

$$|g(s)| \le C_4 e^{-\nu s}, \qquad \text{for } s \in (-\infty, 0].$$

Let $X = L^2(\Omega)$. A family of operators $\{A(t) : t \ge 0\}$ is introduced on $L^2(\Omega)$ by specifying $D = D(A(t)) = H^2(\Omega) \cap H_0^1(\Omega)$ and letting $A(t)u = A(t,x,D)u$ for $u \in D$. If $\alpha > 3/4$, define $g : R^+ \times R \times X_\alpha \to X$ by setting

$$g(t,s,u) = g(t-s) f(\nabla u).$$

X_α will denote the interpolation space obtained from $A^\alpha(0)$. Suppose $\phi \in C_2$ and identify $u(x,t) = x_t(\phi)(x)$, then (8) assumes the form of the function space integrodifferential equation

$$\dot{x}_t(\phi) + Ax_t(\phi) = \int_{-\infty}^{t} f\left(t, s, x_s(\phi)\right) ds + (Bv)(t). \tag{9}$$

If $\phi \in C$ $(\alpha > 3/4)$, then (see [6]) there exists a unique function $x_t(\phi) : R \to X$ which satisfies equation (9) and

$$\|x_t(\phi) - x_t(\psi)\|_{0,\infty} \le C \|x_t(\phi) - x_t(\psi)\|_\alpha, \qquad \text{for some } C > 0.$$

This yields the solution and the system is controllable on J.

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