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Selecting a dynamic and stochastic path method for vehicle routing and scheduling problems

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Abstract

This paper investigates vehicle routing and scheduling which includes dynamic and stochastic elements of travel time on links. In order to consider the characteristics of travel time, three shortest path models were used. In the first model, the path is based on average travel time determined using Dijkstra’s method (1959). The second model is the adaptive least-expected time path model developed by Miller-Hooks (2001) and the third one is the expected shortest path presented by Fu and Rilett (1998). Vehicle routes and schedules were calculated based on link costs by the three shortest path models. Deliveries using the vehicle routes and schedules were simulated and total costs of deliveries were compared. On a test road network, the dynamic and stochastic shortest paths showed good performance in delivery simulations. However, in the road network of the central area of Osaka, average costs in delivery simulations are at the same level for all the shortest path models considered. Therefore, it can be said that the performance of vehicle routing and scheduling in delivery simulations is influenced by characteristics of travel time information, and it is observed that low-cost and stable vehicle routing and scheduling are obtained using dynamic and stochastic shortest path models.

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Keywords: Vehicle routing and scheduling; shortest path problem; dynamic and stochastic network

1. Introduction

In recent days, human lifestyle has changed with the development of a more social based economy. Physical distribution is profoundly affected by this change. For example, due to the spread of the internet it becomes easy to get more information about various things which are made far off in the distance. Similarly, logistics has also advanced and has become more complex. Therefore, it is necessary to improve the efficiency of logistics within a social based economy and transportation systems.

ITS technology enables us to obtain some useful data and construct more efficient transport systems. For logistics companies, real-time travel time information is important for freight transport to adhere to strict time windows and
plan efficient vehicle routing and scheduling. On the other hand, if only the fixed travel time information is available, it would be difficult to determine the optimal vehicle routing and scheduling due to variable nature of travel time.

The Vehicle Routing Problem (VRP) can optimise the logistics costs of vehicle routing. The VRP has been studied in operations research and many rapid and robust solution algorithms have been developed for it (for example see Toth and Vigo, 2002; Kallehauge et al., 2006; Kohl, 1997). However, it is difficult to apply results of these algorithms to real situations. The reason is that, traffic situations change all of the time and travel costs between customers cannot be assumed to be fixed, as is the case with most of the published literature. Taniguchi et al. (2001) developed a vehicle routing and scheduling model which can be applied to real road networks by determining the travel costs between customers using travel time information.

This paper investigates the impacts of the choice of different shortest paths (travel costs) between customers in real road networks. Three types of shortest path models are considered to explore the effects of static or dynamic considerations of the travel time. These models have been used in conjunction with two different VRP models on a hypothetical network as well as on real road network in the central Osaka area.

2. Vehicle Routing and Scheduling Model

This study adopted Probabilistic Vehicle Routing and scheduling Problem with Time Windows (VRPTW-P) model presented by Taniguchi et al. (1999). The VRPTW-P model is defined as follows. A fleet of identical vehicles collect goods from customers and delivers them to the depot or delivers goods to customers from the depot. For each customer a designated time window, specifying the desired time period to be visited is also specified. The VRPTW-P model minimises the total cost of distributing goods with fixed truck capacities and designated time constraints. The total cost is composed of three components; (a) cost of vehicles, (b) vehicle operating costs that are proportional to the time travelled and spent waiting at customers, (c) penalty costs for violations of the designated pickup/delivery time at customers. The VRPTW-P model takes into account the uncertainty of link travel times on the road network to identify the optimal solution. The Vehicle Routing and scheduling Problem with Time Windows Forecasted (VRPTW-F) model does not consider the probability of arrival time and its effects on penalties. The conditions for vehicle routing and scheduling for the VRPTW-P are:

- A vehicle is allowed to make multiple routes per day,
- Each customer must be assigned to exactly one route of a vehicle and all the goods from each customer must be loaded on the vehicle at the same time,
- The total weight of the goods for a route must not exceed the capacity of the vehicle, and
- A vehicle should be operated within the designated time of operation, for instance from 8 a.m. to 5 p.m.

The problem is to determine the optimal assignment of vehicles to customers and the departure time as well as the order of visiting customers for a freight carrier. The model was formulated as follows.

Minimise

$$C(t_0, X) = \sum_{l=1}^{m} c_{f,l} \cdot \delta_l (x_l) + \sum_{l=1}^{m} E[C_{t,l}(t_{l,0}, x_l)] + \sum_{l=1}^{m} E[C_{c,l}(t_{l,0}, x_l)]$$

where

$$E[C_{t,l}(t_{l,0}, x_l)] = c_{t,l} \sum_{i=0}^{N_l} \left\{ T(T_{i,n(i)}, n(i), n(i+1)) + t_{c,n(i+1)} \right\}$$

$$E[C_{p,l}(t_{l,0}, x_l)] = \sum_{i=0}^{N_l} \int_{0}^{t} p_{l,n(i)}(t_{l,0}, t, x_l) \left\{ c_{d,n(i)}(t) + c_{c,n(i)}(t) \right\} dt$$

This paper investigates the impacts of the choice of different shortest paths (travel costs) between customers in real road networks. Three types of shortest path models are considered to explore the effects of static or dynamic considerations of the travel time. These models have been used in conjunction with two different VRP models on a hypothetical network as well as on real road network in the central Osaka area.
Subject to

\[ n_0 \geq 2 \]  \hspace{1cm} (4) \\
\[ n(0) = 0 \]  \hspace{1cm} (5) \\
\[ n(N_i) = 0 \]  \hspace{1cm} (6) \\
\[ \prod_{l=1}^{m} \prod_{i=1}^{N_l} \{n(i) - k\} = 0 \quad \forall k = 1, 2, \ldots, N \]  \hspace{1cm} (7) \\
\[ \sum_{l=1}^{m} N_l = N \]  \hspace{1cm} (8) \\
\[ \sum_{n(i) \in x_l} D(n(i)) = W_{l}(x_l) \]  \hspace{1cm} (9) \\
\[ W_{l}(x_l) \leq W_{c, l} \]  \hspace{1cm} (10) \\
\[ t_s \leq t_{l,0} \]  \hspace{1cm} (11) \\
\[ t'_{l,0} \leq t_e \]  \hspace{1cm} (12) \\

where

\[ t'_{l,0} = t_{l,0} + \sum_{i=0}^{N_l} \left\{ \bar{F}(t_{l,n(i)}, n(i), n(i+1)) + t_{c, n(i+1)} \right\} \]  \hspace{1cm} (13) \\

\[ C(t_0, X) \]  : total cost (yen) \\
\[ t_0 \]  : departure time vector for all vehicles at the depot \\
\[ t_0 = \{t_{l,0} | l = 1, m\} \] \\
\[ X \]  : assignment and order of visiting customers for all vehicles \\
\[ X = \{x_l | l = 1, m\} \] \\
\[ x_l \]  : assignment and order of visiting customers for vehicle \( l \) \\
\[ x_l = \{n(i) | i = 1, N_l\} \] \\
\[ n(i) \]  : node number of \( i \) th customer visited by a vehicle \\
\[ d(j) \]  : number of depot (= 0) \\
\[ N_l \]  : total number of customers visited by vehicle \( l \) \\
\[ n_0 \]  : total number of \( d(j) \) in \( X_l \) \\
\[ m \]  : maximum number of vehicles available \\
\[ c_{f,l} \]  : fixed cost for vehicle \( l \) (yen /vehicle)
\( \delta_l(x_l) \) : = 1; if vehicle \( l \) is used, = 0; otherwise
\( C_{l,0}(t_{l,0}, x_l) \) : operating cost for vehicle \( l \) (yen)
\( C_{p,l}(t_{l,0}, x_l) \) : penalty cost for vehicle \( l \) (yen)
\( c_{l,i,l} \) : operating cost per minute for vehicle \( l \) (yen/min)
\( t_{l,n(i)} \) : departure time of vehicle \( l \) at customer \( n(i) \)
\( \bar{f}(\bar{t}_{l,n(i)}, n(i), n(i + 1)) \) : average travel time of vehicle \( l \) between customer \( n(i) \) and \( n(i + 1) \) at time \( \bar{t}_{l,n(i)} \)
\( t_{e,n(i)} \) : loading/unloading time at customer \( n(i) \)
\( p_{l,n(i)}(t_{l,0}, t, x_l) \) : probability in which a vehicle that departs the depots at time \( t_{l,0} \) arrives at customer \( n(i) \) at time \( t \)
\( c_{d,n(i)}(t) \) : delay penalty cost per minute at customer \( n(i) \) (yen/min)
\( c_{e,n(i)}(t) \) : early arrival penalty cost per minute at customer \( n(i) \) (yen/min)
\( N \) : total number of customers
\( D(n(i)) \) : demand of customer \( n(i) \) (kg)
\( t'_{l,0} \) : last arrival time of vehicle \( l \) at the depot
\( t_s \) : starting of possible operation time of trucks
\( t_e \) : end of possible operation time of trucks
\( W_i(x_l) \) : load of vehicle \( l \) (kg)
\( W_{c,l} \) : capacity of vehicle \( l \) (kg)

The problem specified by equations (1) – (13) involves determining the variable \( X \), that is, the assignment of vehicles and the visiting order of customers and the variable \( t_0 \), the departure time of vehicles from the depot. Note, that \( n(0) \) and \( n(N_i + 1) \) represent the depot in equations (2) and (3).

Figure 1 shows the penalty for vehicle delay and early arrivals at customers. The time period \( (t_{n(i)}', t_{n(i)}') \) of the penalty function defines the width of the soft time window in which vehicles are requested to arrive at customers. If a vehicle arrives at a customer earlier than \( t_{n(i)}' \), it must wait until the start of the designated time window and a cost is incurred during waiting. If a vehicle is delayed, it must pay a penalty proportional to the amount of time it was delayed. This type of penalty is typically observed in goods distribution to shops and supermarkets in urban areas. Multiplying the penalty function and the probability of arrival time as shown in Figure 1, can identify the penalty of early arrivals and delay at customers for the probabilistic model.

The problem described herewith is a NP-hard (Non-deterministic Polynomial-hard) combinatorial optimisation problem. It requires heuristic methods to efficiently obtain a good solution. The model described in this paper uses a Genetic Algorithms (GA) to solve the VRPTW-P. GA was selected because it is a heuristic procedure that can simultaneously determine the departure time and the assignment of vehicles as well as the visiting order of customers.
3. Dynamic and Stochastic Shortest Path

If travel time information can be obtained for several periods in a day, it is necessary to reflect the characteristics of these large amounts of data in the VRP models. In order to leverage some travel time information, we apply this data to dynamic and stochastic shortest path models. We consider that the models including dynamic and stochastic elements are suitable for choosing paths between customers in the vehicle routing and scheduling.

3.1. Adaptive least-expected time path

Miller-Hooks (2001) studied minimum time paths in networks where travel time on each link is time-varying and is known only with uncertainty. In this adaptive least-expected time path problem, it is assumed that the arrival time at the end node is revealed upon arrival at each intermediate node in route to the destination. The adaptive expected time $\lambda_i(t)$ from node i to the given destination node N at departure time t are modelled below.

$$\lambda_i(t) = \min_{j \in \Pi^{-1}(i)} \sum_{p=1}^{K_{p}(t)} (\tau_{ij}^p(t) + \lambda_j(t + \tau_{ij}^p(t)))\rho_{ij}^p(t)$$  \hspace{1cm} (14)$$

with boundary condition, $\lambda_N(t) = 0 \ \forall t \in T$, 
where

\[ \tau_{ij}^k(t) \] the set of non-negative real-valued possible travel times for traversing the link at time \( t \), \( k=1, \ldots, K_{ij}(t) \), where \( K_{ij}(t) \) is the number of possible travel time values on link \((i, j)\) at time \( t \)

\[ \Gamma_{ij}(i) = \{ j | (i, j) \in A \} \] : the set of successor nodes

\( A \) : the set of directed links connecting the nodes

\[ \rho_{ij}^k(t) \] : the probability where travel time \( \tau_{ij}^k(t) \) occurs

Miller-Hooks (2001) introduced two algorithms to determine the adaptive least-expected time (LET) paths and compared the average performance of these algorithms. As a result of the comparison, the Stochastic Decreasing Order of Time (SDOT) algorithm often outperformed the Adaptive Least Expected Time (ALET) algorithm in sparse networks such as transportation networks. Therefore, the SDOT algorithm has been used in this study to determine the adaptive LET paths.

The SDOT algorithm uses the probability related to the travel time, while determining the adaptive LET paths. Therefore, each travel time in the adaptive LET paths includes a distribution of travel times. Furthermore, computational burden of the SDOT algorithm is the same as standard shortest path algorithms.

3.2. Expected shortest path

Fu and Rilett (1998) studied the problem of finding the expected shortest path in a traffic network where the link travel times are modelled as a continuous-time stochastic process. They modelled mean and variance of route travel times as the approximation model of the recursive formula. The second order approximation model of the mean arrival time and the first order approximation model of the variance of the arrival time are given below.

\[
E[Y_i] \equiv E[Y_i] + \mu_{x_a}(E[Y_i]) + \frac{1}{2} \mu_{x_a}^2(E[Y_i]) \cdot Var[Y_i]
\]

\[
Var[Y_i] \equiv \{1 + \mu_{x_a}(E[Y_i]) \cdot Var[Y_i] + \nu_{x_a}(E[Y_i])
\]

where

\( Y_i \) : the arrival time at node \( i \)

\( \mu_{x_a}(t) \) : the mean of the stochastic process \( \{X_a(t), t \in T\} \) corresponding to its first-order probability density function

\( \nu_{x_a}(t) \) : the variance of the random variable \( X_a(t) \)

In the approximation model of mean route travel time, a term containing the variance of the arrival time is considered. So, their model is suitable for choosing a stable route with respect to the varying travel times. Since their model is a mathematical model, it is difficult to obtain optimal expected shortest paths. However, procedures were presented for calculating the derivatives of the mean and variance of the link travel time and for identifying appropriate path by a heuristic algorithm.
4. Case Study on a Test Road Network

4.1. Overview

The test road network, as shown in Figure 2, was used for the initial evaluation of our scheme. In the test road network, the depot was located at node 5 and customers were located at other nodes. Two types of travel times were set based on free speed, viz. 12 minutes and 16 minutes. In this network, travel time information on each link could be obtained by using a traffic simulator. The basic number of OD trips between nodes was fixed and the OD trips for each day were changed randomly within 10%. We simulated the traffic situation for 40 days in this network and travel time information was obtained at each departure time for 1 minute intervals. Travel time information for 30 days was used as historical data for calculating efficient vehicle routing and scheduling. The remaining travel time information for 10 days was used in the delivery simulation for estimating the vehicle routes and schedules based on historical data.

![Test road network](image)

Figure 2 Test road network

In order to calculate the vehicle routes and schedules, a complete network, directly connecting the depot and customers with each other is required. When such a network for vehicle routing and scheduling is reconstructed from the test road network, travel costs between customers have to be determined. Since the VRP is a cost minimisation problem, it is preferable to choose links with smaller cost values between customers. However, it is difficult to choose the shortest path under uncertain traffic situations. So, three shortest path models were applied to the test road network. Two of the models have been already mentioned above i.e. the models given by Miller-Hooks...
and by Fu and Rilett. The third shortest path model is based on average travel time (static travel time) using Dijkstra’s method (Dijkstra, 1959). Using the shortest path results between customers from these three models separate complete networks were developed and the vehicle routing and scheduling was calculated using the VRPTW-F and the VRPTW-P on these complete networks.

4.2. Result

Table 1 shows a comparison of results among each path choice and the VRP model. First, objective function values of the best solution obtained from the VRP model are compared. In this table, the TCs mean estimated total costs, that is the objective function values of the best solution obtained from the VRP model using the historical data for 30 days. On the other hand, the AC is calculated by averaging total costs based on the delivery simulation where vehicles are operated for 10 days according to the vehicle routing and scheduling obtained from the VRP model. Finally, in order to demonstrate differences between the TC and each total cost based on the delivery simulation for 10 days, the AV are defined as the average of variations in the latter costs based on the former cost. In this result, the TCs are same level among each VRP model and path choice model, but the ACs using paths based on average travel times is more than those using other path choice models. Moreover, AVs using paths based on average travel times is also more than those using other path choice models. Comparing the results of the VRPTW-F with the results of the VRPTW-P, the ACs in the VRPTW-P are less than that in the VRPTW-F, so are the AVs. We can infer that this result arises from incorporating dynamic and stochastic elements of the travel times into path choice and VRP models.

Considering all cost components of the objective functions separately, it was observed that the same size and number of vehicles are used in each prior planning. Therefore, the costs of vehicles in all cases are the same. Since the penalty costs per minute are larger than the operating cost per minute, GA tries to find solutions with much less penalty cost as possible. As a result, penalties do not occur at the time of prior planning, but in the delivery simulation, delay and early arrival penalty actually occurred. Particularly, penalties often occur in the prior planning using paths based on average travel times. The travel times on each link based in the traffic simulation vary considerably. This could be the main reason why penalty costs were observed during the delivery simulation. Therefore, we should investigate the performance of the vehicle routing and scheduling using the real travel time data on real road networks.

Finally, it can be concluded that in this case study, path choice models considering dynamic and stochastic elements in travel times are suitable for the stable vehicle routing and scheduling. Expected shortest paths by Fu and Rilett and least-expected time paths by Miller-Hooks choose low-cost and stable paths for the vehicle routing and scheduling. When both shortest path models are compared, the expected shortest paths by Fu and Rilett outperform the least-expected time paths by Miller-Hooks in the AV. The expected shortest path by Fu and Rilett incorporates the effects of the variance of route travel times. Hence, the AV using expected shortest paths is least and therefore the vehicle routing and scheduling based on these paths is most stable. Furthermore, it becomes clear that the variation of travel times in dynamic and stochastic elements have a significant effect on the cost and stability of the vehicle routing and scheduling.

Table 1 Comparison of results among each path choice in VRP models (The test road network)

<table>
<thead>
<tr>
<th>Type of vehicle routing and scheduling model</th>
<th>Type of path choice</th>
<th>TC (yen)</th>
<th>AC (yen)</th>
<th>AV (yen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VRPTW-F</td>
<td>Paths based on average travel times</td>
<td>35,926</td>
<td>51,745</td>
<td>21,533</td>
</tr>
<tr>
<td></td>
<td>Expected shortest paths by Fu and Rilett</td>
<td>36,032</td>
<td>42,862</td>
<td>10,594</td>
</tr>
<tr>
<td></td>
<td>Least-expected time paths by Miller-Hooks</td>
<td>35,648</td>
<td>42,165</td>
<td>13,431</td>
</tr>
<tr>
<td>VRPTW-P</td>
<td>Paths based on average travel times</td>
<td>35,586</td>
<td>45,576</td>
<td>12,576</td>
</tr>
<tr>
<td></td>
<td>Expected shortest paths by Fu and Rilett</td>
<td>35,787</td>
<td>39,766</td>
<td>5,469</td>
</tr>
<tr>
<td></td>
<td>Least-expected time paths by Miller-Hooks</td>
<td>35,717</td>
<td>40,997</td>
<td>6,708</td>
</tr>
</tbody>
</table>
5. Case Study on the Road Network of the Central Area of Osaka

In this case study, the road network of the central area of Osaka is used. Figure 3 shows the road network of the central area of Osaka. The purpose of this case study is to use a realistic road network in terms of travel time data. In this network, travel time data is mainly obtained from VICS (Vehicle Information and Communication Systems). In VICS, travel time information is available at 5 minutes intervals on each links. As is the case with case study on the test road network, travel time information for 30 days was used as historical data and travel time information for subsequent 10 days was used in the delivery simulations. The depot and customers were set hypothetically on 25 nodes of the road network. Similar to the case study based on the test road network, the prior planning and delivery simulations were calculated for the realistic instance as well. The results of this calculation are shown in Table 2. The notation in this table is same as Table 1.

In this case study, total costs of vehicle routing and scheduling in the real road network are compared among each path choice model. The results show that TCs in each path choice model and VRP model are not so different. Furthermore, ACs in the delivery simulation are also very similar to each other for different path choice models. So, path choice models had little effect on the performance of vehicle routing and scheduling. On the other hand, because the AV of expected shortest paths are least in both VRP models, it is considered that the expected shortest path model by Fu and Rilett results in stable vehicle routing and scheduling in terms of fluctuating range in actual deliveries. TCs and ACs are at the same level for both the VRP models. However, AVs are actually different for different VRP models, particularly AVs in the VRPTW-P model are less than those in the VRPTW-F model. Therefore, it is observed that the VRPTW-P model can propose more stable vehicle routing and scheduling than the VRPTW-F model.

Figure 3 The road network of the central area of Osaka
<table>
<thead>
<tr>
<th>Type of vehicle routing and scheduling model</th>
<th>Type of path choice</th>
<th>TC (yen)</th>
<th>AC (yen)</th>
<th>AV (yen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VRPTW-F</td>
<td>Paths based on average travel times</td>
<td>31,345</td>
<td>35,485</td>
<td>4,414</td>
</tr>
<tr>
<td></td>
<td>Expected shortest paths by Fu and Rilett</td>
<td>31,569</td>
<td>35,400</td>
<td>4,204</td>
</tr>
<tr>
<td></td>
<td>Least-expected time paths by Miller-Hooks</td>
<td>31,261</td>
<td>36,320</td>
<td>5,360</td>
</tr>
<tr>
<td>VRPTW-P</td>
<td>Paths based on average travel times</td>
<td>32,143</td>
<td>35,526</td>
<td>3,844</td>
</tr>
<tr>
<td></td>
<td>Expected shortest paths by Fu and Rilett</td>
<td>32,351</td>
<td>35,517</td>
<td>3,644</td>
</tr>
<tr>
<td></td>
<td>Least-expected time paths by Miller-Hooks</td>
<td>32,376</td>
<td>35,701</td>
<td>3,939</td>
</tr>
</tbody>
</table>

### Table 2 Comparison of results among each path choice in VRP models (The road network of the central area of Osaka)

#### 6. Conclusion

In this paper, the effects of travel costs between customers in the VRP were investigated. Path choices between customers on road networks were focused on. In order to choose paths between customers, various shortest path models were employed. The standard shortest path was calculated using average travel times on each link using Dijkstra’s method that considers the static travel time. The dynamic and stochastic shortest paths were calculated using the model given by Fu and Rilett and by the model proposed by Miller-Hooks. After that, vehicle routing and scheduling was calculated based on these shortest paths. In order to estimate the performances of prior vehicle routing and scheduling, deliveries were simulated for 10 operation days. Finally, total costs of the vehicle routing and scheduling were compared for these delivery simulations.

In the case study on the test road network, results of the average total costs of delivery simulations show that the dynamic and stochastic shortest path outperforms the standard shortest path (based on static travel time information) in vehicle routing and scheduling. However, in the case study on the real road network, results of the average total costs of delivery simulations are not so much different. Therefore, from results of the average total costs of delivery simulations, it is confirmed that the performance of the dynamic and stochastic shortest path in vehicle routing and scheduling is as same or better. Therefore, it is important to consider dynamic and stochastic elements of travel time to obtain the effective and stable vehicle routing and scheduling. Furthermore, the average variations in total costs based on the delivery simulation for 10 days are focused on. In both road networks, average variations in the VRPTW-P model are less than those in the VRPTW-F model. Therefore, the VRPTW-P model can obtain more stable vehicle routing and scheduling than the VRPTW-F model.

Computational results show that dynamic and stochastic elements of travel times are important for vehicle routing and scheduling in terms of costs between customers. However, differences among costs of each path choice and of the corresponding VRP models are quite small. This may be due to the fact that only departure time and distribution of travel times is considered and use of limited travel time information (only one month). Future work will investigate the effects of additional dynamic and stochastic elements of the travel time such as incidents and worst traffic situation. Moreover, because it is considered that time windows in vehicle routing and scheduling models have much influence on the routing results, it is necessary to investigate effects of the relationship between time windows and travel time information.

#### References


