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Comparison of the accommodation theories of Coleman and of Helmholtz by finite element simulations

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Abstract

Purpose: The accommodation process of the human eye is still a controversial subject. Coleman assumes that the lens, together with the zonula fibers, forms a diaphragm which is held in a catenary shape due to the pressure difference between the aqueous and vitreous body of the lens. The aim of the paper is to compare the results of two simulations (according to the Helmholtz and to the Coleman theories) with ultrasonographic data.

Methods: An axisymmetric static finite element model of the lens was generated using the literature data for geometry, material properties and loads. The refractive power of the lens was calculated for two different ages (29 and 45 years).

Results: The application of a pressure to the posterior lens surface did not yield an increase in refractive power change during accommodation. Rather a decrease in accommodation related refractive power was found.

Conclusions: Physiologically relevant refractive power changes are obtained by a simulation in accordance with the Helmholtz theory. A simulation in accordance with the Coleman theory does not yield physiological values of refractive power change.

Keywords: Accommodation; Computational model; Age dependence; Modeling; Presbyopia

1. Introduction

The Helmholtz accommodation theory is based on the assumption that the ciliary muscle diameter change during accommodation is responsible for the change in shape of the lens. During accommodation, the ciliary muscle contracts and thus the lens diameter is reduced. In this state, the zonula fibers can relax and the lens shape becomes more spherical. The curvature radii of the anterior and the posterior lens surface both decrease, leading to an increase in refractive power. In cycloplegia, the lens is flattened due to the radial tension of the zonula fibers, and hence the refractive power of the lens is diminished.

Coleman and Fish (2001) assume that the lens, together with the zonula fibers, form a diaphragm, which is held in a catenary shape due to the pressure difference between the aqueous and vitreous bodies of the lens. The diameter change of the ciliary body changes the span of this catenary. Thus, the anterior curvature radius is also changed. According to the Coleman theory, a continuous pressure difference acts on the lens, which was measured (Coleman & Fish, 2001) in primate eyes. The magnitude of this pressure difference is about 2.3 cm of water column, with major changes occurring during the initial seconds of the accommodation phase.

The aim of this paper is to compare the results of finite element model simulations of both the Coleman and the Helmholtz theories with ultrasonographic measurements on human subjects. In order to incorporate the influence of lens age on the refractive power change into

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this study, two models were created using material data for a 29-year-old lens, and a 45-year-old lens.

2. Material and methods

The lens and the zonula fibers were modeled in a two-dimensional axisymmetric model. The model's geometry and the material properties are similar to those of the model developed by Burd, Judge, and Cross (2002). These authors have created a finite element model for the evaluation of age related changes in accommodation behavior. This model is very precisely described by Burd et al. and was used as the basis of our analysis.

Our model consists of the lens nucleus, the lens cortex, the capsular bag and the zonula fibers (Fig. 1). Specific material properties were assigned to these lens components, as described below. The cortex and the nucleus of the lens are modeled by bilinear axisymmetric elements. The capsular bag and the zonular fibers are modeled by linear axisymmetric shell elements. The model was created in the accommodated state, since the zonula fibers and the lens are assumed to be stress-free in the accommodated state. Pre-loads that may exist on the lens and the zonula fibers are neglected. The cycloplegic state is enforced by boundary conditions at the zonula fibers as described below.

The outer contour of the lens in our model is constructed from least square polynomials (Table 1, from Burd et al., 2002). These polynomials are a mathematical approximation of the lens contours as measured by Brown (1973). The capsular bag has a variable thickness which is also given by least square polynomials (Burd et al., 2002) based on measurements by Fisher and Pettet (1972). All material properties of the model assume linear elasticity and isotropy. Furthermore, any time dependence of the material properties, namely viscoelasticity, or other nonlinear influences, such as a change in elastic moduli due to mechanical stress, was disregarded.

The material properties are based on measurements by Fisher (1971). According to Fisher's results, the lens cortex is stiffer than the lens nucleus, a result that is questioned by clinicians. The influence of the material properties on the refractive power change has been studied in the past (Martin, Stachs, Guthoff, & Schmitz, 2003). In this paper, a parametric investigation shows that a physiological refractive power can only be achieved if the lens cortex and the lens nucleus are modeled with different material properties. However, the influence of the viscoelasticity of the lens materials remains to be studied. Essential parameters of the undeformed finite element models as well as the material properties are given in Table 2.

At the zonula fiber end point, the radial displacement is prescribed according to measurements by Strenk et al. (1999): $d\,[\mathrm{mm}] = 0.5129 - 0.00525 \cdot \mathrm{age}\,[\mathrm{years}]$.

The axial displacement at the zonula fiber end point is set to zero. All radial displacements are constrained along the rotation axis of the lens.

This model without further loads simulates precisely the Helmholtz theory. To model the Coleman theory, in addition to the displacements of the zonula fiber end points, a static pressure on the posterior lens surface was applied. It must be pointed out, however, that the magnitude and location of this pressure are not entirely well defined. Coleman uses a mechanical model of the lens in which this pressure is applied as a weight distribution acting on the interior of the balloon representing the lens.

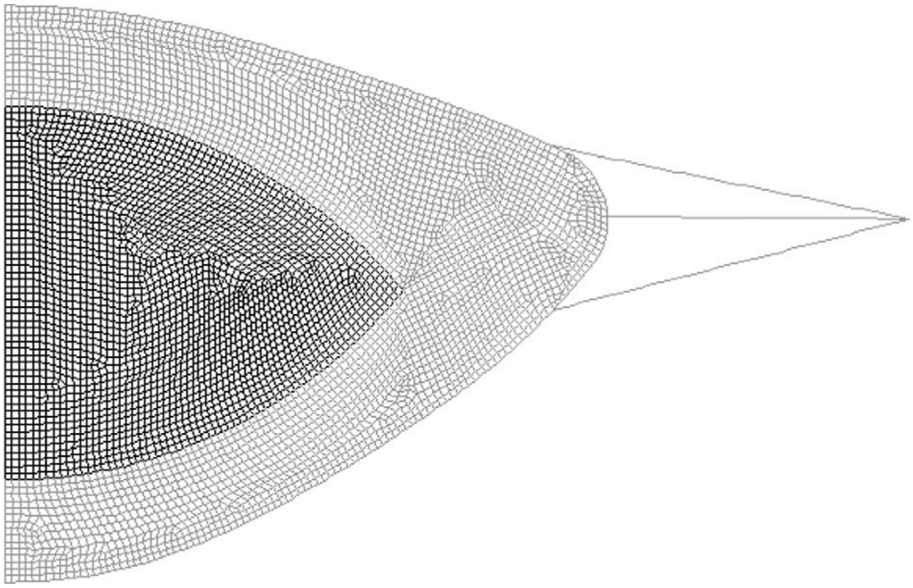

Fig. 1. Undeformed finite element model of the 29-year-old lens.

Table 1
Polynomial coefficients for the description of the undeformed anterior and posterior lens geometry according to $y = aR^5 + bR^4 + cR^3 + dR^2 + f$ (y-coordinate towards anterior along the lens axis, R-coordinate in radial direction of the lens)

	a	*b*	*c*	*d*	*f*
Anterior (29 years)	−0.00153004454939	0.01191111565048	−0.02032562095557	−0. 07692307692308	2.04
Posterior (29 years)	0.00375558685672	−0.03036516318799	0.06955483582257	0. 09433962264151	−2.09
Anterior (45 years)	−0.00026524088453	0.004498662869630	−0.01657250977510	−0. 06578947368421	2.42
Posterior (45 years)	0.00266482873720	−0.02666997217562	0.08467905191557	0. 06172839506173	−2.42

Copied from Burd et al. (2002). All dimensions are in mm. The polynomial coefficients *a*–*d* are given with a high number of decimal places to avoid truncation errors.

Table 2
Geometric parameters and material properties of the undeformed lens model

	29 years	45 years	Reference
Equatorial lens diameter [mm]	8.63	8.90	Burd et al. (2002)
Lens thickness [mm]	4.13	4.84	Burd et al. (2002)
Anterior curvature radius [mm]	5.85	6.66	Burd et al. (2002)
Posterior curvature radius [mm]	3.90	4.62	Burd et al. (2002)
Capsule Young's modulus [N/mm^2]	1.27	1.45	Krag et al. (1996)
Young's moduls lens nucleus [N/mm^2]	0.55E−3	0.10E−2	Fisher (1971)
Young's moduls lens cortex [N/mm^2]	3.42E−3	3.98E−3	Fisher (1971)
anterior zonula stiffness [N/mm]	66E−3	66E−3	Burd et al. (2002)
central zonula stiffness [N/mm]	11E−3	11E−3	Burd et al. (2002)
posterior zonula stiffness [N/mm]	33E−3	33E−3	Burd et al. (2002)
Capsule Poisson's ratio	0.47	0.47	Fisher (1969)
Cortex and nucleus Poisson's ratio	0.49	0.49	Burd et al. (2002)

For a first approach, a pressure difference was applied uniformly over the posterior lens surface. This condition is referred to as case 1 (Fig. 2). The magnitude of the pressure difference (about 225 Pa) was taken from the literature (Coleman & Fish, 2001). To verify the uniform pressure assumption, a second model was analyzed in which the pressure was reduced in steps at the zonula fiber attachments (case 2) (Fig. 3). In this version, the pressure was reduced to 2/3 of the full pressure in the lens area inside the posterior zonula fiber attachment and to 1/3 within the lens segment before the middle zonula fiber attachment.

Coleman further states that ciliary muscle contraction changes the pressure gradient during the initial phase of accommodation. These transient changes have been neglected in this model. There is too little informa-

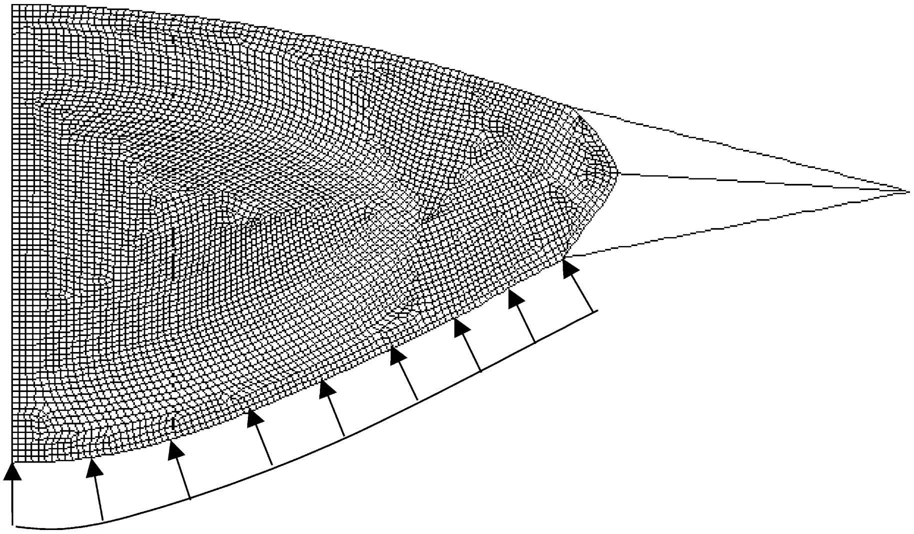

Fig. 2. Finite element model of the 29-year-old lens in the accommodated state with a constant pressure load (case 1).

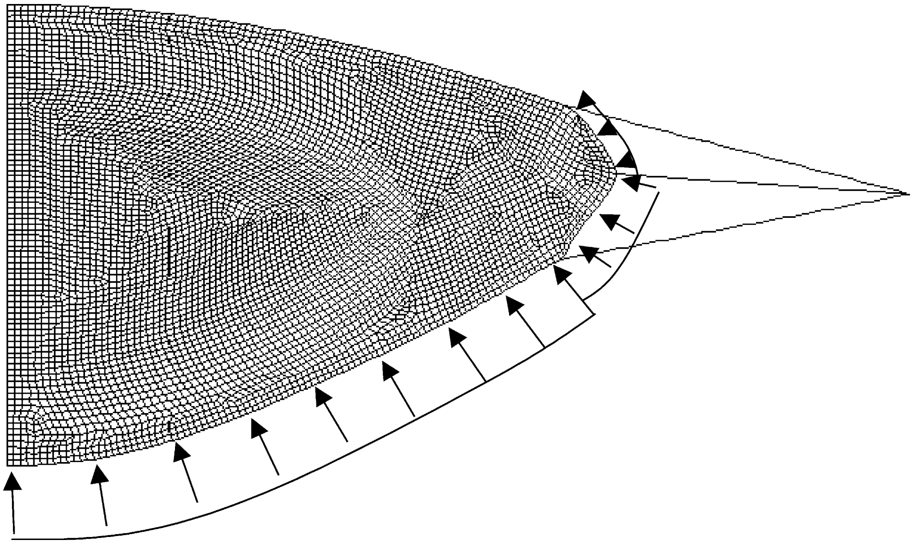

Fig. 3. Finite element model of the 29-year-old lens in the accommodated state with a stepwise reduced pressure load (case 2).

tion about the viscoelastic material properties of the various human lens components to include this refinement into the model.

The model also neglects other elements of the human eye, such as the iris, which may affect the axial shift of the lens during the accommodation. However, earlier work (Drexler, Baumgartner, Findl, Hitzenberger, & Fercher, 1997) has shown that the axial shift of the lens during accommodation is very small. Consequently our model does not consider refractive power changes that may be due to an axial shift during accommodation.

A static finite element analysis was performed using the finite element software package ABAQUS 6.2-7 (ABAQUS Inc., Providence, RI, USA). The calculation was carried out nonlinearly considering large displacements.

The finite element analysis produces the deformed contours of the anterior and the posterior lens surfaces from which the refractive power can be calculated. For this purpose, a least square approximation of the anterior and the posterior lens contour by circular arcs was performed. This approximation was done in the central region of the lens with a radius of 0.8 mm.

The refractive power D was calculated from the anterior and the posterior curvature radii R_a and R_f as follows:

$$D = D_f + D_b - \frac{t_c}{n_l} D_f D_b$$

$$D_f = \frac{n_l - n_{aq}}{R_f}; \quad D_b = \frac{n_{aq} - n_l}{R_b}$$

D_f is the refractive part of the anterior lens surface; D_b is refractive part of the posterior lens surface; n_l is refractive index of the lens ($n_l = 1.42$); n_{aq} is refractive index of the aqueous ($n_{aq} = 1.336$); t_c is central lens thickness.

The lens and the aqueous were assumed to be optically homogeneous. Hence, the refractive indices of the lens and the aqueous were assumed to be constant. Age-related changes of these indices are not considered. This model deals only with the refractive power change due to deformation of the lens and the resulting curvature changes.

The simulation results were compared to data for the refractive power change of the complete eye from ultrasonographic measurements by Beers and van der Heijde (1996). Beers et al., give an accommodation amplitude range of the complete eye from 6 to 7 dpt (29-year-old human subject) and from 2 to 3 dpt (45-year-old human subject). In order to compare the refractive power change values from the isolated lens to those of the complete eye, the lens values are divided by a factor of 1.315 according to Burd et al. (2002).

3. Results

The models of the 29-year-old lens with applied pressure load in the accommodated state and in cycloplegia are displayed in Figs. 3 and 4, respectively. The pressure action causes a motion of the lens towards anterior. The posterior zonula fibers are unloaded and relaxed.

In Table 3, the calculated refractive power changes during cycloplegia, the resultant reaction force on the zonula fiber ends and the equatorial lens radius change are given. All refractive power results are changes relative to the accommodated state. In the case of the additional pressure load, the results represent the relative

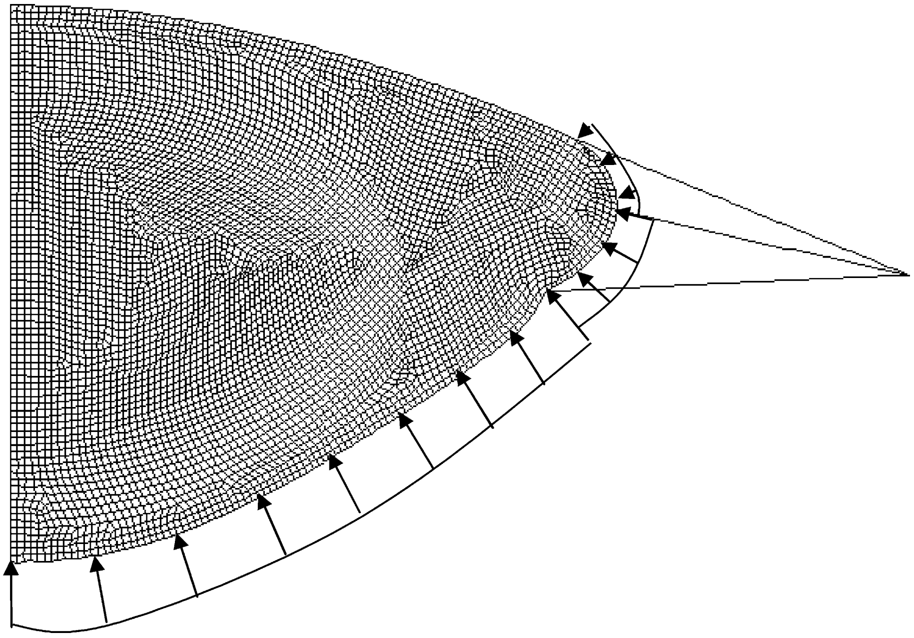

Fig. 4. Finite element model of the 29-year-old lens in cycloplegia with a stepwise reduced pressure load (case 2).

Table 3
Calculated refractive power changes, reaction force and radius changes between cycloplegia and accommodated state

	29 years without pressure	29 years with constant pressure (case 1)	29 years with stepwise reduced pressure (case 2)	45 years without pressure	45 years with constant pressure (case 1)	45 years with stepwise reduced pressure (case 2)
Anterior curvature radius in cycloplegia [mm]	11.51	7.42	7.33	8.91	7.24	6.93
Posterior curvature radius in cycloplegia [mm]	4.44	4.34	4.72	4.57	4.83	5.24
Refractive power change [dpt]	9.10	3.43	1.79	2.71	1.06	0.16
Res. reaction force in cycloplegia [N]	0.097	0.097	0.100	0.080	0.083	0.086
Radius change [mm]	0.286	0.297	0.295	0.222	0.225	0.223

changes to the accommodated state with applied additional pressure load.

In Table 4, the simulated accommodation results of the lens (also calculated for the eye) are presented in comparison with the ultrasonographic accommodation amplitude measurements of human eyes by Beers and van der Heijde (1996).

4. Discussion and conclusion

The model reproduces the magnitude and the tendency of refractive power change to diminish with age, as is shown in the comparison of the 29- and 45-year-old lens. The results without pressure load correspond to the Helmholtz theory and agree very well with the values given by Burd et al. (2002) and Duane (1912).

The model does neither consider the local differences nor the age-related changes of the refractive indices of the lens. The effect on the refraction power due to the axial displacement of the lens is not considered by the current model. The refractive power change of the lens is achieved only by the curvature change of the lens surface. Further studies should include other parts of the eye such as the iris and the vitreous body.

Table 4
Comparison of the simulated accommodation amplitude values with measurements from Beers and van der Heijde (1996) (All values given in dpt.)

Human eye [age]	29 years		45 years	
Theory	Helmholtz	Coleman	Helmholtz	Coleman
Accommodation amplitude of the lens (simulation results)	9.1	3.4/1.8	2.7	1.1/0.2
Accommodation amplitude of the eye (calculated from the simulation results)	6.9	2.6/1.4	2.0	0.8/0.1
Measured accommodation amplitude of the human eye (Beers & van der Heijde, 1996)	6–7		2–3	

The geometry and the material data of the model could certainly be refined by measurements using current methods, such as ultrasound biomicroscopy. Furthermore, the material behaviors of the model are very simplifying.

With the applied pressure load according to Coleman, the model calculations show a comparatively low refractive power change. Simulation according to the Helmholtz theory for both the 29- and the 45-year-old lenses result in accommodation amplitudes for the eyes that are of the same magnitude as measured by Beers and van der Heijde (1996) on human subjects.

The model shows that a pressure difference causes a decrease of the calculated refractive power change. This is due to the bending load generated by the assumed pressure difference that leads to an increase in curvature of the anterior lens surface, resulting in a higher refractive power of the lens in cycloplegia. Conversely, in the accommodated state, the curvature of the anterior lens surface decreases due to the applied pressure. The simulation, therefore, shows that the influence of the applied pressure load on the posterior lens surface (independent of its detailed shape in the lens periphery) is not essential for the accommodation process. Otherwise an increase of the refractive power change due to the applied pressure should be expected.

The simulation with its limitations and simplifying assumptions nevertheless demonstrates that the Helmholtz theory gives a more accurate description of human accommodation than the Coleman theory.

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